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# PARTICLE ROTATIONS IN GRANULAR MATERIALS

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## ABSTRACT

Particle rotations can have a dominant influence on the behavior of granular materials, particularly in materials with circular or spherical particles. The paper briefly reviews experimental evidence of the magnitude and variability of particle rotations and their effect on a granular material's stiffness and strength. Evidence of rotational patterning is reported from DEM simulations of a large square assembly of multi-sized circular particles. The translational velocities of the particles were used to compute the rotation and deformation rates within small polygonal regions of material. The material rotation could then be compared with the rotations of the particles themselves. Two meso-scale phenomena are described in the paper: rotating clusters and rotation chains. During biaxial tests, the local deformation rates are highly variable, with banded regions having deformation rates much greater than the mean rate, and with clustered regions in which very little deformation occurs. Material within a nearly-rigid region usually rotates in either an entirely clockwise or entirely counter-clockwise manner. The direction of this material rotation matches the direction of particle rotations in these regions. In a second rotational pattern, the most rapidly rotating particles are aligned in chain-like patterns oblique to the principal stress directions. These rotation chains are usually located within intensely deforming band-like regions.

## INTRODUCTION

Particle rotations are known to have an important influence on the mechanical behavior of granular materials, especially when the particles are circular or spherical. The importance of particle rotations was established in experiments by Oda et al. (1982) on plastic disks, in which they discovered that the peak strength was nearly independent of the inter-particle friction angle. This unusual result was attributed to the dominance of inter-particle rolling as a micro-scale mechanism in granular materials. Following these early experiments, other investigators have characterized the extent of particle rotations, their effects upon the macro-scale strength, and the importance of the micro-scale rolling mechanism. Some of these experimental results are summarized in the following paragraphs, and such results provide a general understanding of the effects of particle rotations and of their marked importance. There is presently no general continuum framework, however, for incorporating the large spatial variability of particle rotations at moderate and large strains, although Cosserat and micro-polar models have been used to model more gradual variations (Mühlhaus and Vardoulakis 1987). Our plan is not to bridge this formidable gap. To the contrary, we present experimental evidence that discloses

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further complexities, in the form of long-range organizations of the particle rotations. The experiments reveal rotation-related phenomena that appear at scales larger than the micro-scale of two-particle interactions, but at scales smaller than what would be considered macro-scale continuum behavior. Two such meso-scale phenomena are presented in the paper: rotating clusters and rotation chains. We begin with a brief account of the experimental record.

Experiments have consistently shown that, although particle rotations are large, the mean rotation of the particles within an assembly is nearly equal to the mean, continuum spin of the assembly (Bardet 1994; Calvetti et al. 1997; Misra and Jiang 1997). This apparent order in the particle rotations is confuted by observations of substantial fluctuations of individual rotations from the mean (Bagi 1993). As an example, Dedecker et al. (2000) measured the particle spin rates in numerical (DEM) biaxial tests on an assembly of circular disks. They found that the spins of some particles were as large as twenty times the overall strain rate. Although modest at small strains, the rotation fluctuations increase with increasing strain (Bardet 1994; Calvetti et al. 1997).

The net result of particle rotations is a reduction in strength and stiffness. These mechanical effects have been established in two types of numerical experiments. In one approach, the rotations are restricted with the use of rotational springs or prevented altogether (Bardet 1994; Calvetti et al. 1997; Iwashita and Oda 1998). The resulting strength and stiffness exceed those in control experiments with unrestrained rotations. In a second approach, measurements are taken of the incremental effect of rotations upon the contact forces (Kuhn 1991). These measurements also suggest a softening effect of the particle rotations.

Particle rolling appears to be a dominant micro-mechanism between particle pairs, and rolling is likely responsible for reducing the strength and stiffness of particle assemblies. Oda et al. (1982) and Bardet (1994) measured the rolling between between contacting particle pairs, and they reported that far more particles are engaged in rolling than in movements that produce sliding at the contacts. Their work also suggests that the rolling between particle pairs is organized at a larger scale, and such meso-scale patterns are considered in the remainder of the paper.

## MESO-SCALE ROTATION PATTERNING

Cundall et al. (1982) observed that particle rotations are spatially organized, and in 2D numerical experiments, they reported that the spins of particles within the predominant force-carrying chains were arranged in an alternating clockwise/counter-clockwise fashion. We consider two other meso-scale patterns of particle rotation: rotating clusters and rotation chains. These patterns are studied within a numerical 2D assembly of 4008 densely packed circular particles (Fig. 1a). The disk sizes ranged from $0.45D_{50}$ to $1.40D_{50}$, where $D_{50}$ is the weighted median particle diameter. The square assembly was isotropically compacted to a void ratio of 0.1803, and it is surrounded on all sides with periodic boundaries. Biaxial compression tests were performed by vertically compressing the assembly at a constant rate while maintaining constant horizontal stress ($\dot{\epsilon}_{22} = \mathrm{constant}$, $\overline{\sigma}_{11} = \mathrm{constant}$), conditions in which the mean-field spin is zero. A simple mechanism was employed between contacting particles. Linear normal and tangential springs were assigned equal stiffnesses, and slipping between particles would occur whenever the contact friction coefficient of 0.50 was attained. Unlike the models of Iwashita and Oda (1998) and Bardet (1994), no rotational resistance was included in the contact mechanism nor was any external constraint applied to the particle rotations.

The stress-strain behavior is shown in Fig. 1b, where the change in vertical stress $\Delta\sigma_{22}$ has been normalized by dividing by the initial mean stress, $p_{\mathrm{o}}$. In the paper, we confine our attention

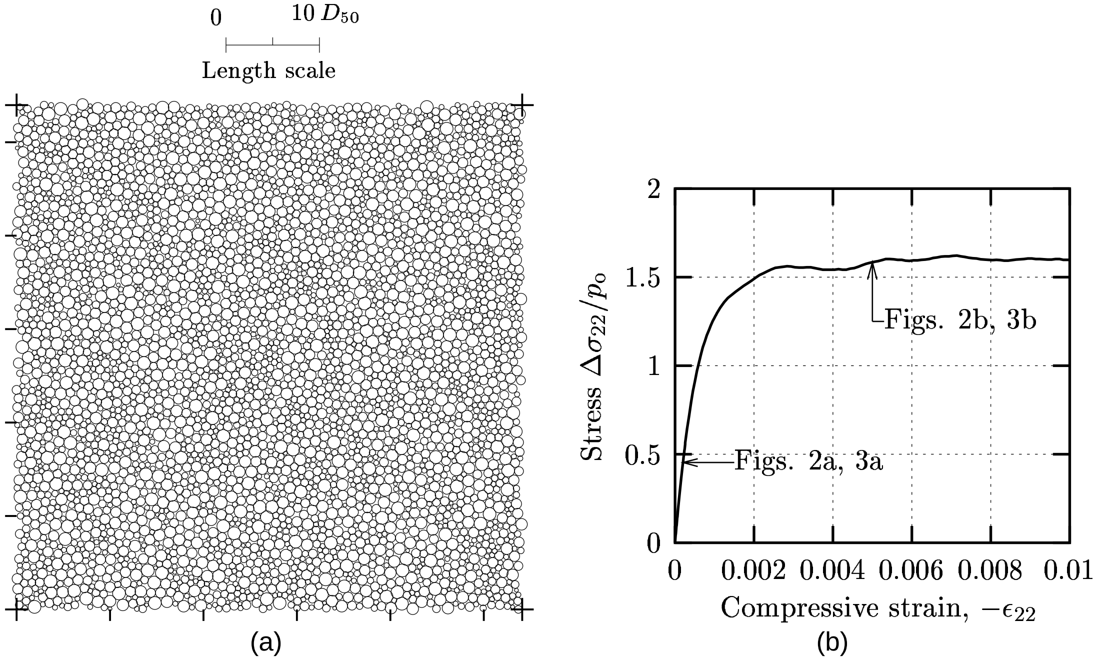



**FIG. 1. (a) Assembly of 4008 circular disks and (b) stress-strain results for the biaxial compression test.**

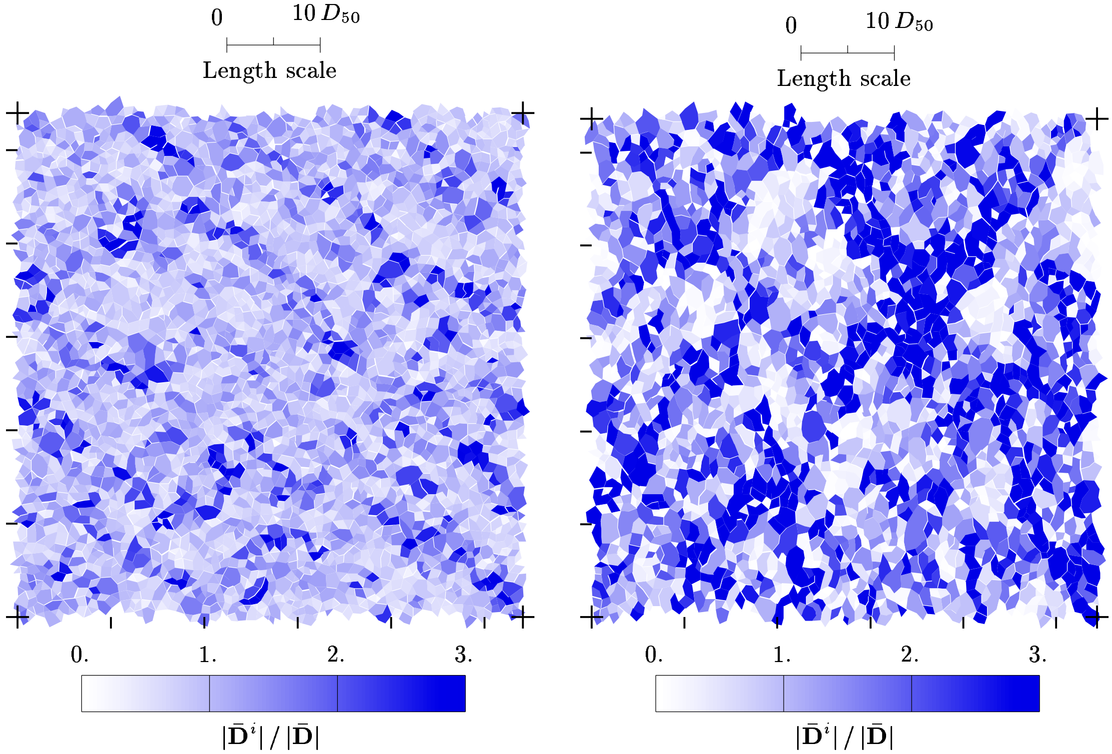
to the patterning of rotations at the vertical strains of $\epsilon_{22} = -0.0002$ and $\epsilon_{22} = -0.005$, at which the behavior is almost entirely elastic and almost entirely plastic, respectively (Fig. 1b).

**Rotating Clusters**

Deformation within granular materials is known to be non-uniform, and the heterogeneous deformations are also spatially organized. This organization is expressed, in part, in deformation structures such as microbands and shear bands (Kuhn 1999). In this section, we consider one simple measure of deformation intensity, identify regions within which the material is nearly rigid, and then investigate the rotations of both particles and material within these nearly-rigid regions. As a measure of deformation intensity, we consider the rate of deformation $\mathbf{D}$, the symmetric part of the velocity gradient $\mathbf{L}$,

$$\mathbf{L} = \mathbf{D} + \mathbf{W} \,, \tag{1}$$

where $\mathbf{W}$ is the material spin tensor. Each of these quantities can be locally measured within small polygonal regions (or *void cells*) inside of a two-dimensional assembly. The edges of each void cell are the branch vectors of contacting particle pairs, and the vertices are the centers of these particles. The initial assembly of 4008 disks could be partitioned into 3859 such polygonal regions, so that an average polygon is only somewhat larger than a single particle. The velocity gradient $\mathbf{L}^i$ within each, $i^{\text{th}}$, polygon can be computed from the relative velocities of its particle pairs (Kuhn 1999). In DEM simulations, the instantaneous velocities can oscillate, a characteristic which might obscure the general trend of particle movements. Such oscillatory effects were extenuated by two computational means. The assembly deformation was advanced in small increments, and each successive deformation increment was applied only after a sequence of time steps had allowed the material to equilibrate within the




(a) $\epsilon_{22} = -0.0002$ (b) $\epsilon_{22} = -0.005$

**FIG. 2. The local deformation rates $|\mathbf{D}^i|/|\overline{\mathbf{D}}|$ within polygonal void cells: (a) strain $\epsilon_{22} = -0.0002$, (b) strain $\epsilon_{22} = -0.005$.**

current deformation increment. This approach assured that the particles would remain in near-equilibrium throughout the simulation. Instead of using instantaneous velocities, we captured the more representative particle motions by computing the differences in particle positions at two deformation states that are separated by many deformation increments. These techniques produced stable and consistent motion and deformation measurements. In the current work, we conceal explicit reference to time rates, by dividing the local rates within polygons, $\mathbf{L}^i$ or $\mathbf{D}^i$, by an average rate, either $\overline{\mathbf{L}}$ or $\overline{\mathbf{D}}$, of the entire assembly.

Figure 2 shows the local deformation rates $|\mathbf{D}^i|/|\overline{\mathbf{D}}|$ within the numerous void cells at vertical strains of $\epsilon_{22} = -0.0002$ and $\epsilon_{22} = -0.005$. The deformation norms $|\mathbf{D}^i|$ and $|\overline{\mathbf{D}}|$ are computed as

$$|\mathbf{D}^i| = \sqrt{D^i_{jk} D^i_{jk}}\,, \quad |\overline{\mathbf{D}}| = \sqrt{\overline{D}_{jk} \overline{D}_{jk}}\,, \tag{2}$$

with summation over indices $j$ and $k$. A local region "$i$" is darkly shaded when its normed deformation $|\mathbf{D}^i|/|\mathbf{D}|$ is large, without regard to whether the local region is compressing, dilating, or shearing in any direction. Deformation heterogeneity is apparent in the figure. Some (dark) regions are deforming at rates many times greater than the average continuum rate $|\overline{\mathbf{D}}|$; other (white) regions are nearly rigid. Similar, nearly-rigid regions have also been reported by Misra and Jiang (1997). We describe several aspects of this deformation heterogeneity:

1. The heterogeneity is modest at small strains and grows progressively larger as the strain

increases to at least $\epsilon_{22} = -0.005$.

2. The particle packing is denser within the (white) nearly-rigid regions. The average valence (the number of edges per polygonal void cell) is 3.61 among the 10% of void cells that are deforming the least; the average valence is 5.40 among the 10% of void cells that are most vigorously deforming (at $\epsilon_{22} = -0.005$).
3. The (white) nearly-rigid regions average about 7–8 particle diameters ($D_{50}$) in width. These regions often have a central zone of a few void cells in which the deformation $|\mathbf{D}^i|$ is almost zero. The deformation increases further from this center and blends into the more actively deforming material.
4. The most intensely deforming regions are organized as bands that trend diagonally through the material—oriented oblique to the directions of principal stress. These regions are microbands, within which the shearing, particle rotations, and material rotation are most intense (Kuhn 1999).

Our primary interest is in the particle and material rotations that occur within the nearly-rigid clusters. The *material rotation* is the spin $\mathbf{W}^i$ of a polygonal void cell region. This spin is computed from the translational velocities of the particle centers, which lie at the polygon corners. The material spin $\mathbf{W}^i$ is measured independently of the particle spins $\omega^k$ of individual particles. The particle spins were separately plotted and compared with material spins $\mathbf{W}^i$. The following rotation characteristics were observed within the nearly-rigid regions of material:

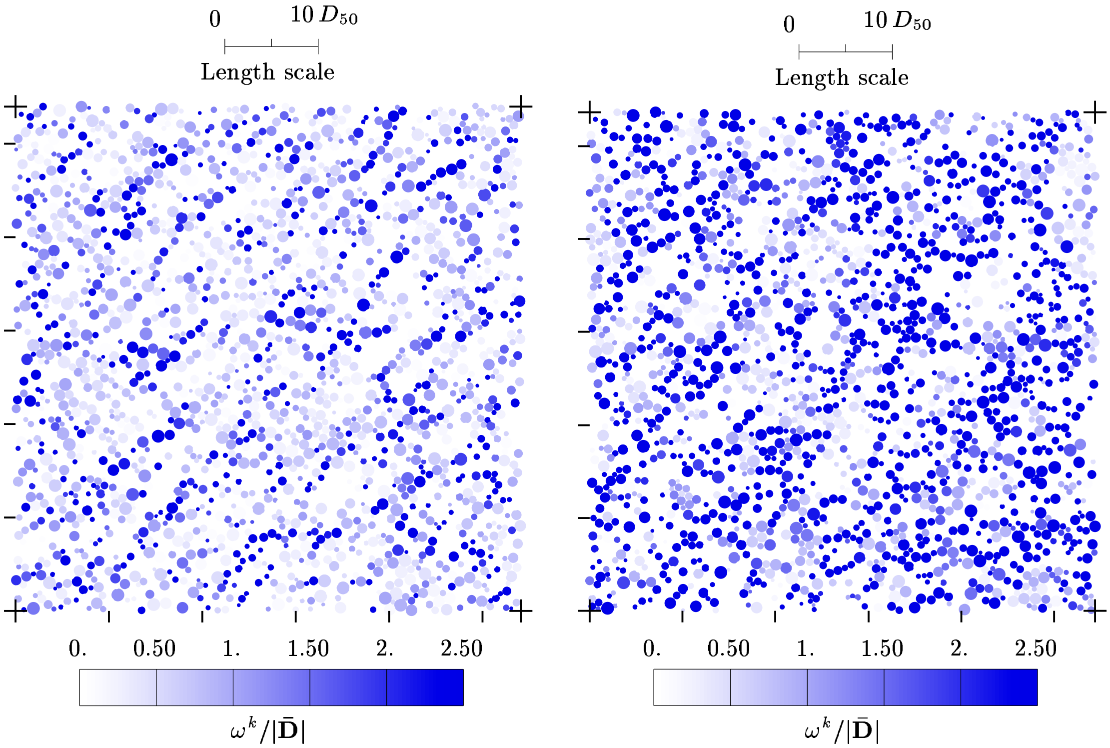
1. The least-deforming, nearly-rigid regions usually have a small material rotation $\mathbf{W}^i$ among their many polygonal void cells. Each nearly-rigid region has a rotation that is either entirely clockwise or entirely counter-clockwise. These regions can be described as *rotating clusters*.
2. Within the rotating clusters, particle rotations $\omega^k$ are strongly correlated with material rotation $\mathbf{W}^i$: particles rotate in the same direction as their encompassing region. Although we have not yet compared rotation magnitudes, the *direction* of the particle rotations is consistent with the material rotation.

### Rotation Chains

Figure 3 shows the counter-clockwise rotations of particles at the vertical strains $\epsilon_{22} = -0.0002$ and $\epsilon_{22} = -0.005$. *Only counter-clockwise* rotations are shown in these monochrome plots, where the shading depends upon the dimensionless rotation rate $\omega^k/|\overline{\mathbf{D}}|$. The most rapidly rotating particles are usually aligned in chain-like patterns oblique to the principal stress directions. These *rotation chains* are somewhat more sinuous at the larger strain. Allowing for their frequent crooks and staggers, some rotation chains can be traced across the full height and width of the assembly. The particles along counter-clockwise rotation chains are usually not touching each other, as this would produce intense sliding between particles, but they are instead arranged with oppositely rotating particles in a manner that produces a roller-bearing effect. The chains are closely associated with microbands, as can be seen by comparing Figs. 2 and 3 (Kuhn 1999). As with rotating clusters, the material spin $\mathbf{W}^i$ along the rotation chains is strongly correlated with the particle rotation $\omega^k$.

## CONCLUSION

We have identified two patterns of particle and material rotation: rotating clusters and rotation chains. In each pattern, rotations are organized across distances of many particle diameters. Particle rotations in both patterns are strongly correlated with material rotation. Rotation




(a) $\epsilon_{22} = -0.0002$ (b) $\epsilon_{22} = -0.005$

**FIG. 3. Counter-clockwise particle rotation rates $\omega^i/|\overline{\mathrm{D}}|$: (a) strain $\epsilon_{22} = -0.0002$, (b) strain $\epsilon_{22} = -0.005$.**

patterning reflects the complex organization of heterogeneous deformations within granular materials.

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