Continuum Models of Discrete Particle Systems with Particle Shape Considered

Matthew R. Kuhn[1] Ching S. Chang[2]

[1]University of Portland

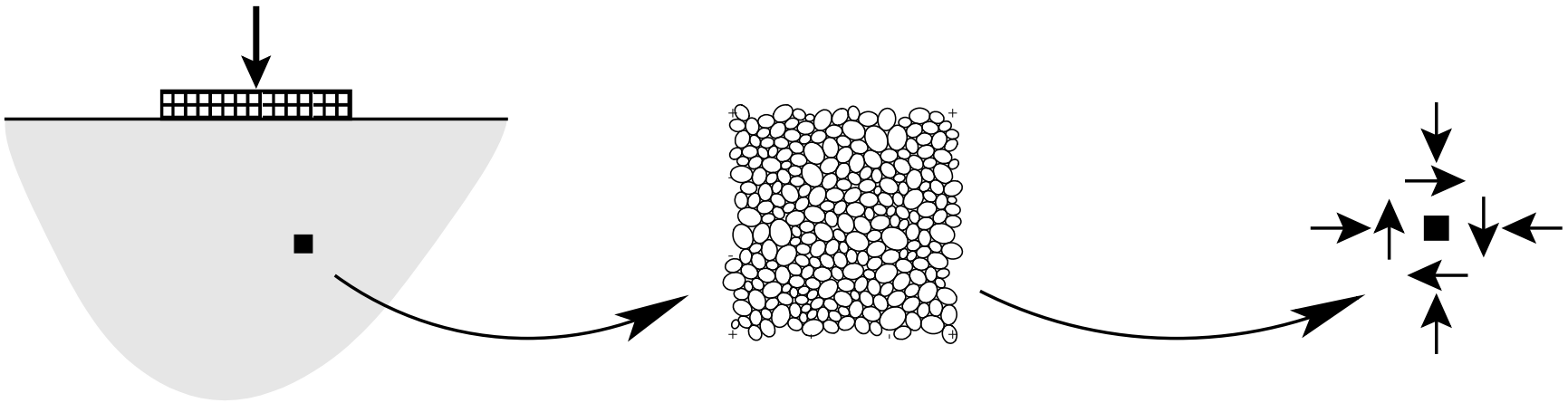
[2]University of Massachusetts

McMAT Mechanics and Materials Conference 2005

Outline

1. Introduction
2. Quantifying high-gradient behavior
   - DEM “bending” experiments
   - Questions about granular behavior
   - Experiment results

Continuum vs. Discrete Frameworks



Continuum

Small (but finite!) granular sub-region

Continuum point

Classical vs. Generalized Continua

Continuum representations. . .

Classical continuum or Generalized continua

1) Micro-polar
2) Strain gradient dependent
3) Non-local

Uniform deformation

$$\frac{\partial \epsilon / \partial \mathbf{x}}{\epsilon} \ll \frac{1}{D}$$

High-gradient deformation

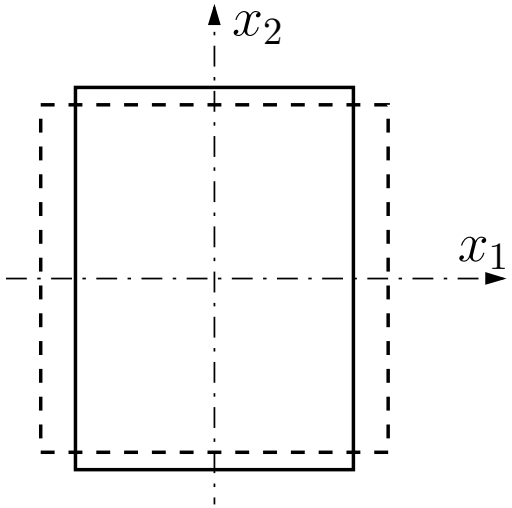
$$\frac{\partial \epsilon / \partial \mathbf{x}}{\epsilon} \approx \frac{1}{D}$$

Outline

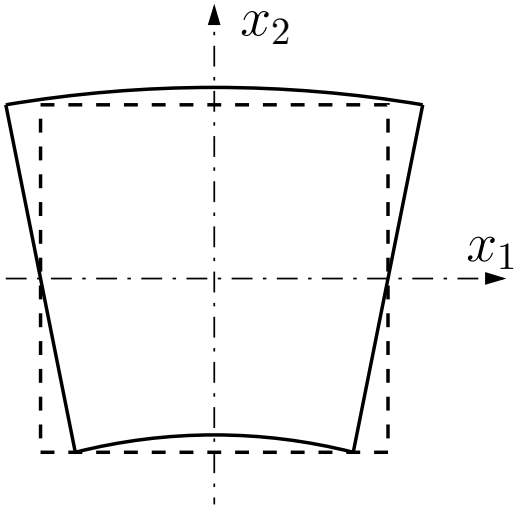
1 Introduction

2 Quantifying high-gradient behavior
- DEM “bending” experiments
- Questions about granular behavior
- Experiment results

DEM "Bending" Experiments — 2D



"Uniform" deformation



"Bending" deformation

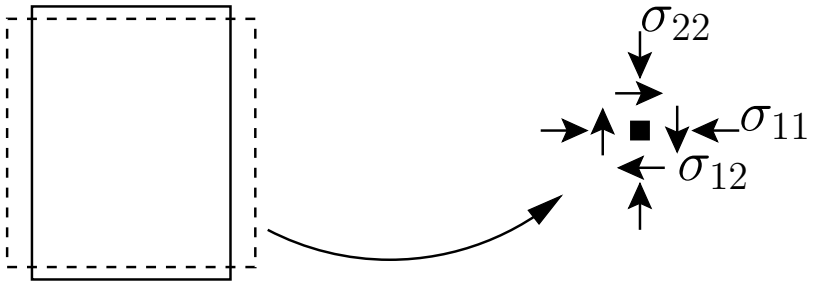
Strain: Horiz. strain $\rightarrow | \, | \leftarrow \; \epsilon_{11}$ Vert. gradient $\Updownarrow \frac{d\epsilon_{11}}{dx_2}$

Rotation: Rotation $\curvearrowleft | \, | \curvearrowright$ Horiz. gradient $\Leftrightarrow \frac{d\theta}{dx_1}$

Generalized Continuum Stresses

Continuum representation of stress . . .

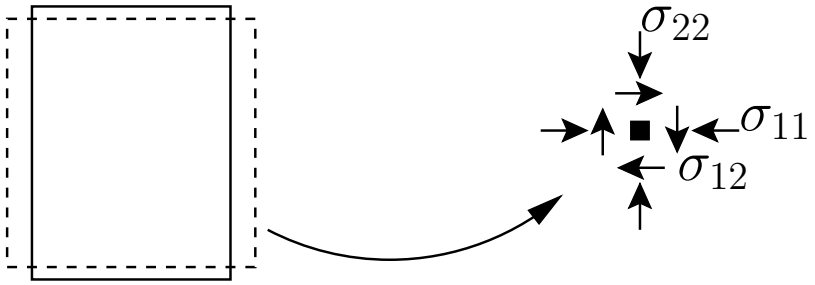
$$\delta W_{\text{Internal}} = \boldsymbol{\sigma}_{ji}\,\delta u_{i,j} + T_{ji}\,\delta\theta_{i,j} + \boldsymbol{\sigma}_{jki}\,\delta u_{i,jk}$$



Generalized Continuum Stresses

Continuum representation of stress . . .

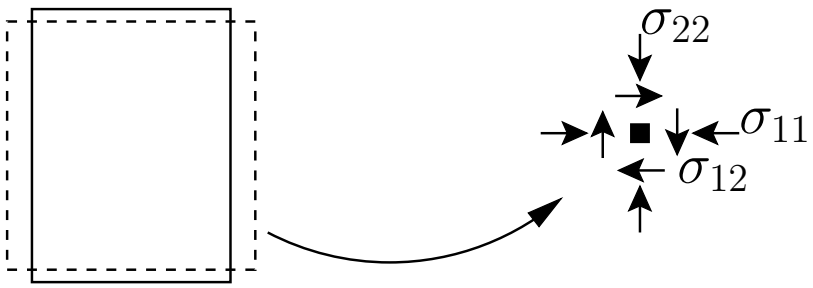
$$\delta W_{\text{Internal}} = \boldsymbol{\sigma}_{ji}\,\delta u_{i,j} + T_{ji}\,\delta\boldsymbol{\theta}_{i,j} + \boldsymbol{\sigma}_{jki}\,\delta u_{i,jk}$$

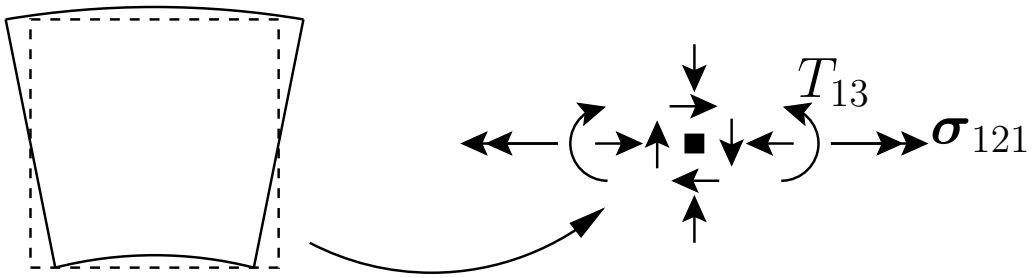


Generalized Continuum Stresses

Continuum representation of stress . . .

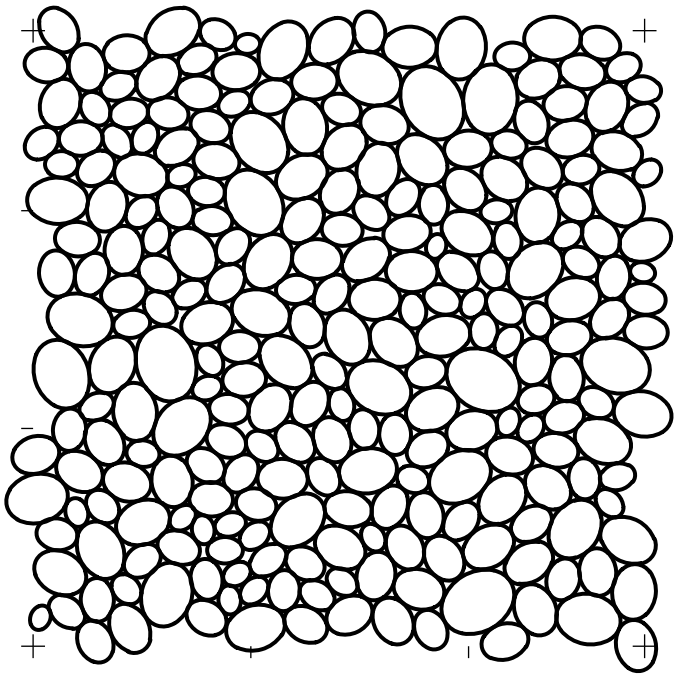
$$\delta W_{\text{Internal}} = \boldsymbol{\sigma}_{ji}\,\delta u_{i,j} + T_{ji}\,\delta\boldsymbol{\theta}_{i,j} + \boldsymbol{\sigma}_{jki}\,\delta u_{i,jk}$$





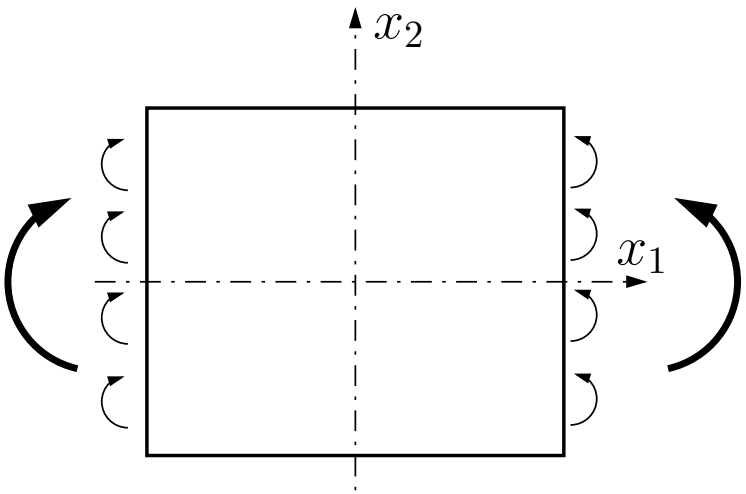
Discrete Region

DEM Simulations — 256 Particles — Circles or Ovals



Bending Resistance in a Discrete Region

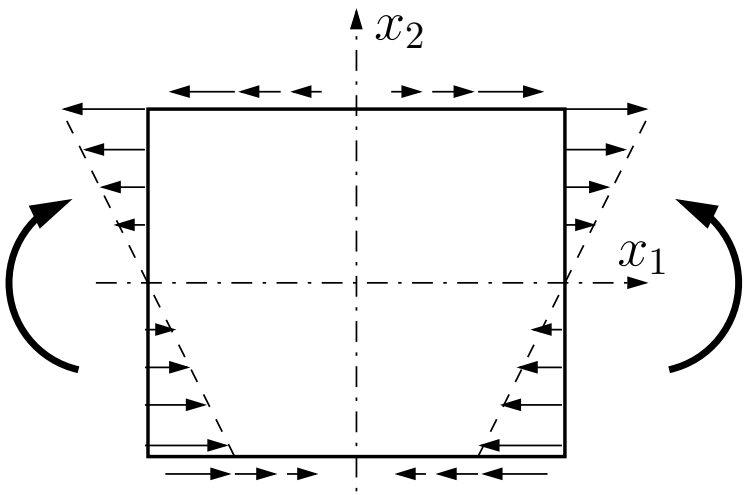
Boundary Moments:



$\Downarrow$

T_{13}

Boundary Forces:



$\Downarrow$

$\boldsymbol{\sigma}_{121}$

$$\text{Bending Moment} \quad = \quad T_{13} \ (+) \ \boldsymbol{\sigma}_{121}$$

Outline

1 Introduction

2 Quantifying high-gradient behavior
- DEM “bending” experiments
- Questions about granular behavior
- Experiment results

Granular Behavior ⇔ Questions

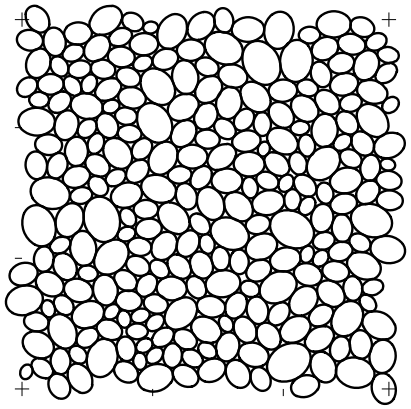
Questions:

1. Are the boundary moments significant?

   ¿ $\boxed{|T_{13}| > 0}$?

2. Are boundary forces consistent with classical beam theory?

   ¿ $\boxed{\sigma_{121} \longrightarrow E\,I\,\frac{d^2u_1}{dx_1 dx_2}}$?



Granular Behavior ⇔ Questions

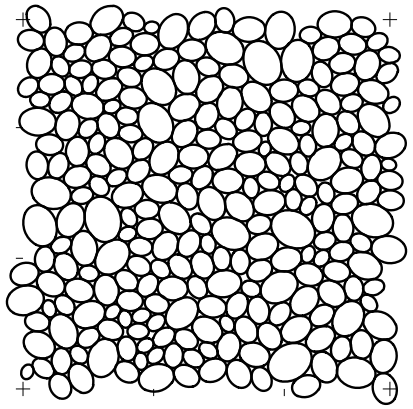
Questions:

1. Are the boundary moments significant?

$$¿ \boxed{|T_{13}| > 0} ?$$

2. Are boundary forces consistent with classical beam theory?

$$¿ \boxed{\sigma_{121} \rightarrow E\,I\,\frac{d^2 u_1}{dx_1 dx_2}} ?$$



Outline

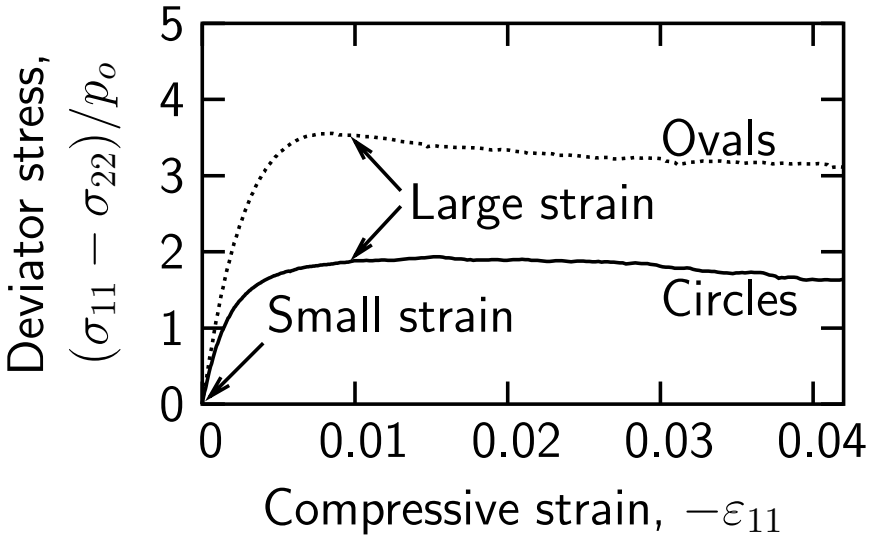
1 Introduction

2 Quantifying high-gradient behavior
- DEM “bending” experiments
- Questions about granular behavior
- Experiment results

Results Summary

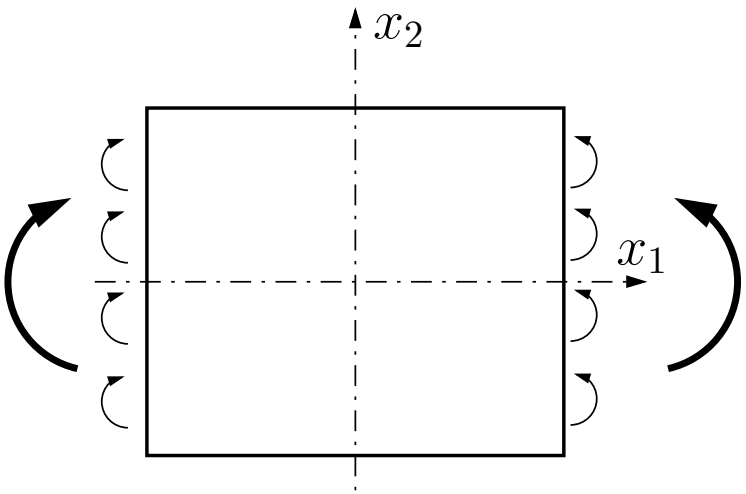
Experiment results — incremental response:

Question	Small strain	Large strain
1) $\lvert T_{13} \rvert > 0$?	No	No
2) $\boldsymbol{\sigma}_{121} \rightarrow EI \frac{d^2 u_1}{dx_1 dx_2}$?	Yes	No



Results Summary

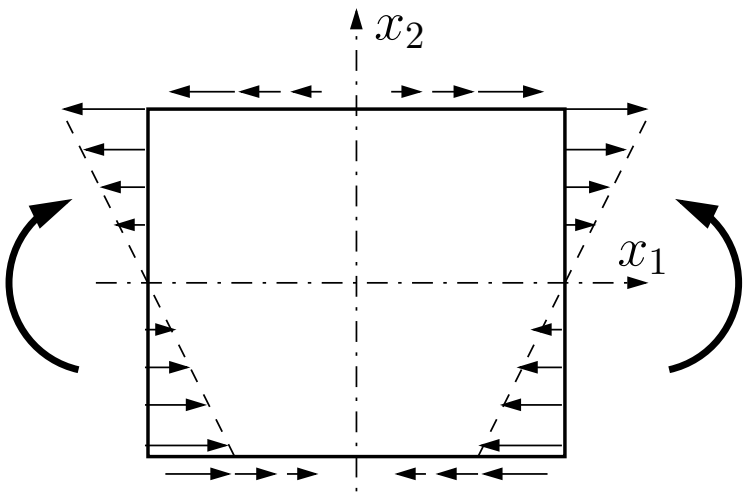
Boundary Moments:



$\Downarrow$

T_{13}

Boundary Forces:



$\Downarrow$

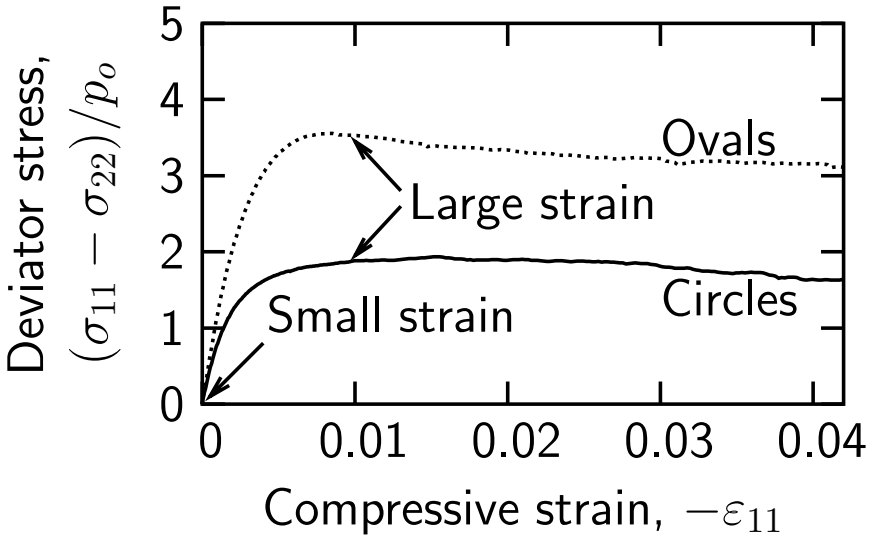
$\boldsymbol{\sigma}_{121}$

Bending Moment $= T_{13} \; (+) \; \boldsymbol{\sigma}_{121}$

Results Summary

Experiment results — incremental response:

Question	Small strain	Large strain
1) $\lvert T_{13} \rvert > 0$?	No	No
2) $\sigma_{121} \rightarrow EI \frac{d^2 u_1}{dx_1 dx_2}$?	Yes	No



Results Details

DEM Simulation Results

Dimensionless Bending Stiffnesses

256 particles — 50 assemblies

Large Strain

		Circles	Ovals
$\lvert T_{13} \rvert$	Boundary moments	-0.01	-0.01
σ_{121}	Boundary forces	0.60	1.16
$EI\,u''$	"Beam theory"	0.25	0.65

Summary

- DEM simulations can probe the response of small regions to high strain gradients.
- Cosserat-type torque stress does not contribute to incremental bending stiffness.
- A generalized stiffness is associated with the 1st gradient of strain. Stiffness is larger for oval particles.

Further Reading I

M. R. Kuhn 2005.
Are granular materials simple? An experimental study of strain gradient effects and localization.
Mechanics of Materials, 37(5):607–627.

C. S. Chang and M. R. Kuhn 2005.
On virtual work and stress in granular media.
Int. J. Solids and Structures, 42(13):3773–3793.