Measurement of Micro-Polar Effects in Granular Materials

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1 An Unusual 2D Experiment

Stage I: Horizontal compression to the peak stress

Stage II: Flexure increment

General conditions:
- DEM simulations: three 2D particle shapes
  - Circles
  - Ovals
  - “Nobbies”
- 256 – 1024 – 4096 particles
- Flexible boundaries
- Linear(frictional contacts. No contact moments.
- Quasi-static deformation

Stage II flexure:
The boundary displacements produce an incremental curvature $d\psi$,
$$du_i = d\psi x_1 x_2 \Rightarrow d\psi$$
which creates gradients of strain and rotation:
$$\frac{\varepsilon_{12}}{x_2} = d\psi \quad \text{and} \quad \frac{\partial \theta}{\partial x_1} = d\psi$$

2 Three Questions

1. Is the incremental “flexural stiffness” consistent with a simple / classical material?
   $$\frac{dM}{d\psi} \approx I \cdot \frac{1}{2} \left( E_{\text{loading}} + E_{\text{unloading}} \right)$$
   where $I = \frac{1}{12} \ell^3$. The “$E$” values are those measured during uniform (Stage I) compression and extension — with no gradients of strain or rotation.

2. Does the particle shape affect this relative bending stiffness?
3. Does the assembly size affect the relative bending stiffness?

3 Results

1. A simple / classical material? No! Not at the peak state.

<table>
<thead>
<tr>
<th>Stiffness 256 circles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E/k$, loading</td>
</tr>
<tr>
<td>$E/k$, unloading</td>
</tr>
<tr>
<td>$(E_{\text{loading}} + E_{\text{unloading}})/2k$</td>
</tr>
<tr>
<td>Flexural tests (Stage II)</td>
</tr>
<tr>
<td>$\approx \times 2.00$ increase</td>
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</tbody>
</table>

   Stiffness is greater when deformation includes strain and rotation gradients. This observation is inconsistent with a simple / classical material.


   - Increased flexural stiffness
     - Circles × 2.00 increase
     - Ovals × 1.73 increase
     - Nobbies × 1.75 increase

   The three shapes exhibit similar increases in flexural stiffness.


<table>
<thead>
<tr>
<th>Increased flexural stiffness, Assembly size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circles</td>
</tr>
<tr>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>256 particles × 2.00 increase</td>
</tr>
<tr>
<td>1024 particles × 2.09 increase</td>
</tr>
<tr>
<td>4096 particles × 2.14 increase</td>
</tr>
</tbody>
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   Small and large assemblies exhibit about the same increases in flexural stiffness.

4 Conclusions and Implications

- At the start of loading (small strains), granular stiffness nearly conforms to that of a simple material within a classical continuum.
- At the peak state (large strains), granular stiffness does not conform to that of a simple material within a classical continuum.
- Oddly, particle shape has very little effect on the relative increase in stiffness.
- Possible continuum models:
  (1) Strain-gradient dependent material within a classical continuum.
  (2) Cosserat continuum.
- Either continuum model would predict a more elevated flexural stiffness for small assemblies than for large assemblies. Oddly, the experiments contradict this prediction.