1 Problem Summary

- **Regime of behavior**: Dense granular flow of 2-dimensional material (i.e., “steady-state” or “critical state” flow)
- **Phenomena of interest**: Probability distributions of the local void topology (void cell valence) and particle topology (coordination number)
- **Hypothesis**: these probability distributions can be estimated by applying a maximum entropy principle.

2 Background

Dense granular flow at the “critical state”
- Characterized by constant bulk characteristics: constant volume, constant mean stress, constant shear stress, constant fabric anisotropy, etc.
- Yet, at the micro-scale, change is rapid and pervasive.
- Our focus is on the continually changing contact topology.

Topology of two-dimensional flow

- **Bulk measures** of topology:
  - Mean coordination no. \( \langle l \rangle = n \)
  - Mean void valence \( \langle l \rangle = m \)

The particle graph is a planar graph that represents the contacts (edges), particles (vertices), voids (faces, polygons).

The particle graph continually changes:
- New contacts “split” existing polygons
- Lost contacts “merge” existing polygons

These changes of topology will completely alter the particle graph during small increments of deformation.

3 Solution

- **Objective**: theory-based probability distributions of the void valences and coordination numbers

- **Premise**: topology is maximally disordered during granular flow
- **Step 1**: Categorize the individual void polygons (i.e., develop a taxonomy of void type).

Any planar graph can be constructed from a “journal” of pairs \( (l_i, \Delta M_i) \):

Each pair describes an additional void that is appended to the previous planar graph:
- \( l_i \) = valence of the appended polygon
- \( \Delta M_i \) = no. of added edges to create the polygon

- **Step 2**: Discrete probability distribution by type:

  \[
  P_{l,\Delta M} = \frac{1}{Z(\lambda_1, \lambda_2)} \exp \left( -\lambda_1 l - \lambda_2 \Delta M \right)
  \]

  with a partition function \( Z(\lambda_1, \lambda_2) \) and Lagrange multipliers \( \lambda_1 \) and \( \lambda_2 \) that satisfy the mean coordination number and void valence.

- **Step 6**: use the duality principle to derive the corresponding probability distribution of coordination numbers.

4 Verification

Comparison of predicted probability distributions with discrete element (DEM) simulations: