Rolling and Friction in Discrete Element Simulations

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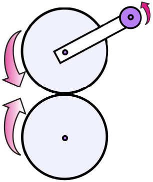
Classification of Rolling Resistance

Rolling Resistance — Two Categories	
Velocity-preserving (with *contact torque*)	Torque-preserving (with *contact creep*)
$M_{\text{input}} > M_{\text{output}}$ $\dot{\theta}_{\text{input}} = \dot{\theta}_{\text{output}}$	$M_{\text{input}} = M_{\text{output}}$ $\dot{\theta}_{\text{input}} > \dot{\theta}_{\text{output}}$



$$\dot{W}_{\text{input}} = M_{\text{input}} \cdot \dot{\theta}_{\text{input}}$$

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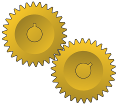
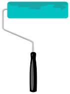
Gears Adhesive surfaces Visco-elastic materials Elasto-plastic materials	





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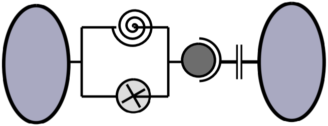

Iwashita & Oda (1998)

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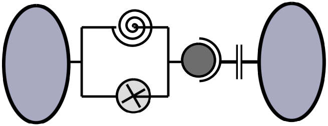
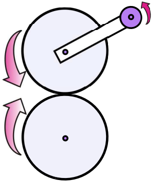
Gears Adhesive surfaces Visco-elastic materials Elasto-plastic materials	"Creep-friction"
	
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Creep-Friction: Definition

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- Rolling resistance and dissipation in the absence of a contact moment
- Contact mechanics are different than for Cattaneo-Mindlin sliding

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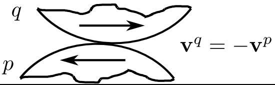
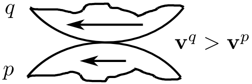
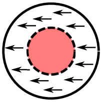
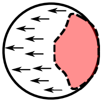
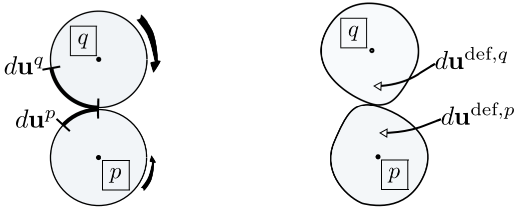
Creep-Friction vs. Cattaneo-Mindlin Friction

	Cattaneo-Mindlin Zero-Rolling Contact	Creep-Friction Rolling Contact
Used in DEM?	Widespread	Rare
Genealogy	Cattaneo (1938) Mindlin (1949) Deresiewicz (1953)	Carter(1926) Poritsky(1950) Kalker (1967)
Original problem	Machinery sliding (transient sliding)	Rail-wheel interaction (steady-state rolling)

Creep-Friction vs. Cattaneo-Mindlin Friction

	Cattaneo-Mindlin Zero-Rolling Contact	Creep-Friction Rolling Contact
Normal stress	Hertz	Hertz
Elastic-frictional?	Yes	Yes
Particle motions		
Slip & stick within contact area between spheres	Symmetric 	Non-symmetric 
Reference frame	Lagrangian	Eulerian

Eulerian kinematics of a moving contact



Rigid displacement

$$d\mathbf{u}^{\text{disp}} \equiv d\mathbf{u}^q - d\mathbf{u}^p$$

Particle deformation

$$d\mathbf{u}^{\text{def}} \equiv d\mathbf{u}^{q,\text{def}} - d\mathbf{u}^{p,\text{def}}$$

Rolling

$$d\mathbf{u}^{\text{roll}} \equiv \tfrac{1}{2}(d\mathbf{u}^q + d\mathbf{u}^p)$$

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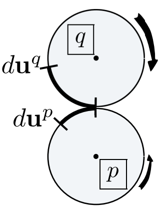

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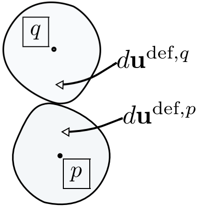

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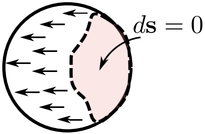

Contact area

Slip $d\mathbf{s}$ within the contact area:

$$d\mathbf{s} = d\mathbf{u}^{disp} - \left(\frac{\partial \mathbf{u}^q}{\partial \mathbf{x}} - \frac{\partial \mathbf{u}^p}{\partial \mathbf{x}}\right) \cdot d\mathbf{u}^{roll} - \frac{d\mathbf{u}^{def}}{\partial t} dt$$

Stick zone: $d\mathbf{s} = \mathbf{0}$

Transient vs. Steady-State Conditions

Cattaneo-Mindlin problem — pure sliding

"Transient" conditions

$$\mathbf{0} = d\mathbf{u}^{\text{disp}} - \left(\frac{\partial \mathbf{u}^q}{\partial \mathbf{x}} - \frac{\partial \mathbf{u}^p}{\partial \mathbf{x}} \right) \cdot d\mathbf{u}^{\text{roll}} - \frac{d\mathbf{u}^{\text{def}}}{\partial t} dt$$

Creep-friction problem — steady rolling

"Steady-state" creep-friction

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General problem for DEM — transient sliding & rolling

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Current situation

Cattaneo-Mindlin sliding:

- Solved!

Creep-friction rolling:

- Steady-state rolling of two cylinders: solved, Carter (1926)
- Steady-state rolling of two spheres: no exact solution!

  Only analytical approximations of the form

$$\xi_{ss} \equiv \frac{v^{disp}}{v^{roll}} = \mathcal{F}_{ss}\left(G, \mu, a, \frac{|Q|}{\mu P}\right)$$

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Recent approximation of the transient problem

Dahlberg & Alfredsson (2009) approximation:

- Non-steady state (transient) rolling of cylinders
- Start with the general 2-D problem

$$\mathbf{0} = d\mathbf{u}^{\text{disp}} - \left(\frac{\partial \mathbf{u}^q}{\partial \mathbf{x}} - \frac{\partial \mathbf{u}^p}{\partial \mathbf{x}} \right) \cdot d\mathbf{u}^{\text{roll}} - \frac{d\mathbf{u}^{\text{def}}}{\partial t} dt$$

- Express in an approximate 1-D scalar form

$$du^{\text{disp}} = \mathcal{C}_{\text{C-M}}(dQ) + \mathcal{F}_{\text{ss}} \left(\frac{|Q|}{\mu P} \right) \frac{du^{\text{roll}}}{du^{\text{disp}}} du^{\text{disp}}$$

- Invert the Cattaneo-Mindlin contact compliance "$\mathcal{C}_{\text{C-M}}$" to find force increment:

$$dQ = \mathcal{C}_{\text{C-M}}^{-1} \left[du^{\text{disp}} \left(1 - \mathcal{F}_{\text{ss}} \left(\frac{|Q|}{\mu P} \right) \frac{du^{\text{roll}}}{du^{\text{disp}}} \right) \right]$$

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Transient rolling of spheres

3-D Transient rolling of spheres. Reduce to a 2-D form:

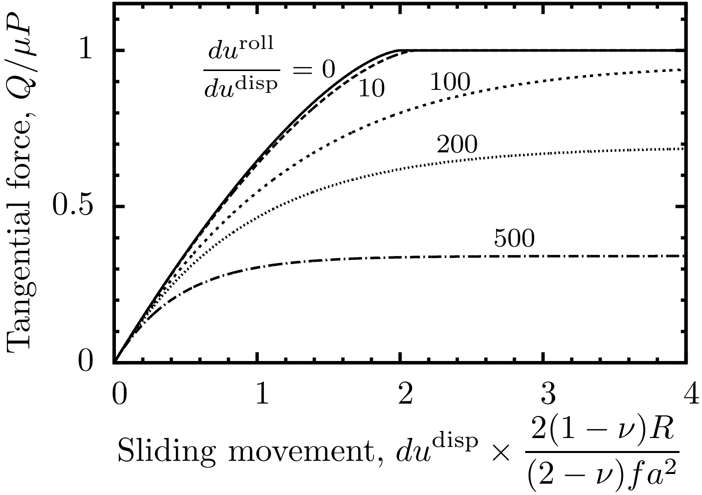
$$d\mathbf{Q} = \mathcal{C}_{\text{C-M}}^{-1}\left(\cancel{d\mathbf{u}^{\text{disp}}}\right) \;\Rightarrow\; \mathcal{C}_{\text{C-M}}^{-1}\left(d\mathbf{u}^{\text{disp}} - \mathcal{F}_{\text{ss}}\left(\frac{|\mathbf{Q}|}{\mu P}\right)\frac{\mathbf{Q}}{|\mathbf{Q}|}\left|d\mathbf{u}^{\text{roll}}\right|\right) \tag{1}$$

using Kalker's (2000) approximation of the steady-state creepage:

$$\mathcal{F}_{\text{ss}} = \frac{3\mu P}{Ga^2 C_{11}}\left[1 - \left(1 - \frac{|\mathbf{Q}|}{\mu P}\right)^{1/3}\right]$$

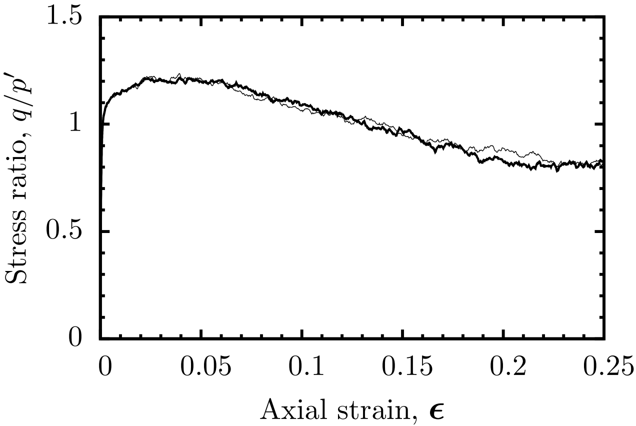
In a displacement-driven DEM code, compute the force increment $d\mathbf{Q}$ by using a standard Cattaneo-Mindlin algorithm for $\mathcal{C}_{\text{C-M}}^{-1}$. But use (1) instead of $d\mathbf{u}^{\text{disp}}$.

Effect of rolling upon tangential force



Effect of rolling in DEM simulation

Drained triaxial compression of 4096 spheres



Conclusions

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- Creep-friction: a type of rolling resistance without rotational "springs".
- A new approximate solution to the creep-friction problem for 3D transient rolling & sliding of spheres.
- Unless the rolling rate is large (relative to sliding rate), the effect of creep-friction is small.
- An example simulation with a dense packing of spheres reveals a minimal effect of creep-friction.

Questions?

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Rolling contact phenomena — linear elasticity.
Rolling Contact Phenomena, Jacobsen, B. & Kalker, J.J. (eds.), 1–84.

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Wear, 266(1-2):316–326.