Entropy model for dense steady state flow of granular materials

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1 Problem Summary

- Regime of behavior: Dense granular flow of 2-dimensional material (i.e., "steady-state" or "critical state" flow)
- Phenomena of interest: Probability distributions of contact quantities (contact forces, movements, & orientations)
- Hypothesis: these probability distributions can be estimated by applying a maximum entropy principle.

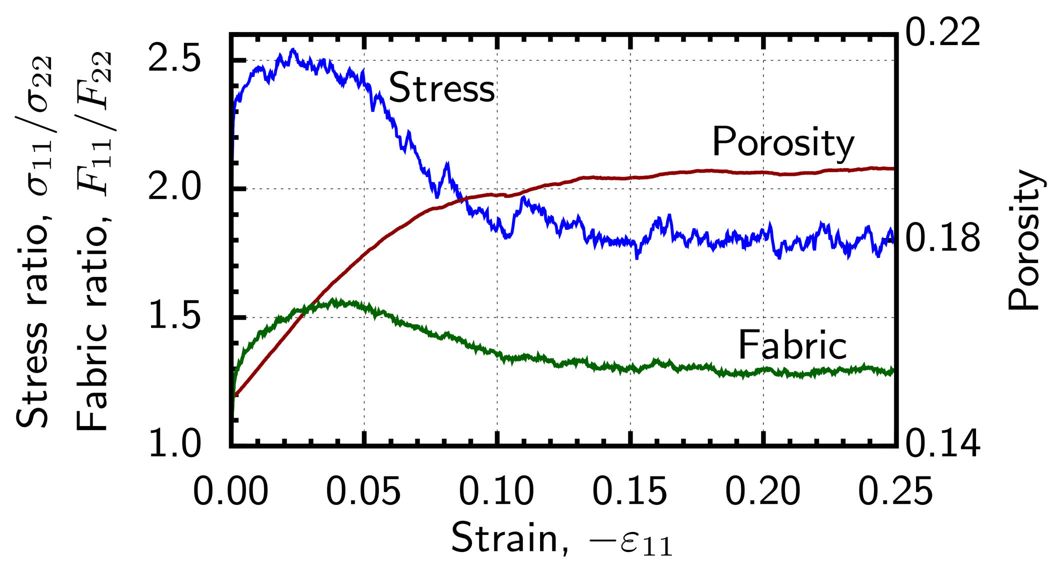
2 Background

Dense granular flow at the "critical state":

- Characterized by constant bulk characteristics: constant volume, constant mean stress, constant shear stress, constant fabric anisotropy, etc.
- Yet, at the micro-scale, change is rapid and pervasive.
- The critical state exhibits a "convergent" character: materials with different initial densities and arrangements converge toward the same density and fabric.

Typical results

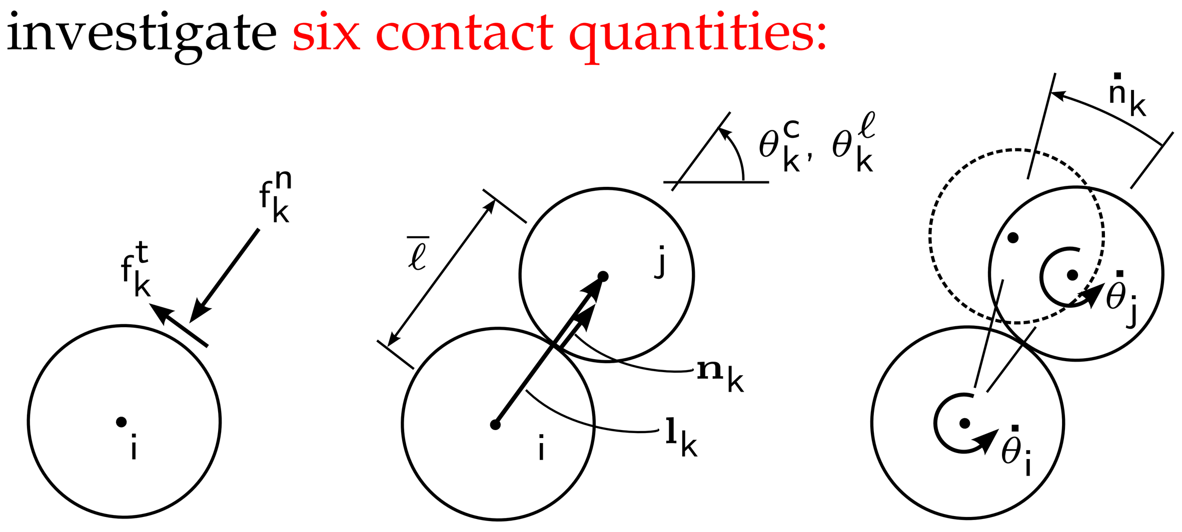
Biaxial compression with constant mean stress: DEM simulations.



Note that stress deviatoric stress ratio, q/p, converges toward a steady-state value.

3 Entropy Model

- Objective: theory-based probability distributions of the contact forces, movements, and orientations
- We investigate six contact quantities:



Quantity	Description
f^{n}	Compressive normal contact force
f^{t}	Tangential contact force
θ^{c}	Orientation of contact normal vector $\mathbf{n}^{c}$
θ^{ℓ}	Orientation of branch vector $\mathbf{l}^{\ell}$
$\dot{n}$	Relative angular shift of particles' centers
$\dot{\phi}$	Contact movement from particle rotations, $\frac{1}{2}(\dot{\theta}_i + \dot{\theta}_j)$

- Rigid-frictional contact law

$$f^{n}_{k} \in \mathbb{R}^{+} \quad \text{and} \quad f^{t}_{k} \in \begin{cases} -\mu f^{n}_{k} & \Leftarrow \quad \dot{n}_k - \dot{\phi}_k < 0 \\ (-\mu f^{n}_{k},\, \mu f^{n}_{k}) & \Rightarrow \quad \dot{n}_k - \dot{\phi}_k = 0 \\ \mu f^{n}_{k} & \Leftarrow \quad \dot{n}_k - \dot{\phi}_k > 0 \end{cases}$$

- Probability density of the six contact quantities:

$$p\left(f^{n}, f^{t}, \theta^{c}, \theta^{\ell} \dot{n}, \dot{\phi}\right) \quad \text{or} \quad p(\cdots)$$

- Four constraints on the density:
  1. Mean stress $= p_o$
  2. Isochoric flow
  3. Frictional dissipation consistent with work $\sigma : \mathbf{D}$
  4. Fraction of sliding contacts, $\eta = 11.2\%$
- Disorder (Shannon entropy):

$$H\left(p(\cdots)\right) = -\int \cdots \int_{\{f^{n}, f^{t}, \theta^{c}, \theta^{\ell}, \dot{n}, \dot{\phi}\}} p(\cdots) \ln\left(p(\cdots)\right)$$

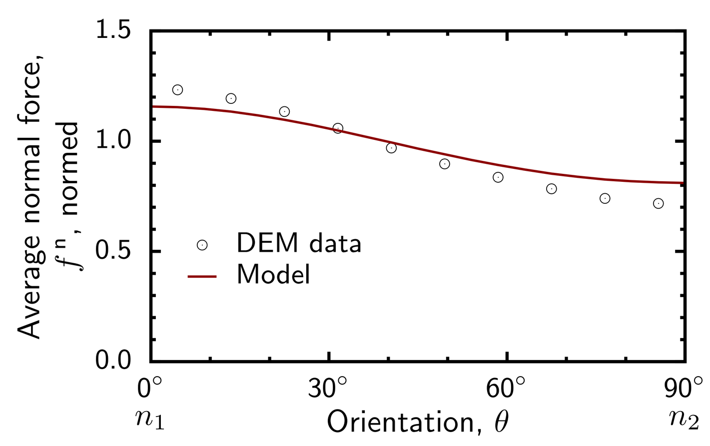
- Solution of maximum entropy — Jaynes' formalism — with Lagrange multipliers λ_i to enforce the four constraints:

$$p(\cdots) = \frac{1}{Z(\cdots)} \exp\left(-\sum_{i=1}^{4} \lambda_i \Gamma_i(\cdots)\right)$$

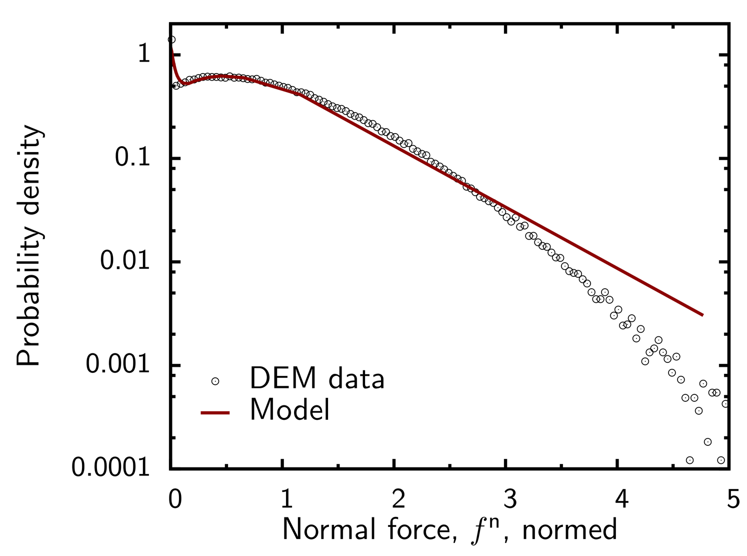
4 Verification

Comparison of predicted distributions with discrete element (DEM) simulations:

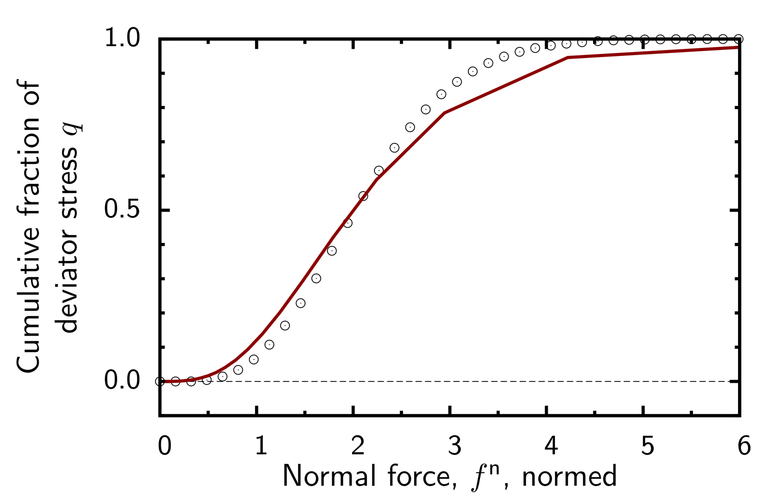
- Anisotropy of contact force:



- Probability density of contact forces:



- Contribution of weak and strong contacts to deviator stress q:



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