1 Problem Summary

- Regime of behavior: Dense granular flow of 2-dimensional material (i.e., “steady-state” or “critical state” flow)
- Phenomena of interest: Probability distributions of contact quantities (contact forces, movements, & orientations)
- Hypothesis: these probability distributions can be estimated by applying a maximum entropy principle.

2 Background

Dense granular flow at the “critical state”:

- Characterized by constant bulk characteristics: constant volume, constant mean stress, constant shear stress, constant fabric anisotropy, etc.
- Yet, at the micro-scale, change is rapid and pervasive.
- The critical state exhibits a “convergent” character: materials with different initial densities and arrangements converge toward the same density and fabric.

Typical results

Bi-axial compression with constant mean stress: DEM simulations.

Note that stress deviatoric stress ratio, \(q/p\), converges toward a steady-state value.

3 Entropy Model

- Objective: theory-based probability distributions of the contact forces, movements, and orientations
- We investigate six contact quantities:

<table>
<thead>
<tr>
<th>Quantity Description</th>
<th>(f_n^k)</th>
<th>(f_t^k)</th>
<th>(\theta^k)</th>
<th>(\phi^k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive normal contact force</td>
<td>Tangential contact force</td>
<td>Orientation of contact normal vector (n^k)</td>
<td>Orientation of branch vector (l^k)</td>
<td></td>
</tr>
<tr>
<td>(n_k)</td>
<td>(f_n^k)</td>
<td>(\theta^k)</td>
<td>(\phi^k)</td>
<td></td>
</tr>
<tr>
<td>Relative angular shift of particles’ centers</td>
<td>Contact movement from particle rotations, (\dot{\theta} + \dot{\phi})</td>
<td></td>
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</tr>
</tbody>
</table>

- Rigid-frictional contact law

\[ f_n^k \in \mathbb{R}^+ \quad \text{and} \quad f_t^k \in \{-\mu f_n^k, \mu f_n^k\} \quad \Rightarrow \quad n_k - \phi_k = 0 \]
\[ |f_n^k| \quad \Rightarrow \quad n_k - \phi_k > 0 \]

- Probability density of the six contact quantities:

\[ p \left( f_n, f_t, \theta, \phi, n, \phi \right) \quad \text{or} \quad p(\cdots) \]

- Four constraints on the density:
  1. Mean stress = \(\mu_0\)
  2. Isochoric flow
  3. Frictional dissipation consistent with work \(\sigma \cdot D\)
  4. Fraction of sliding contacts, \(\eta = 11.2\%\)

- Disorder (Shannon entropy):

\[ H (p(\cdots)) = - \int \cdots \int p(\cdots) \ln (p(\cdots)) \]

- Solution of maximum entropy — Jaynes’ formalism — with Lagrange multipliers \(\lambda\), to enforce the four constraints:

\[ p(\cdots) = \frac{1}{Z(\cdots)} \exp \left( - \sum_{i=1}^{4} \lambda_i \Gamma_i(\cdots) \right) \]

4 Verification

Comparison of predicted distributions with discrete element (DEM) simulations:

- Anisotropy of contact force:

- Probability density of contact forces:

- Contribution of weak and strong contacts to deviator stress \(q\):