Bulk Evolution of Fabric and Stress Derived from Contact Migration Rules

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Outline

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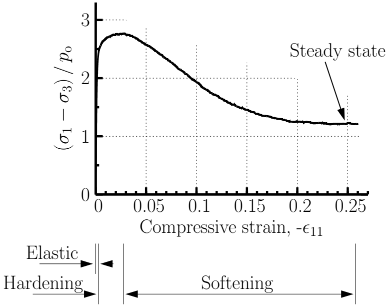
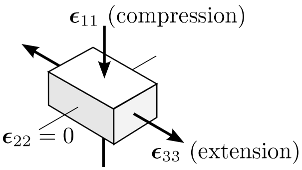
1. Introduction & Scope
   - Range of loading behavior
   - Granular fabric
   - The current study
2. Principles
3. Model & Calibration
4. Applications

Introduction — Range of behavior

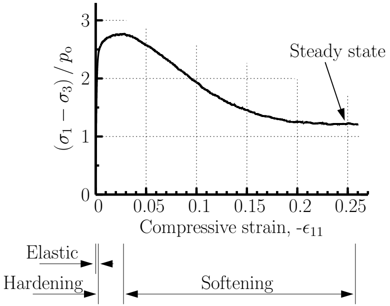
Behavior — plane strain biaxial compression:

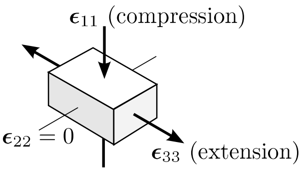




Introduction — Range of behavior

Behavior — plane strain biaxial compression:

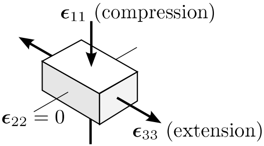




Intended range of the model: Deviatoric post-peak behavior

Introduction — Granular fabric

Fabric anisotropy after loading:



Plane strain biaxial loading

Introduction — Granular fabric

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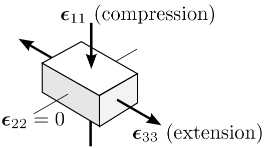
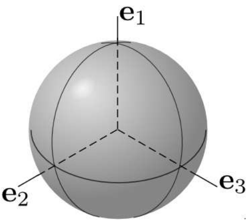


Plane strain biaxial loading



Unit sphere, Ω

Introduction — Granular fabric

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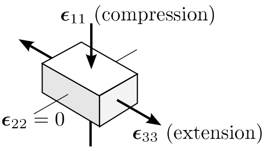

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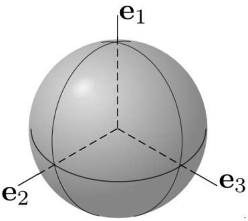
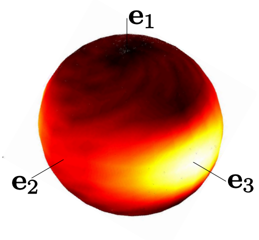


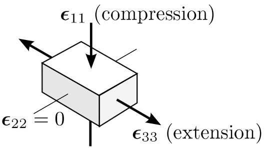
Unit sphere, Ω



Contact density

Introduction — Granular fabric

Fabric anisotropy after loading:



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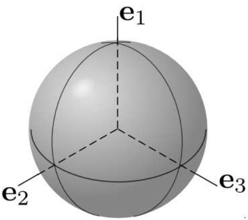

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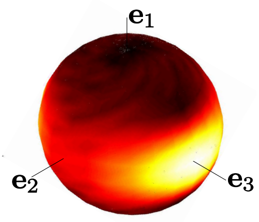


Contact density

Fabric anisotropy $\begin{cases} \text{contact density} \\ \text{contact force density} \end{cases}$

Introduction — Current study

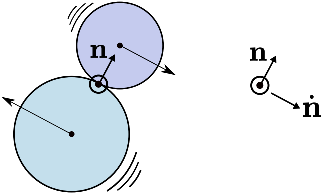
Questions:

- How/why does granular fabric evolve?
- Relationship between the changing fabric and the bulk stress?

Outline

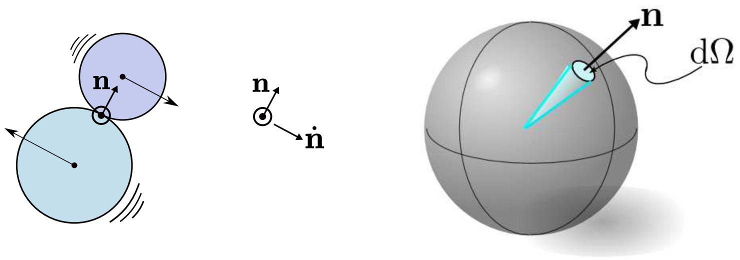
Principles — Contact movement

Contact movement — Movement of a unit normal vector:



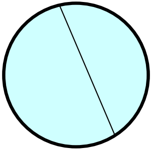
Principles — Contact movement

Contact movement — Movement of a unit normal vector:



Principles — Contact movement

Contact movements through $d\Omega$:



Principles — Contact movement

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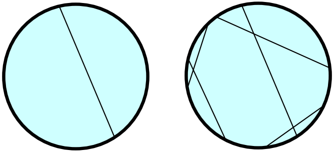

Principles — Contact movement

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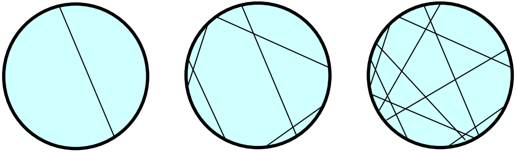

Principles — Contact movement

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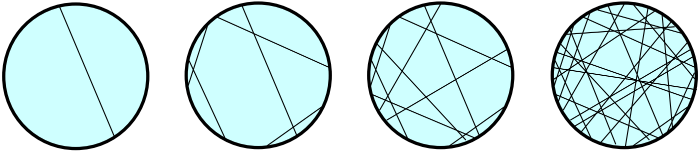
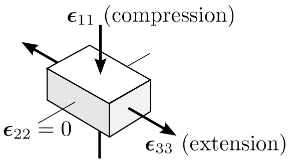


Principles — Contact movement

A "prevailing migration" of contacts ?



Principles — Contact movement

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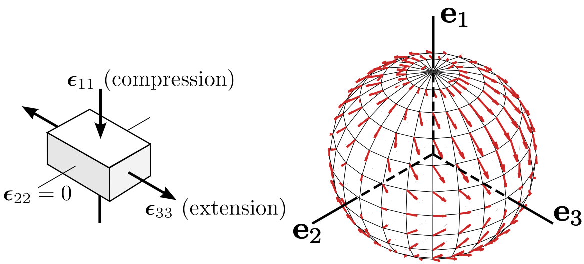


Prevailing migration rate $\dot{\mathbf{n}}(\mathbf{n})$

Principles — Contact movements

Prevailing contact movement — DEM results:

Bulk deformation

$$\dot{\mathbf{n}}(\mathbf{n}) \approx \alpha \underbrace{(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})}_{} \cdot \underbrace{\mathbf{D} \cdot \mathbf{n}}_{}$$

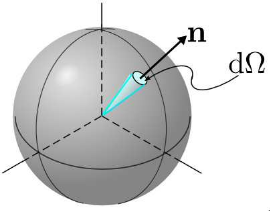
Movement vector

Projection tensor

$\alpha = 1.5$

Principles — Fabric evolution

Fabric densities:



Unit sphere

Principles — Fabric evolution

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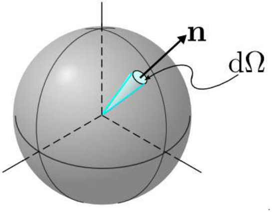
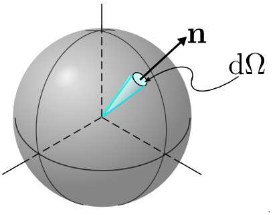


Unit sphere

$\widehat{g}(\mathbf{n})$	contact density
$\widehat{f^n}(\mathbf{n})$	normal force density
$\widehat{f^t}(\mathbf{n})$	tangential force density

Principles — Fabric evolution

Fabric densities:



Unit sphere

$\widehat{g}(\mathbf{n})$	contact density
$\widehat{f^n}(\mathbf{n})$	normal force density
$\widehat{f^t}(\mathbf{n})$	tangential force density

Example: $\widehat{g}(\mathbf{n}) =$ number of contacts per *particle* per *area* of unit sphere

Principles — Fabric evolution

Fabric evolution:

$$\left.\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = \textbf{?}$$

Principles — Fabric evolution

Fabric evolution:

$$\left.\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = \left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{matl}} - \boldsymbol{\nabla} \cdot \left(\dot{\mathbf{n}}\, \widehat{g}(\mathbf{n})\right) + \left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{diffusion}}$$

1	2	3
Matl. rate	Divergence / convection	Diffusion

Principles — Fabric evolution

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$$\left.\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = \left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{matl}} - \boldsymbol{\nabla} \cdot (\dot{\mathbf{n}}\, \widehat{g}(\mathbf{n})) + \left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{diffusion}}$$

1	2	3
Matl. rate	Divergence / convection	Diffusion

Ma and Zhang (2006)

Principles — Fabric evolution

Material (source) density rate:

$$\left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t} \right)_{\text{matl}} \longrightarrow$$

$$\boxed{1}$$

Principles — Fabric evolution

Divergence & convection rates:

$$\left.\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = \cdots$$

$$\left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{matl}} - (\nabla \cdot \dot{\mathbf{n}})\,\widehat{g}(\mathbf{n}) - \dot{\mathbf{n}} \cdot (\nabla \widehat{g}(\mathbf{n})) + \left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{diffusion}}$$

1	2a	2b	3
Matl. rate	Divergence	Convection	Diffusion

Principles — Fabric evolution

Contact divergence rate:

$$(\boldsymbol{\nabla} \cdot \dot{\mathbf{n}})\,\widehat{g}(\mathbf{n}) \qquad \longrightarrow$$

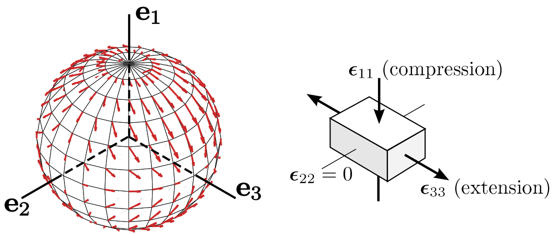
2a

Principles — Fabric evolution

Contact divergence rate:

$$(\boldsymbol{\nabla} \cdot \dot{\mathbf{n}})\, \widehat{g}(\mathbf{n}) \longrightarrow$$

2a



Principles — Fabric evolution

Contact convection rate:

$$\dot{\mathbf{n}} \cdot \left(\nabla \widehat{g}(\mathbf{n})\right) \quad \longrightarrow$$

$\boxed{\text{2b}}$

Principles — Fabric evolution

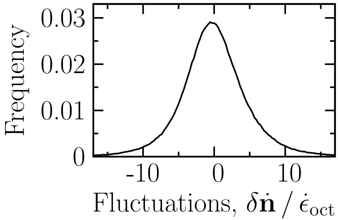
Contact convection rate:

$$\dot{\mathbf{n}} \cdot \left(\nabla \widehat{g}(\mathbf{n}) \right) \longrightarrow$$

2b

Principles — Fabric evolution

Contact diffusion due to velocity fluctuations — DEM results:



Principles — Fabric evolution

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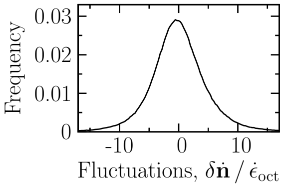

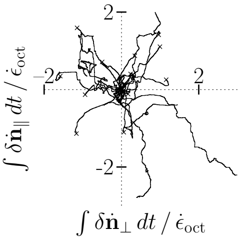

Principles — Fabric evolution

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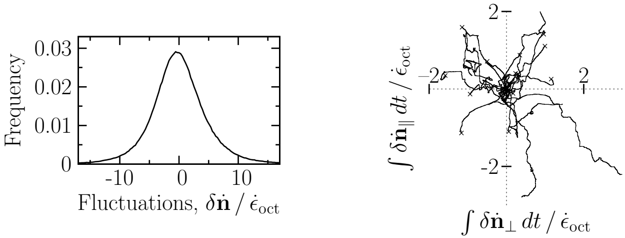

$$\left(\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right)_{\text{diffusion}} = D_g\, \nabla^2 \widehat{g}(\mathbf{n})\, \dot{\epsilon}\,, \qquad D_g = 0.03$$

3

Outline

1. Introduction & Scope
2. Principles
3. Model & Calibration
   - Force material rates
4. Applications

Calibration — force rates

Evolution of normal force density:

$$\left.\frac{\partial \widehat{f^{\mathrm{n}}}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = \left(\frac{\partial \widehat{f^{\mathrm{n}}}(\mathbf{n})}{\partial t}\right)_{\mathrm{matl}} - \boldsymbol{\nabla} \cdot \left(\dot{\mathbf{n}}\, \widehat{f^{\mathrm{n}}}(\mathbf{n})\right) + \left(\frac{\partial \widehat{f^{\mathrm{n}}}(\mathbf{n})}{\partial t}\right)_{\mathrm{diffusion}}$$

1	2	3
Matl. rate	Divergence / convection	Diffusion

Calibration — force rates

Normal force material rate — DEM results:

$$\left(\frac{\partial \widehat{f^{\mathrm{n}}}}{\partial t} \right)_{\mathrm{matl}} \approx \beta \quad k^{\mathrm{n}} \quad \underbrace{\mathbf{n} \left(\mathbf{D} \cdot \mathbf{n} \right)}$$

Bulk deformation ($\mathbf{D}$)

Mean-field rate ($\mathbf{n} \left(\mathbf{D} \cdot \mathbf{n} \right)$)

$$k^{\mathrm{n}} = \left[\frac{3 G^2 \overline{\ell}^4}{(1-\nu)^2} \widehat{f^{\mathrm{n}}}(\mathbf{n}) \left(\widehat{g}(\mathbf{n}) \right)^2 \right]^{1/3}$$

Hertz stiffness

$$\beta \approx 0.0024$$

Outline

1 Introduction & Scope

2 Principles

3 Model & Calibration

4 Applications
- Possible applications
- Intermediate principal stress

Possible applications:

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1) Effect of the intermediate principal stress
2) Non-coaxial stress & strain increments
3) Effect of spatial gradients of strain
4) Softening / hardening rates

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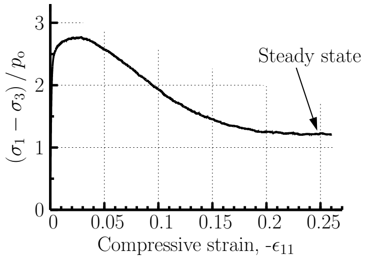
Application — Intermediate principal stress

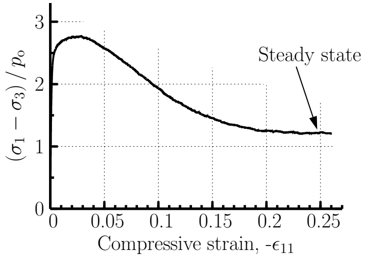
Effect of the intermediate principal stress — DEM results:



Plane strain biaxial compression

Application — Intermediate principal stress

Effect of the intermediate principal stress — DEM results:



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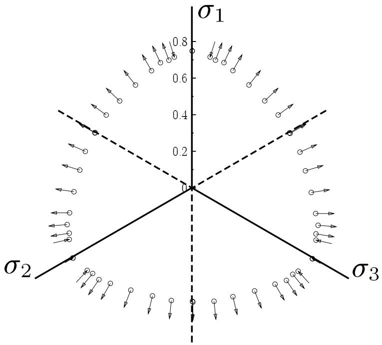


Application — Intermediate principal stress

At the steady state ⟶ stationary densities:

$$\left.\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0$$

$$\left.\frac{\partial \widehat{f^{\mathrm{n}}}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0$$

$$\left.\frac{\partial \widehat{f^{\mathrm{t}}}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0$$

Application — Intermediate principal stress

$$\left.\frac{\partial \widehat{f^{\mathrm{n}}}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0.0024 \left[\frac{3G^2 \bar{\ell}^4}{(1-\nu)^2} \widehat{f^{\mathrm{n}}}(\mathbf{n}) \left(\widehat{g}(\mathbf{n}) \right)^2 \right]^{1/3} \mathbf{n} \cdot (\mathbf{D} \cdot \mathbf{n})$$

$$+ 0.68\, \widehat{f^{\mathrm{n}}}(\mathbf{n}) \frac{\left| 1.5\, (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}) \cdot (\mathbf{D} \cdot \mathbf{n}) \right|^2}{\dot{\epsilon}}$$

$$+ (3 \times 1.5)\, \mathbf{n} \cdot (\mathbf{D} \cdot \mathbf{n})\, \widehat{f^{\mathrm{n}}}(\mathbf{n})$$

$$- 1.5\, (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}) \cdot (\mathbf{D} \cdot \mathbf{n}) \cdot \left(\boldsymbol{\nabla} \widehat{f^{\mathrm{n}}}(\mathbf{n}) \right)$$

$$+ 0.03 \left(\nabla^2 \widehat{f^{\mathrm{n}}}(\mathbf{n}) \right) \dot{\epsilon}$$

$$= 0$$

Application — Intermediate principal stress

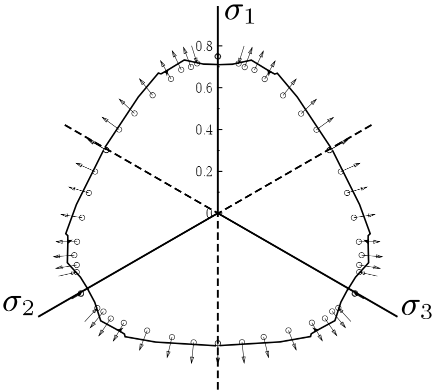
At the steady state $\longrightarrow$ stationary densities:

$$\left.\frac{\partial \widehat{g}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0\,, \qquad \left.\frac{\partial \widehat{f^{\mathrm{n}}}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0\,, \qquad \left.\frac{\partial \widehat{f^{\mathrm{t}}}(\mathbf{n})}{\partial t}\right|_{\mathbf{n}} = 0$$

- Coupled non-linear PDEs on the 2D unit sphere
- Solution depends on the “data” D: the bulk deformation
- Solve for the density distributions $\widehat{g}(\mathbf{n})$, $\widehat{f^{\mathrm{n}}}(\mathbf{n})$, and $\widehat{f^{\mathrm{t}}}(\mathbf{n})$
- Use $\widehat{f^{\mathrm{n}}}(\mathbf{n})$ and $\widehat{f^{\mathrm{t}}}(\mathbf{n})$ to find the corresponding stress $\boldsymbol{\sigma}$

Application — Intermediate principal stress

Solution of PDEs:



Conclusions

Conclusions:

- A model for fabric evolution
- Based on transport of contact and force densities
- Calibrated with DEM
- Reasonable prediction of effect of intermediate principal stress

Questions?