Strain Gradient Effects in Granular Materials and Their Relation to Shear Bands

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Symposium on Multiscale Aspects of Stability in Granular Media
In Memory of Ioannis Vardoulakis

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Los Angeles, California

ENGINEERING MECHANICS INSTITUTE

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Introduction — Ioannis Vardoulakis Symposium

Continuum models with inherent length scale:

1. Cosserat / micropolar continua
   - Muehlhaus H.-B. and Vardoulakis I. (1987). The thickness of shear bands in granular materials. Geotechnique, 37, 271-283.
   - Vardoulakis I. (1989). Shear banding and liquefaction in granular materials on the basis of a Cosserat continuum theory. Ingenieur Archiv, 59, 106-113.
2. Strain gradient-dependent material models
   - Vardoulakis I. and Aifantis E. (1989). Gradient dependent dilatancy rule and its implications on shear banding and liquefaction. Ingenieur Archiv, 59, 197-208.
   - Vardoulakis I. and E. Aifantis (1994). On the role of microstructure in the behavior of soils: Effects of higher order gradients & internal inertia. Mechanics of Materials, 18, 151-158.
3. Non-local material models
   - Vardoulakis, I. and Aifantis E. (1991). A gradient flow theory of plasticity for granular materials. Acta Mechanica, 87, 197-217.

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   $\boldsymbol{\tau} = f\,(\epsilon, \dot{\epsilon})$ Simple material

   $\boldsymbol{\tau} = f\,(\epsilon, \dot{\epsilon},\ \nabla\epsilon, \nabla(\nabla\epsilon), \ldots)$ Gradient-dependent material
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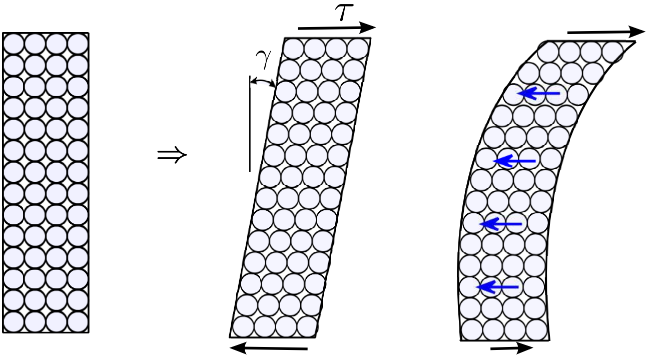
   Does stress really depend upon the spatial gradients of strain?
3. Non-local material models

Outline:

1. Introduction — gradient-dependent materials
2. DEM simulations to explore the dependence of stress on the spatial gradients of strain
3. Relation of gradient-dependent behavior to shear bands

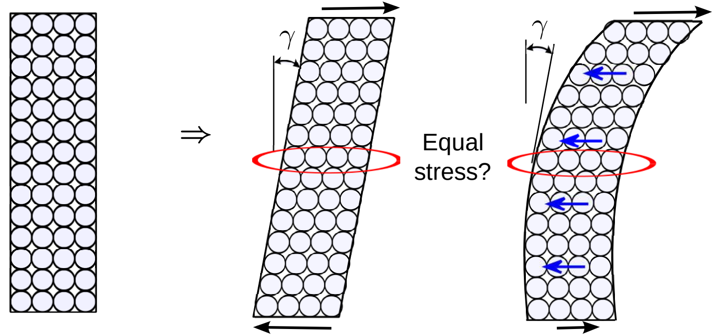
Gradient-dependent material

Does stress depend upon the spatial gradient of strain?

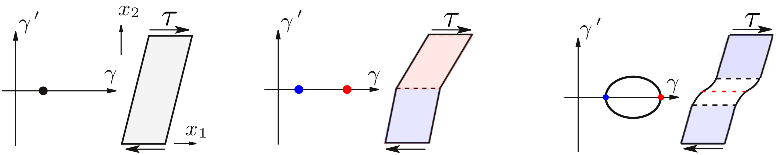


Gradient-dependent material

Does stress depend upon the spatial gradient of strain?

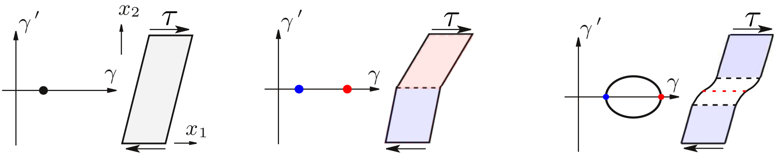


Phase-plane representation



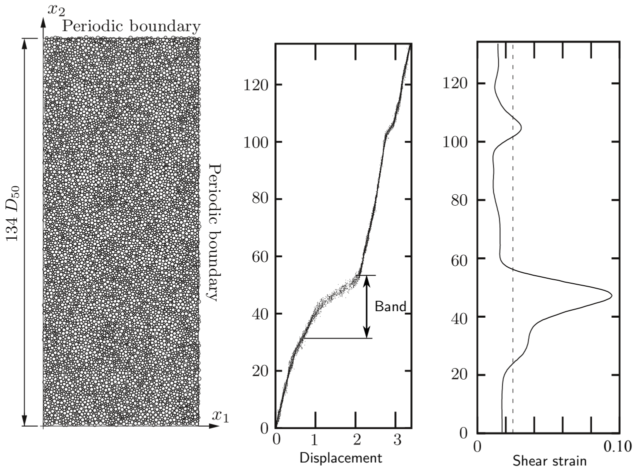
$$\tau = \mathcal{H} : \gamma$$

Phase-plane representation

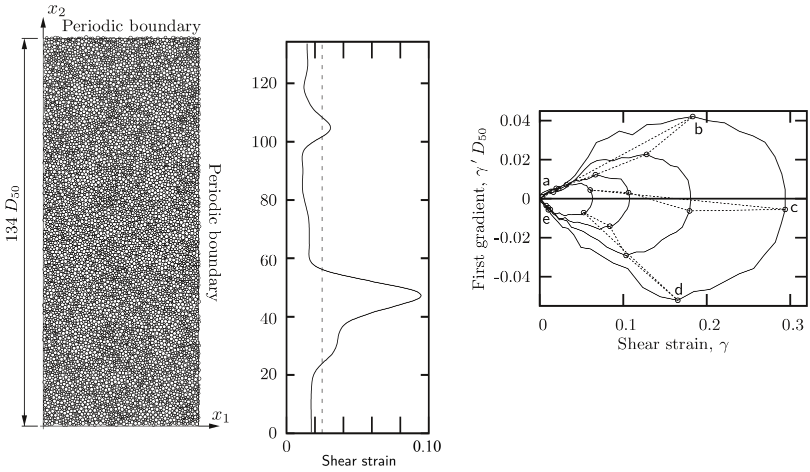


$$\tau = \mathcal{H} : \gamma \qquad\qquad \tau = \boldsymbol{f}(\gamma, \nabla\gamma, \nabla^2\gamma, \ldots)$$

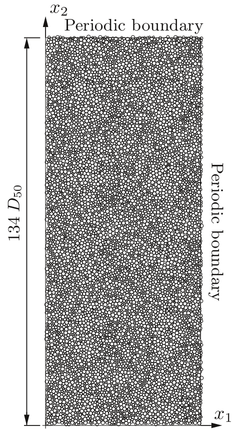
Shear bands in DEM simulation — free deformation

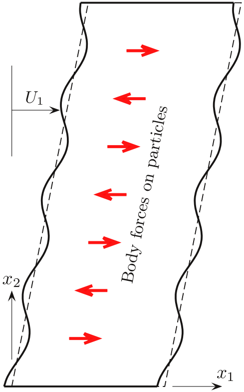


Shear bands in DEM simulation — free deformation

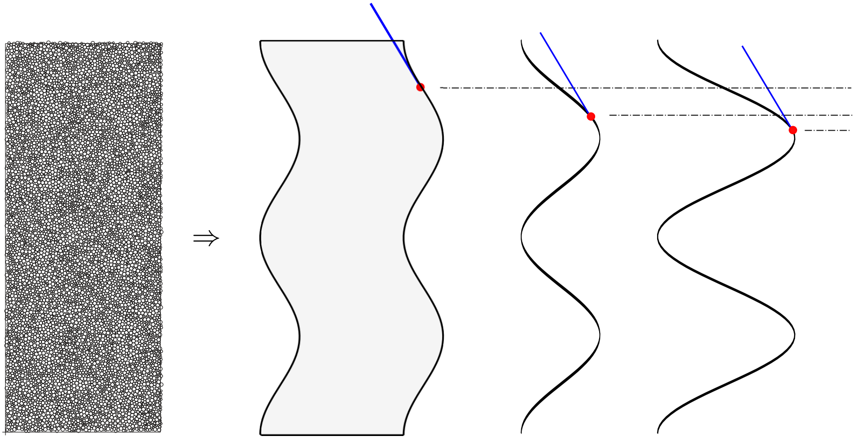


Constrained deformation using body forces



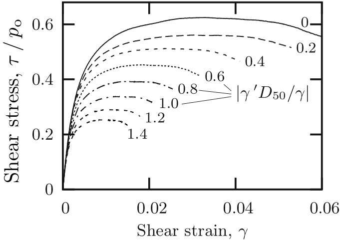


Effect of the first strain gradient



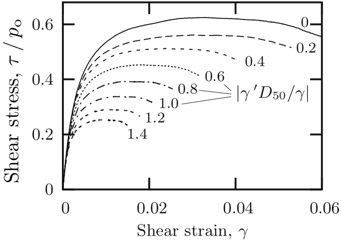
Same γ but different γ'

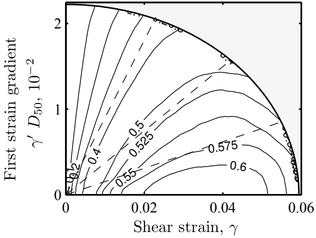
Effect of the first strain gradient



1. An increasing first gradient, γ', has a softening effect.
2. Circuits in $\gamma - \gamma'$ phase plane

Effect of the first strain gradient





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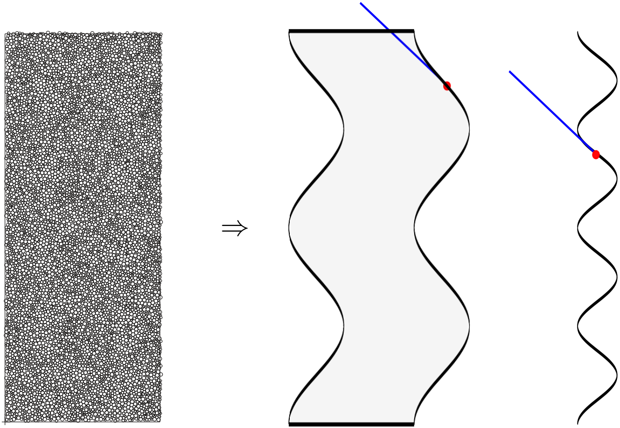
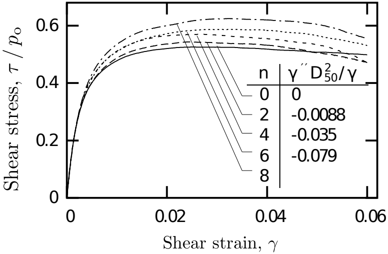
Effect of the second strain gradient



Same γ but different γ''

Effect of the second strain gradient



An increasing second gradient, γ'', has a hardening effect.

Shear bands

Shear bands and gradient-dependent behavior:

- Persistent bands develop near the peak stress state
- Shear strain γ is non-uniform within a shear band
- Shear stress depends upon γ, γ', and γ''
- In incremental form, $d\tau = f(d\gamma, d\gamma', d\gamma'')$
- Shear stress is constant within a shear band: $d\tau / dx_2 = 0$
- Can an incremental model explain the profile of strain within a shear band?

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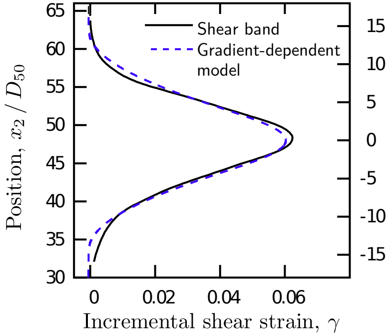
Incremental behavior

Incremental behavior near the peak stress state:

$$
\begin{aligned}
d\tau &= f(d\gamma, d\gamma', d\gamma'') \\
&= G\, d\gamma + B_1 \left| \frac{d\gamma}{dx_2} \right| + B_2 \frac{d^2\gamma}{dx_2^2} \\
\frac{d\tau}{dx_2} &= 0
\end{aligned}
$$

Use DEM simulations of constrained deformation to measure G, B_1, and B_2

Solution of the incremental model:

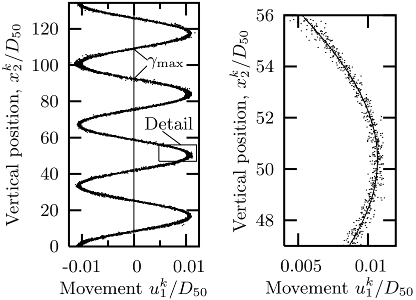


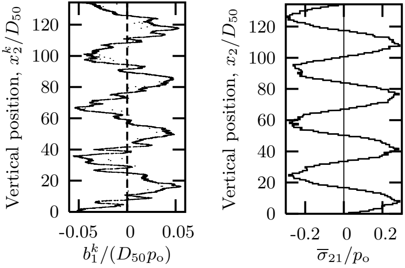
Summary

- DEM simulations can model the appearance and evolution of shear bands
- The effect of strain gradients can be measured with DEM simulations
  - The first gradient $\left|\frac{d\gamma}{dx}\right|$ has softening effect
  - The second gradient $-\frac{d^2\gamma}{dx^2}$ has a hardening effect
- A gradient-dependent incremental model can capture scale-dependent features of shear bands

Further Reading I

- M. R. Kuhn 2005.
  Are granular materials simple? An experimental study of strain gradient effects and localization.
  Mechanics of Materials, 37(5):607–627.
- M. R. Kuhn 2003.
  An experimental method for determining the effects of strain gradients in a granular material.
  Comm. Numer. Methods Engrg., 19(8):573–580.


Vertical position, x_2^k/D_{50}
120
100
80
60
40
20
0
γ_{max}
Detail
-0.01
0
0.01
Movement u_1^k/D_{50}
56
54
52
50
48
0.005
0.01
Movement u_1^k/D_{50}



(a) Body forces

(b) Shear stress