Particle Rolling and Its Effects in Granular Materials

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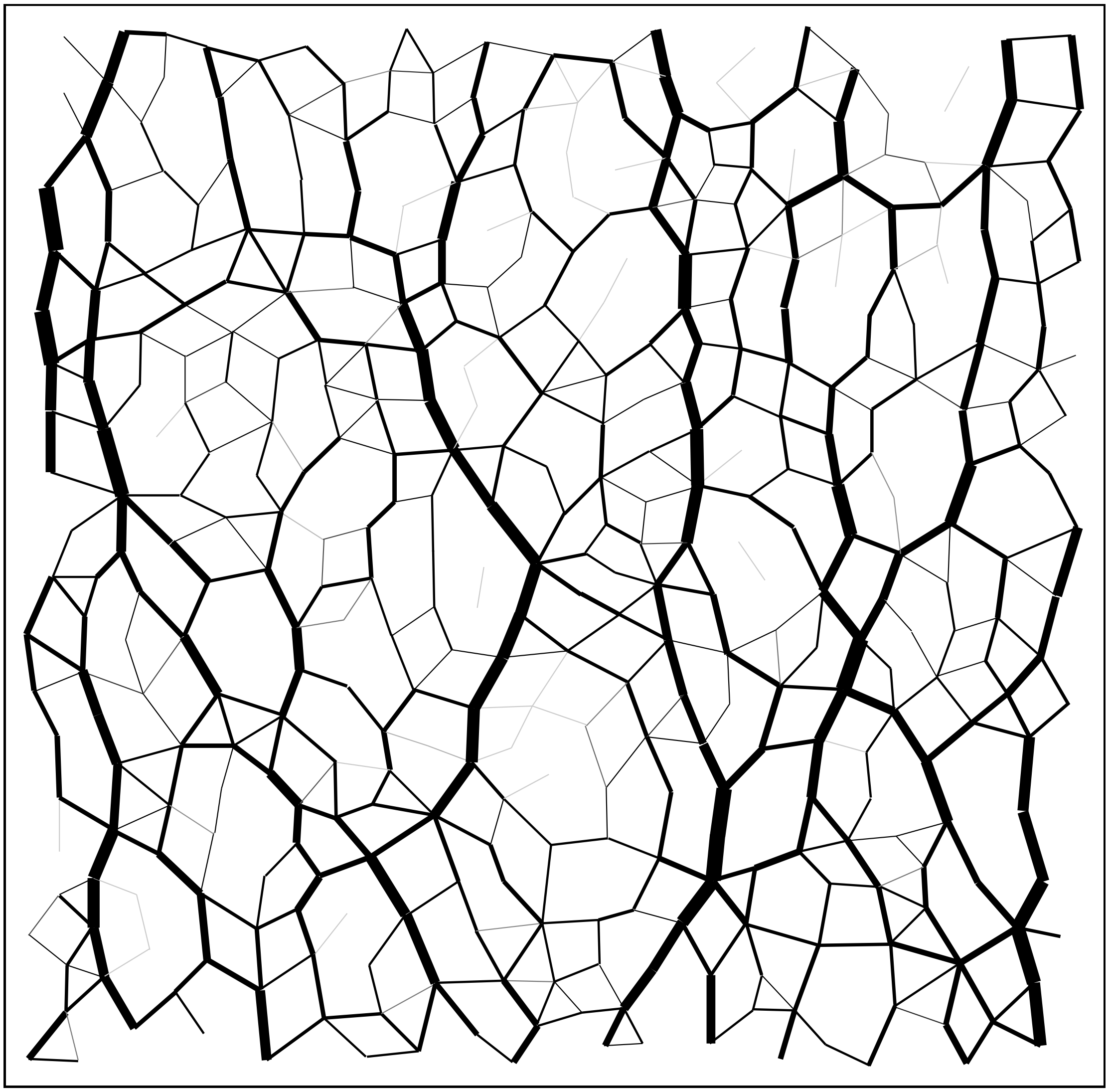
www.egr.up.edu/contrib/kuhn/QuaDPM.pdf

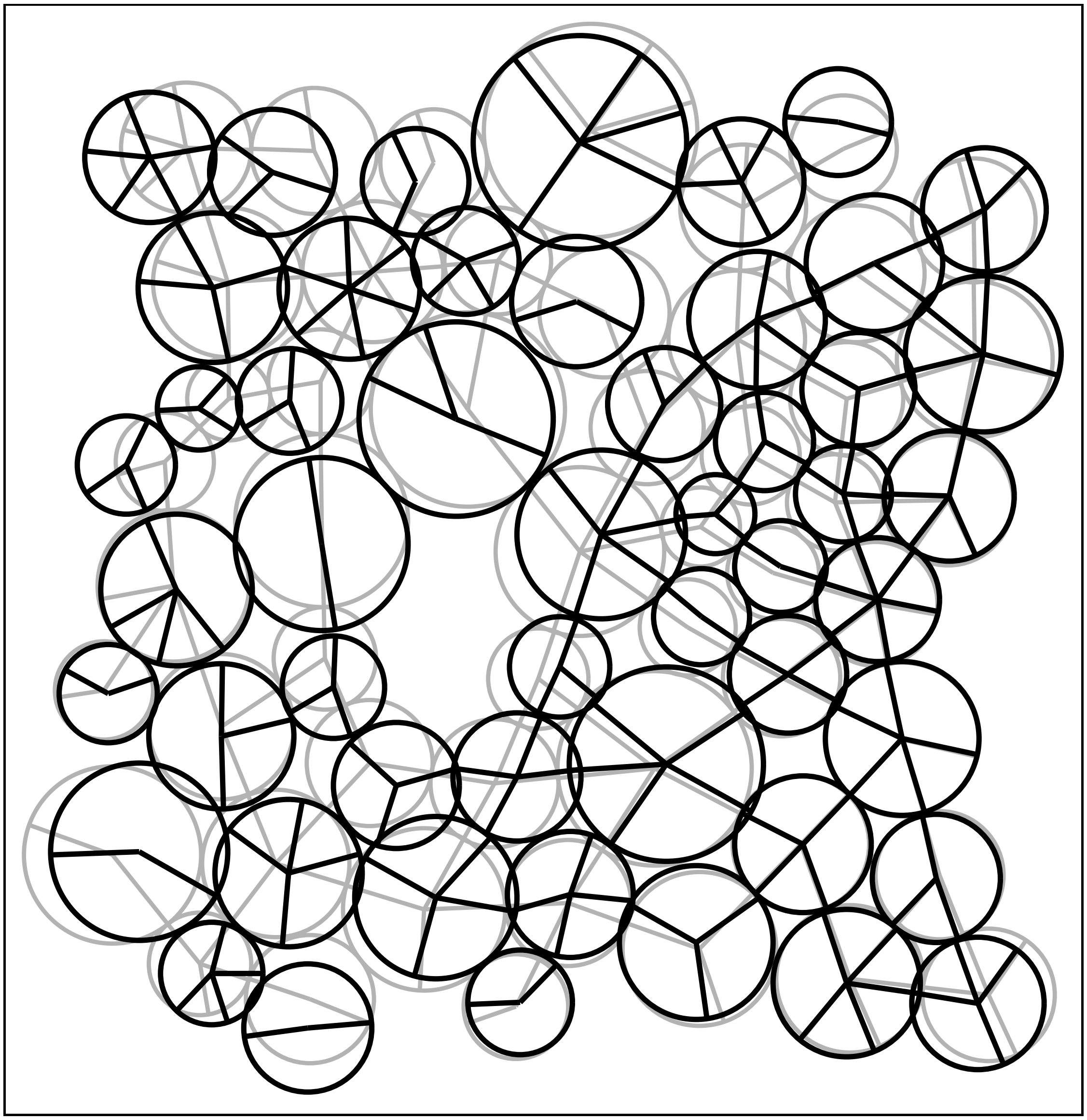
Topics

1. **Stiffness effects**
2. **Rolling definitions**
3. **Rolling measurements in DEM simulations**
4. **Spatial patterning**
5. **An alternative rolling measure ?**

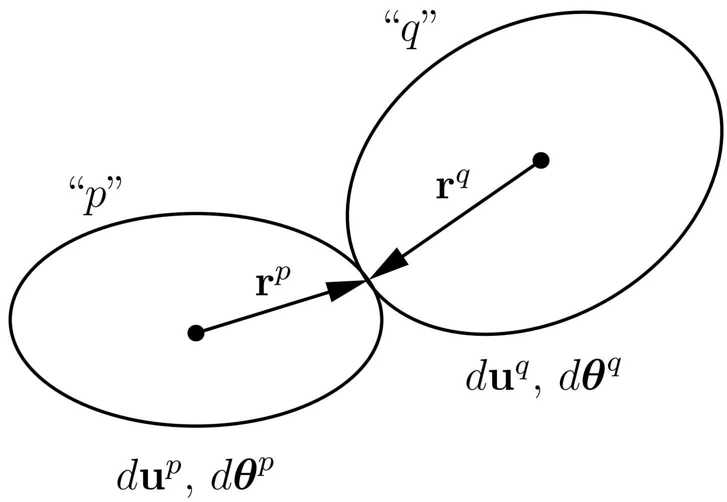
Example: Force chain



Example, continued: Collapsing force chain



Particle Rotations and Material Stiffness



$$d\mathbf{u}^{\mathbf{def}} = (d\mathbf{u}^q - d\mathbf{u}^p) \qquad \text{“ } d\mathbf{u}^{\textbf{def, translation}} \text{ ”}$$

$$+ (d\boldsymbol{\theta}^q \times \mathbf{r}^q - d\boldsymbol{\theta}^p \times \mathbf{r}^p) \qquad \text{“ } d\mathbf{u}^{\textbf{def, rotation}} \text{ ”}$$

Correlations ?

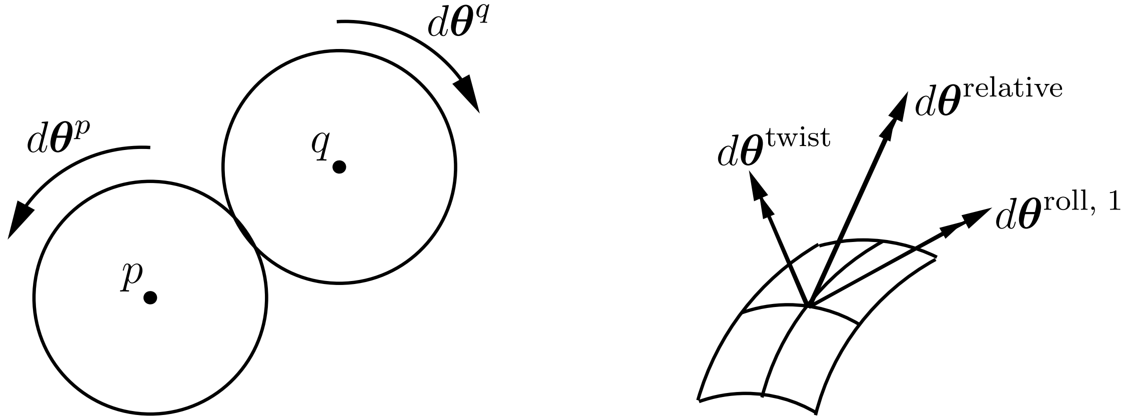
$$d\mathbf{u}^{\textbf{def, translation}} \longleftrightarrow d\mathbf{u}^{\textbf{def, rotation}}$$

$$= -0.2 \textbf{ to } -0.7$$

$$d\mathbf{u}^{\mathbf{def}} \longleftrightarrow d\mathbf{u}^{\textbf{def, rotation}}$$

$$= 0 \textbf{ to } +0.7$$

Rolling Definition Number 1



$$d\boldsymbol{\theta}^{\textbf{relative}} = d\boldsymbol{\theta}^q - d\boldsymbol{\theta}^p$$

$$\Downarrow$$

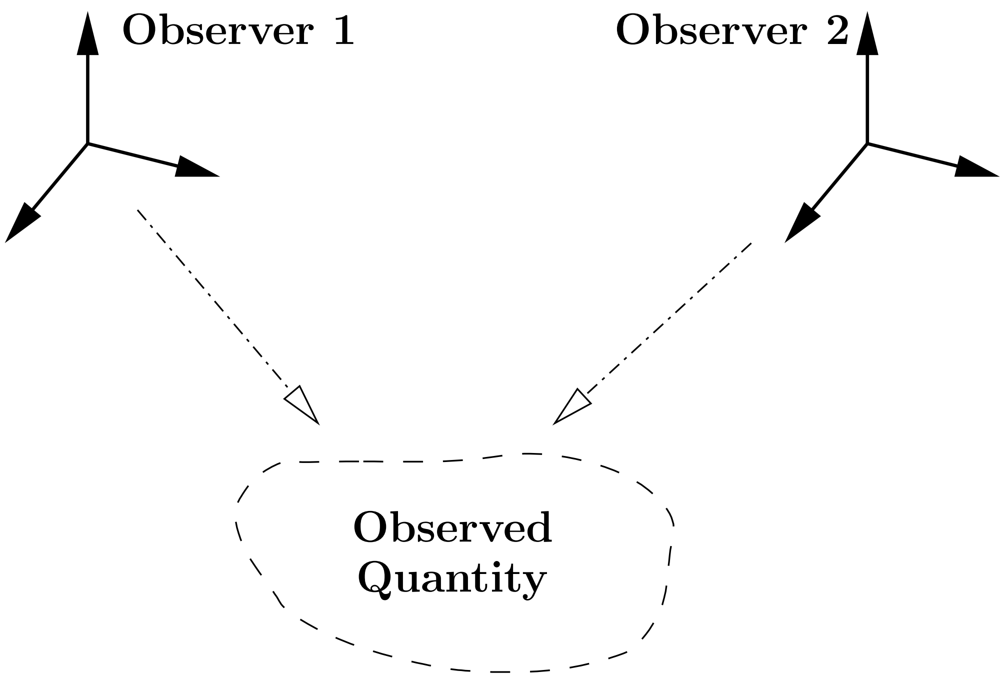
$$d\boldsymbol{\theta}^{\textbf{twist}} = \left(d\boldsymbol{\theta}^q - d\boldsymbol{\theta}^p\right) \cdot \mathbf{n}$$

$$d\boldsymbol{\theta}^{\textbf{roll, 1}} = d\boldsymbol{\theta}^{\textbf{relative}} - d\boldsymbol{\theta}^{\textbf{twist}}$$

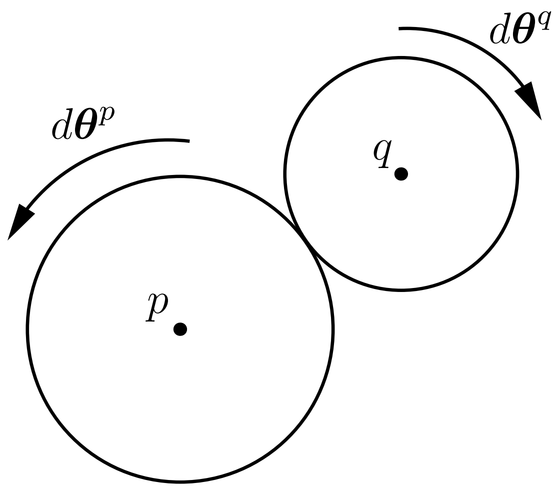
Notes:

- **Applicable in numerical simulations when contacts resist relative rotation**
- **Conjugate internal force: contact moment**
- **Objective**

Objectivity



Rolling Definition Number 2



$$\textbf{Rolling} \stackrel{?}{=} r^q d\boldsymbol{\theta}^q - r^p d\boldsymbol{\theta}^p$$

Not objective !

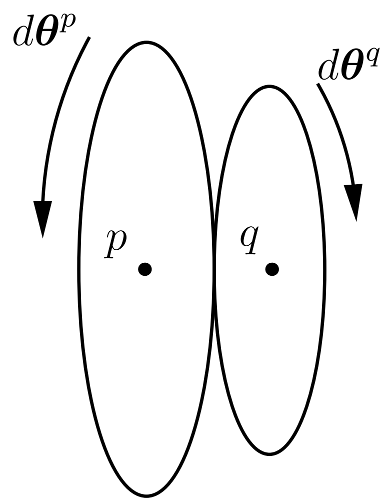
An objective form:

$$du^{\textbf{roll, 2}} = \frac{1}{2}\Bigg[(d\boldsymbol{\theta}^p \cdot \mathbf{z}^{\mathrm{t}})(\mathbf{r}^p \cdot \boldsymbol{\lambda}^{\mathrm{t}}) + (d\boldsymbol{\theta}^q \cdot \mathbf{z}^{\mathrm{t}})(\mathbf{r}^q \cdot \boldsymbol{\lambda}^{\mathrm{t}}) - \frac{(d\mathbf{u}^q - d\mathbf{u}^p)\cdot \mathbf{t}}{\ell^{\perp \mathrm{t}}}(\mathbf{r}^p + \mathbf{r}^q)\cdot \boldsymbol{\lambda}^{\mathrm{t}}\Bigg]$$

Notes:

- **A scalar translation**
- **Arbitrary shapes**
- **Objective**

Rolling Definition Number 3

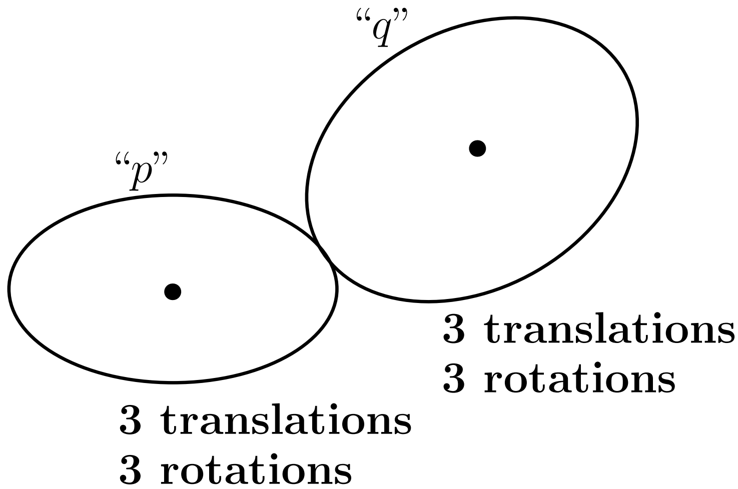


$$d\mathbf{u}^{\textbf{roll, 3, t}} = -(\mathbf{K}^p + \mathbf{K}^q)^{-1}\left[d\boldsymbol{\theta}^{\textbf{relative}} \times \mathbf{n} + \frac{1}{2}(\mathbf{K}^p - \mathbf{K}^q)\, d\overline{\mathbf{u}}^{\textbf{def}}\right]$$

Notes:

- **A tangent translation vector**
- **Arbitrary shapes**
- **Objective**

Classifying the Motions of a Contacting Pair



$$\Downarrow$$

12 particle motions

$$\Downarrow$$

$$\left[\quad \mathsf{A} \quad\right] \left[\begin{array}{c} 12 \\ \textbf{particle} \\ \textbf{motions} \end{array}\right] = \left[\begin{array}{c} 12 \\ \textbf{contact} \\ \textbf{motions} \end{array}\right]$$

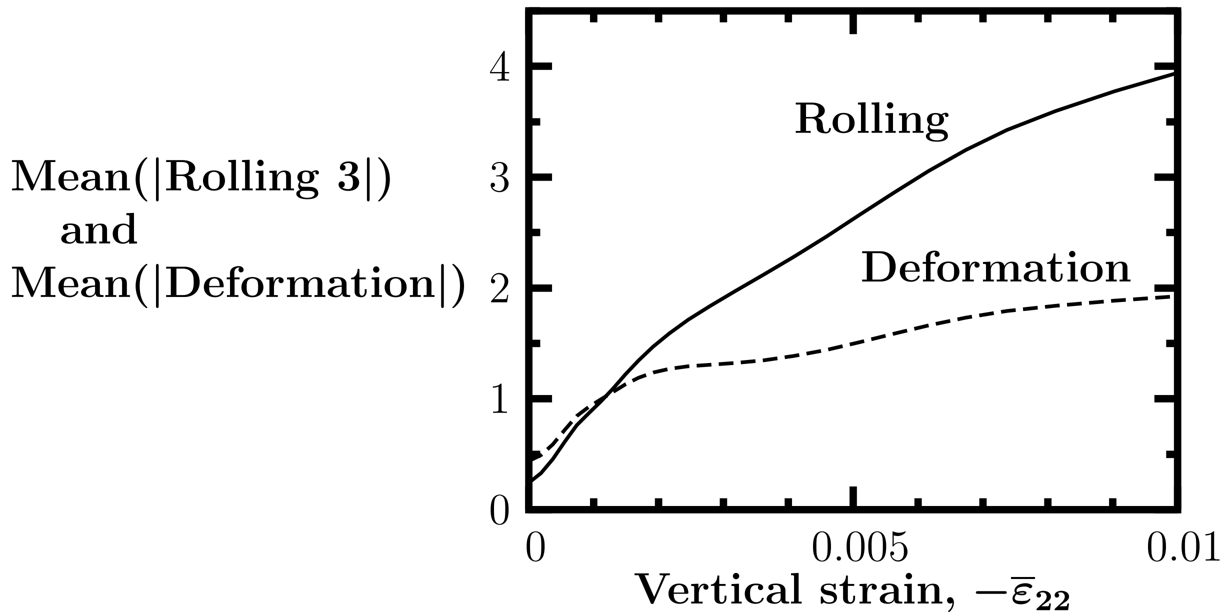
$$\Downarrow$$

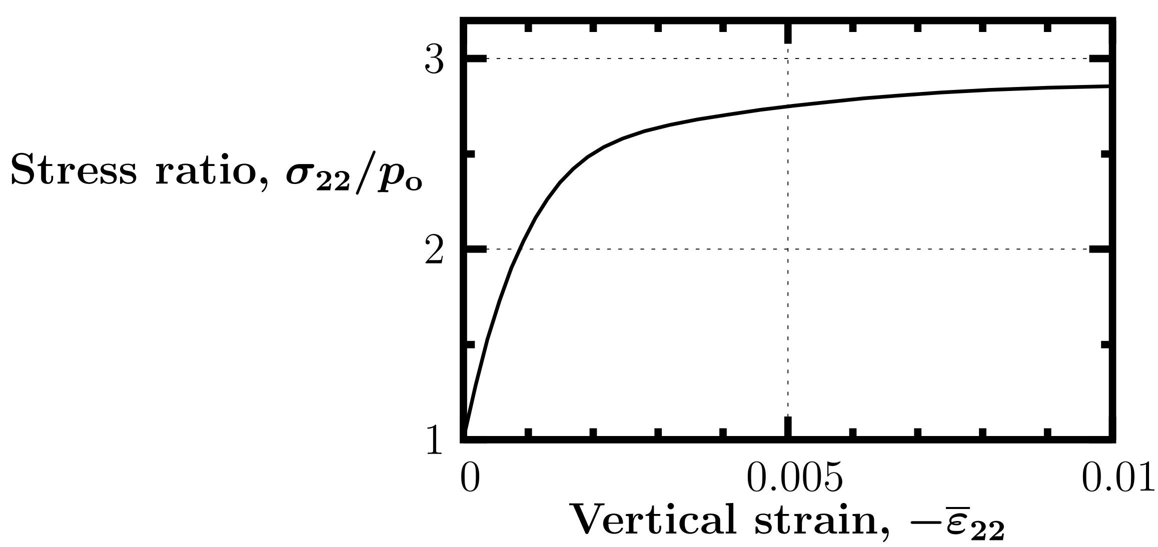
12 contact motions ?

12 Contact Motions:

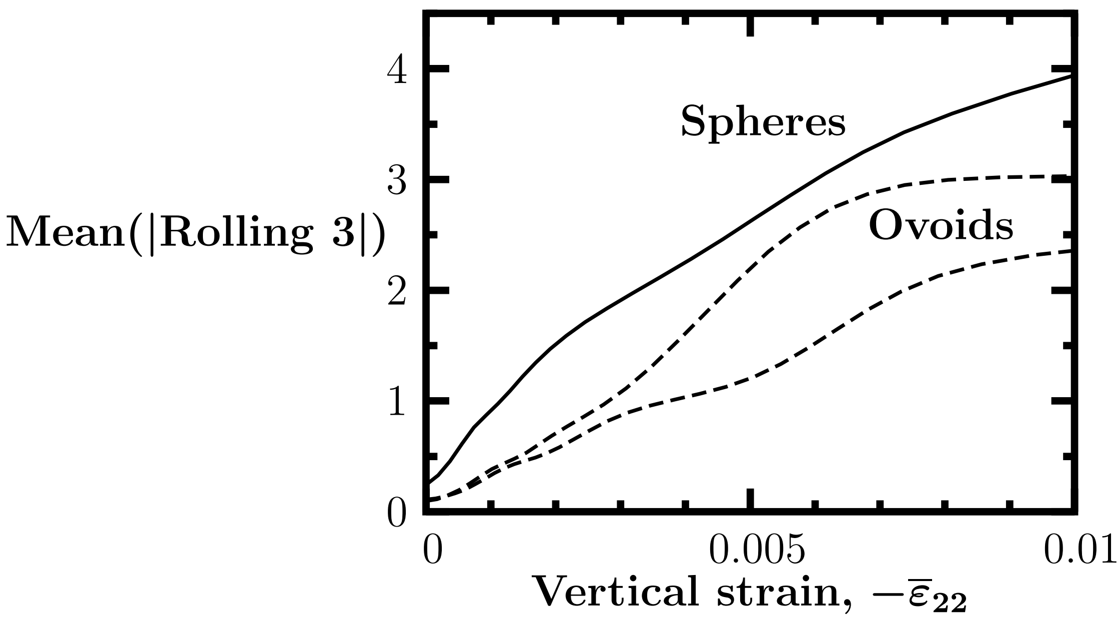
	3	**contact deformations,** $d\mathbf{u}^{\mathbf{def}}$
+	**3**	**contact rolling motions, for example:** $d\mathbf{u}^{\mathbf{relative}} = d\boldsymbol{\theta}^q - d\boldsymbol{\theta}^p$
+	**3**	**rigid translations**
+	**3**	**rigid rotations**
=	**12**	**contact motions**

Comparison of Rolling and Deformation Movements



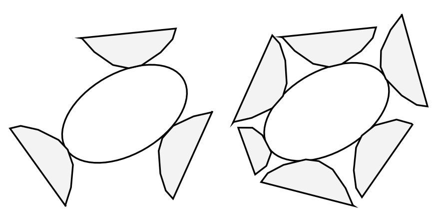


Comparison of Rolling Among Spheres and Non-Spheres

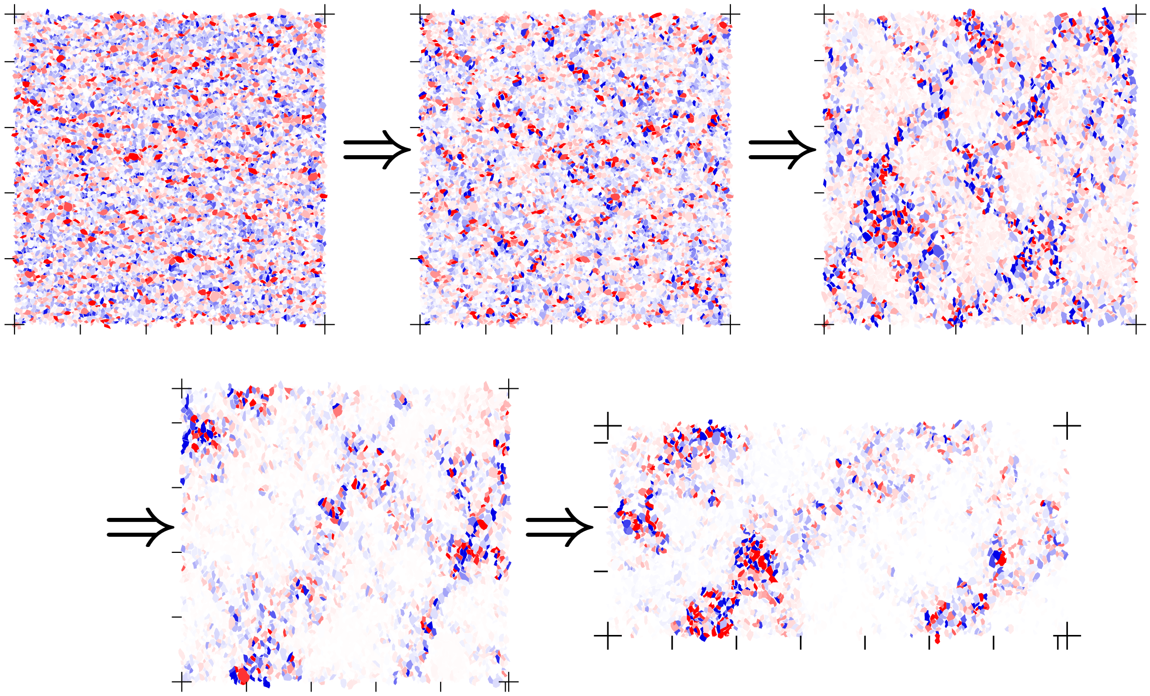


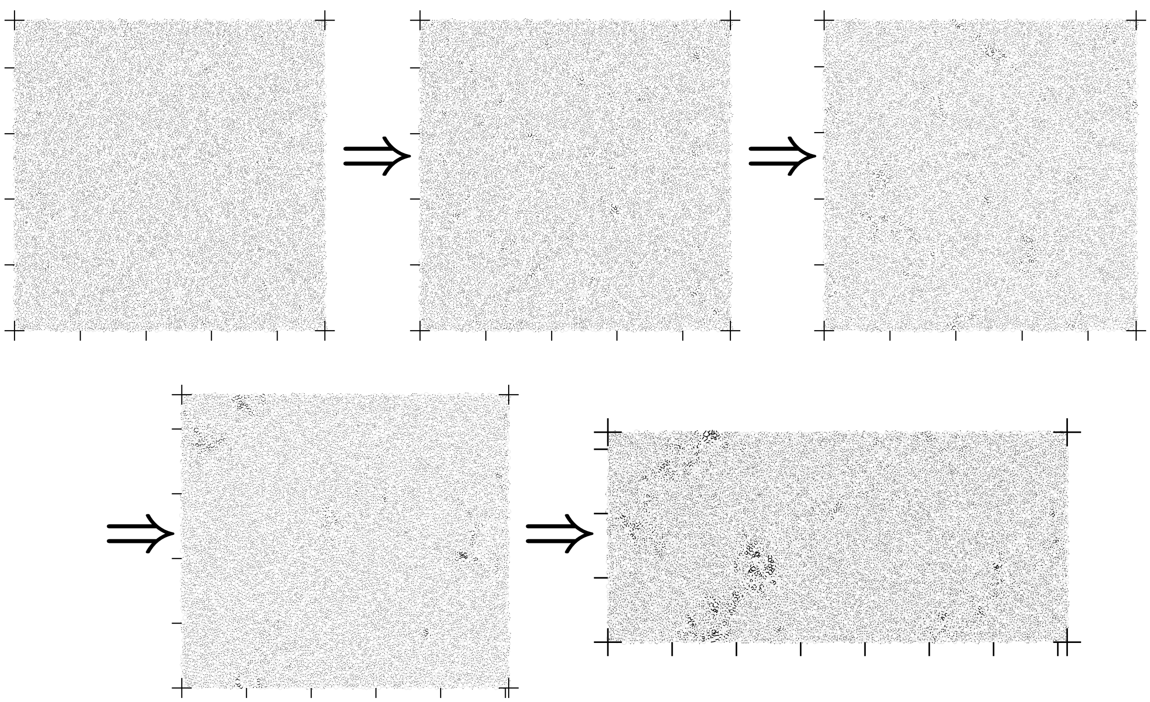
Characteristics of Particle Rolling in Dense Packings

- **Types 1, 2, and 3 rolling are strongly correlated.**
- **Rolling motions are large when compared with deformation motions.**
- **Rolling increases with strain.**
- **Rolling is smaller within non-circular and non-spherical particle assemblies.**
- **Rolling intensity is similar in all "directions."**
- **Rolling is uncorrelated with the normal contact force, f^n (at least, with linear contacts).**
- **Rolling has same intensity for slipping and non-slipping contacts.**
- **Contact density inhibits rolling.**



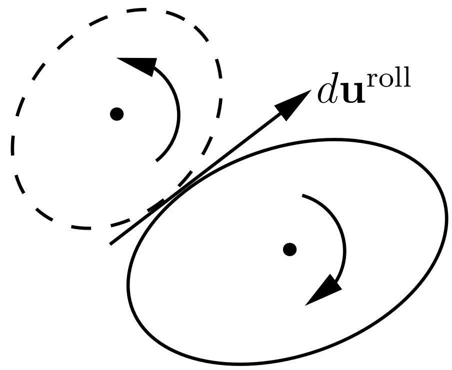
Patterning: Dilation and Rolling Motions



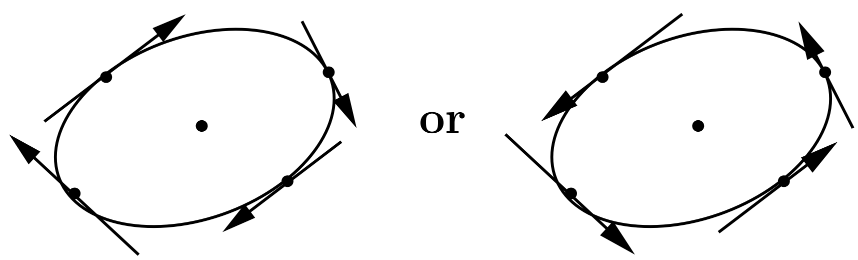


Patterning of the Rolling Motions, continued

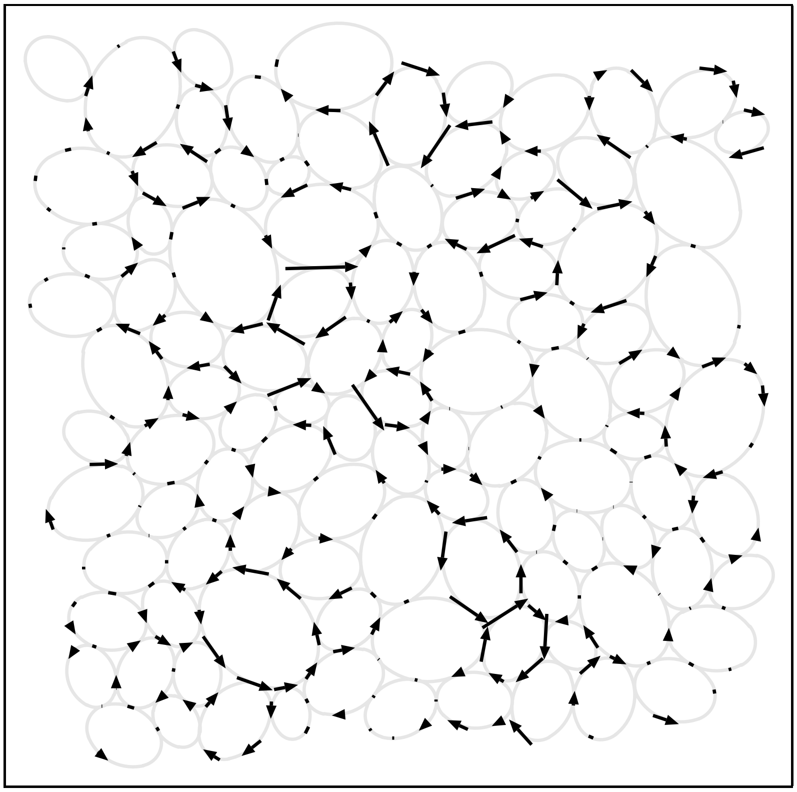
1. **Represent the rolling motions as a vector:**



2. **The predominant pattern for the rolling motions:**

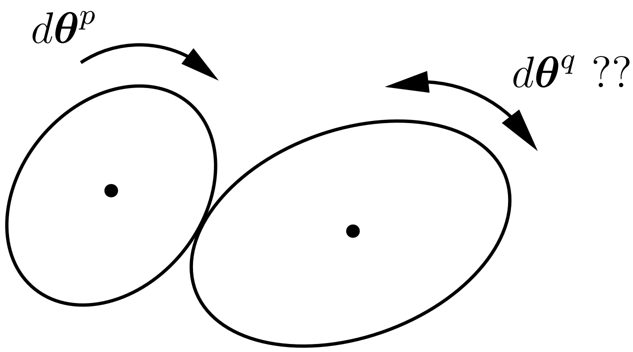


Patterning of Rolling Motions



Gear-like Motions in 3D ?

1. **Compare $d\boldsymbol{\theta}^p$ and $d\boldsymbol{\theta}^q$**



   Correlation = -0.1 **to** -0.5

2. **Consider all spheres having 6 contacts.**

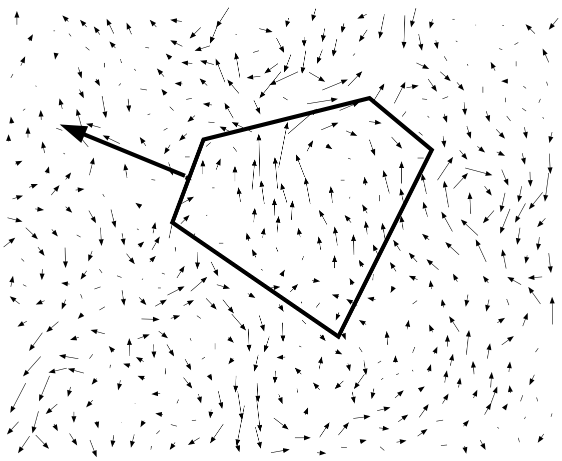
   Percent % of these spheres with all 6 rolling vectors in the same rotational direction, say $\mathbf{e}_1$**:**

   % = xx% to xx%

An alternative measure of rolling:

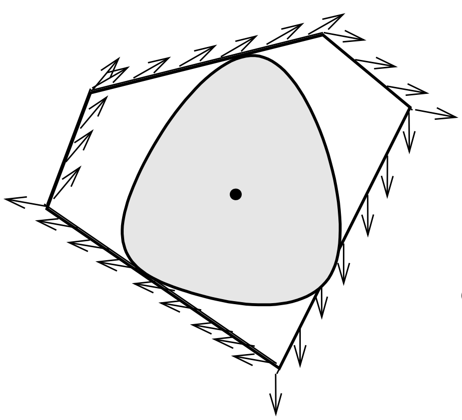
The Curl of rolling translations within an assembly

1. Continuous vector field



$$\mathsf{curl}(\mathbf{u}) = \lim_{V \to 0} \frac{1}{V} \oiint_S \mathbf{n} \times \mathbf{u} \, dS$$

2. Average rolling curl within a material cell



$$\mathsf{curl}(\mathbf{u}^{\mathbf{cell}}) = \frac{1}{V^{\mathbf{cell}}} \oiint_{S^{\mathbf{cell}}} \mathbf{n} \times \mathbf{u}^{\mathbf{roll}} \, dS$$

Particle curls