Measurement of Micro-Polar Effects in Granular Materials

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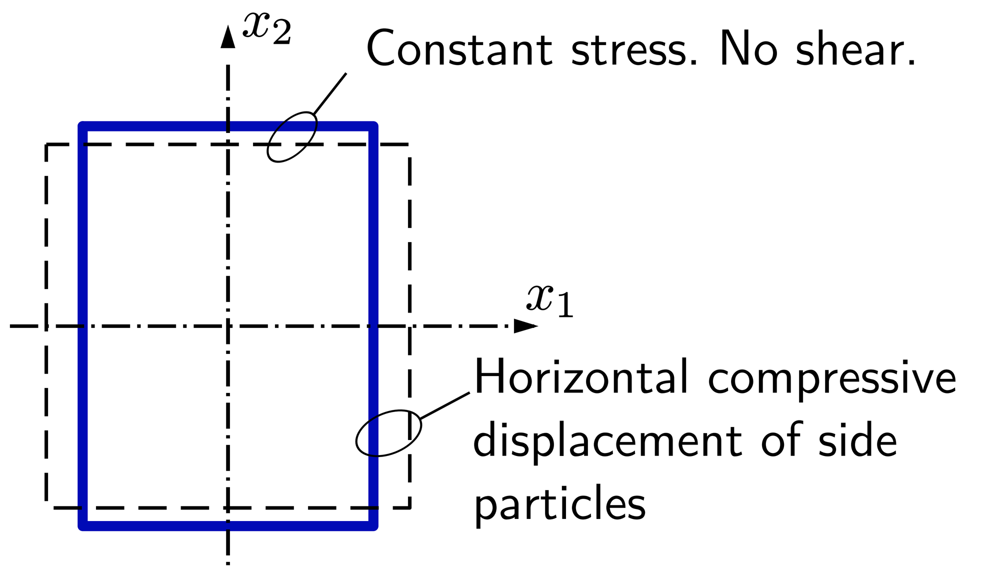
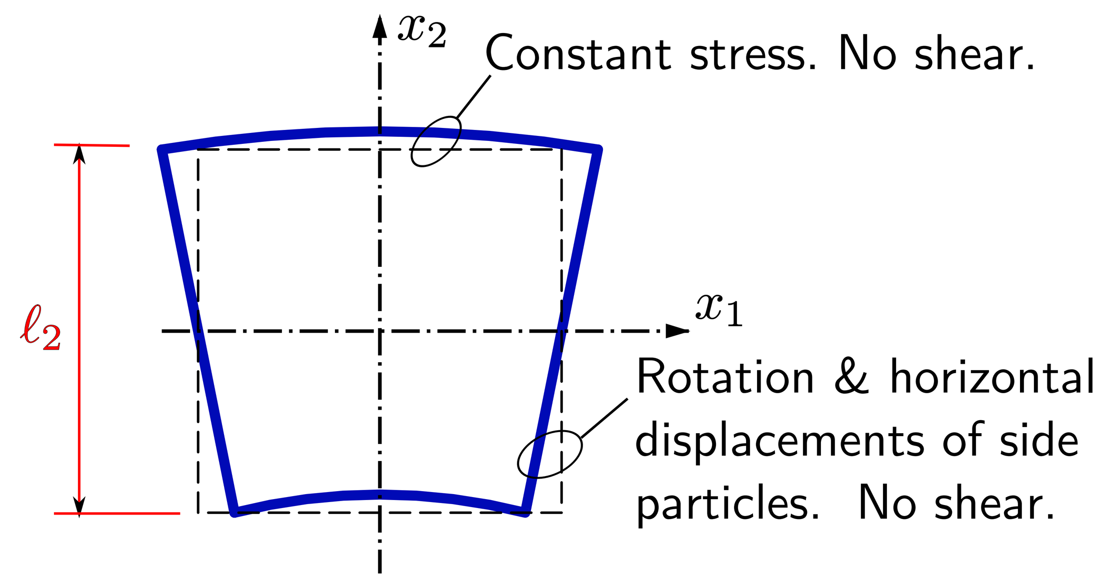
1 An Unusual 2D Experiment

Stage I:
Horizontal compression to the peak stress

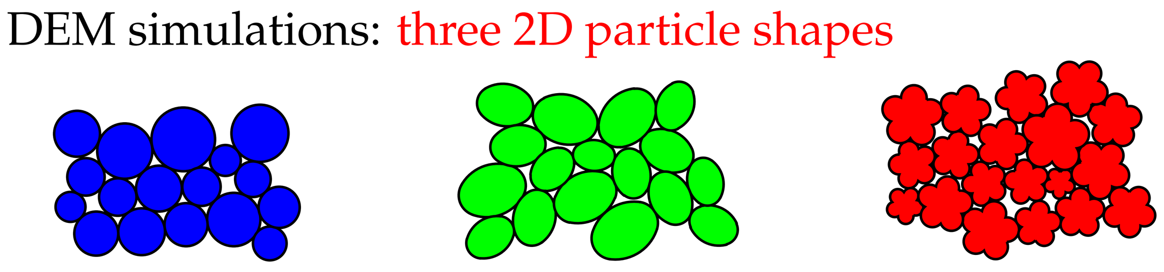


Stage II:
Flexure increment



General conditions:

- DEM simulations: three 2D particle shapes



Circles Ovals "Nobbies"

- 256 – 1024 – 4096 particles
- Flexible boundaries
- Linear/frictional contacts. No contact moments.
- Quasi-static deformation

Stage II flexure:

The boundary displacements produce an incremental curvature $d\psi$,

$$du_1 = d\psi \; x_1 x_2 \; \Rightarrow \; d\psi$$

which creates gradients of strain and rotation:

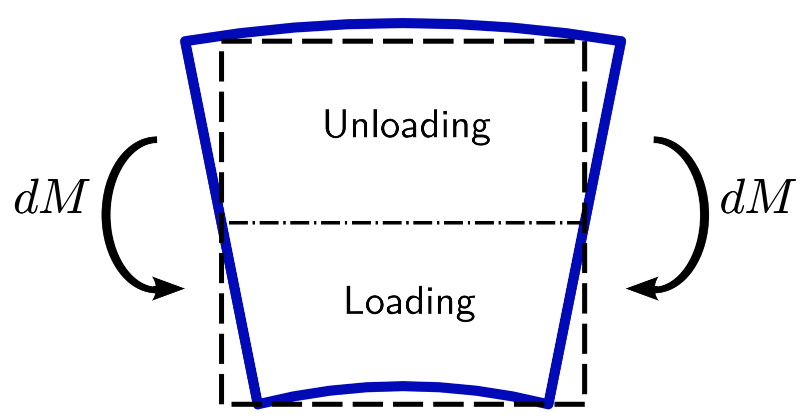
$$\frac{\epsilon_{11}}{x_2} = d\psi \quad \text{and} \quad \frac{\partial\theta}{\partial x_1} = d\psi$$

2 Three Questions

1. Is the incremental "flexural stiffness" consistent with a simple / classical material ?

$$\frac{dM}{d\psi} \stackrel{?}{=} I \cdot \frac{1}{2}\left(E_{\text{loading}} + E_{\text{unloading}}\right)$$

where $I = \frac{1}{12}\ell_2^3$. The "E" values are those measured during uniform (Stage I) compression and extension — with no gradients of strain or rotation.



2. Does the particle shape affect this relative bending stiffness ?
3. Does the assembly size affect the relative bending stiffness ?

3 Results

1. A simple / classical material ? *No* ! Not at the peak state.

	Stiffness 256 circles	
Compression tests (Stage I)		
E/k, loading	0.00	
E/k, unloading	0.38	
$(E_{\text{loading}} + E_{\text{unloading}})/2k$	0.19	$\approx \times 2.00$ increase
Flexural tests (Stage II)		
$(dM/d\psi) \cdot (1/I\,k)$	0.38	

Stiffness is greater when deformation includes strain and rotation gradients. This observation is inconsistent with a simple / classical material.

2. Influence of particle shape ? *No*.

Shape	Increased flexural stiffness 256 particles
Circles	× 2.00 increase
Ovals	× 1.73 increase
Nobbies	× 1.75 increase

The three shapes exhibit similar increases in flexural stiffness.

3. Influence of assembly size ? *No*.

Assembly size	Increased flexural stiffness, Circles
256 particles	× 2.00 increase
1024 particles	× 2.09 increase
4096 particles	× 2.14 increase

Small and large assemblies exhibit about the same increases in flexural stiffness.

4 Conclusions and Implications

- At the start of loading (small strains), granular stiffness nearly conforms to that of a simple material within a classical continuum.
- At the peak state (large strains), granular stiffness *does not* conform to that of a simple material within a classical continuum.
- Oddly, particle shape has very little effect on the relative increase in stiffness.
- Possible continuum models:
  (1) Strain-gradient dependent material within a classical continuum.
  (2) Cosserat continuum.
- Either continuum model would predict a more elevated flexural stiffness for small assemblies than for large assemblies. Oddly, the experiments contradict this prediction.

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