Continuum Models of Discrete Particle Systems with Particle Shape Considered

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Outline

1. Introduction

2. Quantifying high-gradient behavior
   - DEM “bending” experiments
   - Questions about granular behavior
   - Experiment results
Continuum vs. Discrete Frameworks

Continuum Small (but finite!) granular sub-region Continuum point

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Continuum-Discrete Granular Models with Bending
### Classical vs. Generalized Continua

Continuum representations...

<table>
<thead>
<tr>
<th>Classical continuum</th>
<th>Generalized continua</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1) Micro-polar</td>
</tr>
<tr>
<td></td>
<td>2) Strain gradient dependent</td>
</tr>
<tr>
<td></td>
<td>3) Non-local</td>
</tr>
</tbody>
</table>

#### Uniform deformation

\[
\frac{\partial \epsilon / \partial \mathbf{x}}{\epsilon} \ll \frac{1}{D}
\]

#### High-gradient deformation

\[
\frac{\partial \epsilon / \partial \mathbf{x}}{\epsilon} \approx \frac{1}{D}
\]
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Continuum-Discrete Granular Models with Bending
DEM “Bending” Experiments — 2D

“Uniform” deformation

“Bending” deformation

Strain:

Horiz. strain \( \epsilon_{11} \)

Vert. gradient \( \frac{d\epsilon_{11}}{dx_2} \)

Rotation:

Horiz. gradient \( \frac{d\theta}{dx_1} \)
Generalized Continuum Stresses

Continuum representation of stress . . .

$$\delta W_{\text{Internal}} = \sigma_{ji} \delta u_{i,j} + T_{ji} \delta \theta_{i,j} + \sigma_{jki} \delta u_{i,jk}$$
Continuum representation of stress...

\[ \delta W_{\text{Internal}} = \sigma_{ji} \delta u_{i,j} + T_{ji} \delta \theta_{i,j} + \sigma_{jki} \delta u_{i,jk} \]
Generalized Continuum Stresses

Continuum representation of stress . . .

\[ \delta W_{\text{Internal}} = \sigma_{ji} \delta u_{i,j} + T_{ji} \delta \theta_{i,j} + \sigma_{jki} \delta u_{i,jk} \]

\[ \sigma_{11} \quad \sigma_{12} \quad \sigma_{22} \]

\[ T_{13} \quad \sigma_{121} \]
DEM Simulations — 256 Particles — Circles or Ovals
**Bending Resistance in a Discrete Region**

**Boundary Moments:**

\[
T_{13}
\]

**Boundary Forces:**

\[
\sigma_{121}
\]

Bending Moment = \(T_{13} \quad (+) \quad \sigma_{121}\)
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Questions:

1. Are the boundary moments significant?
   \[ |T_{13}| > 0 \]

2. Are boundary forces consistent with classical beam theory?
   \[ \sigma_{121} \rightarrow E I \frac{d^2 u_1}{dx_1 dx_2} \]
Questions:

1. Are the boundary moments significant?
   \[ |T_{13}| > 0 ? \]

2. Are boundary forces consistent with classical beam theory?
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Introduction

Quantifying high-gradient behavior

DEM “bending” experiments

Questions about granular behavior

Experiment results
## Results Summary

Experiment results — incremental response:

<table>
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<tr>
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<th>Small strain</th>
<th>Large strain</th>
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<td>1) $</td>
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<td>Yes</td>
<td>No</td>
</tr>
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### Graph

- **Deviator stress** $\frac{(\sigma_{11} - \sigma_{22})}{p_0}$
- **Compressive strain** $-\varepsilon_{11}$

- **Small strain**
  - Circles
- **Large strain**
  - Ovals
  - Large strain

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Continuum-Discrete Granular Models with Bending
Results Summary

Boundary Moments:

\[ T_{13} \]

Bending Moment = \( T_{13} (\sigma_{121}) \)

Boundary Forces:

\[ \sigma_{121} \]
Experiment results — incremental response:

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![Graph of deviator stress vs. compressive strain](image)

- Large strain
- Small strain
- Circles
- Ovals

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Continuum-Discrete Granular Models with Bending
Results Details

DEM Simulation Results

*Dimensionless Bending Stiffnesses*

256 particles — 50 assemblies

*Large Strain*

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<tr>
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<th>Ovals</th>
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<tr>
<td>$</td>
<td>T_{13}</td>
<td>$ Boundary moments</td>
</tr>
<tr>
<td>$\sigma_{121}$ Boundary forces</td>
<td>0.60</td>
<td>1.16</td>
</tr>
<tr>
<td>$EIu''$ “Beam theory”</td>
<td>0.25</td>
<td>0.65</td>
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Continuum-Discrete Granular Models with Bending
Summary

- **DEM simulations** can probe the response of small regions to high strain gradients.
- **Cosserat-type torque stress** does not contribute to incremental bending stiffness.
- **A generalized stiffness** is associated with the 1st gradient of strain. Stiffness is larger for oval particles.
Are granular materials simple? An experimental study of strain gradient effects and localization.  

On virtual work and stress in granular media.  