Multi-scale Phenomena in Granular Materials

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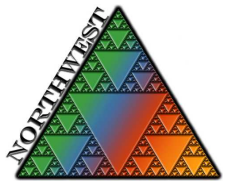
Workshop on Multi-scale Modeling of Materials

University of Puget Sound, Tacoma, WA



May 25–30, 2006

Outline

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Introduction

Granular materials:

- Assemblies of particles
- Bulk behavior is a consequences of particle interactions
- The “micro-scale” is well defined: individual particles
- The good news: amenable to direct, computational simulation
- The bad news: bulk behavior is complex, with multiple domains of behavior.

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Behavior domains

Domains of granular behavior:

Slow deformation	or	Rapid flow
Small-strain behavior	or	Large-strain behavior
Unbonded particles	or	Bonded particles
Contact interaction	or	Long-range interaction
Soft particles	or	Hard particles
Single phase	or	Multiple phases
2D	or	3D

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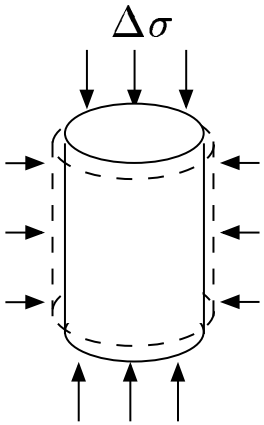
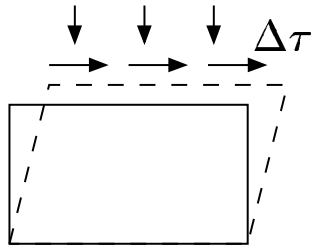
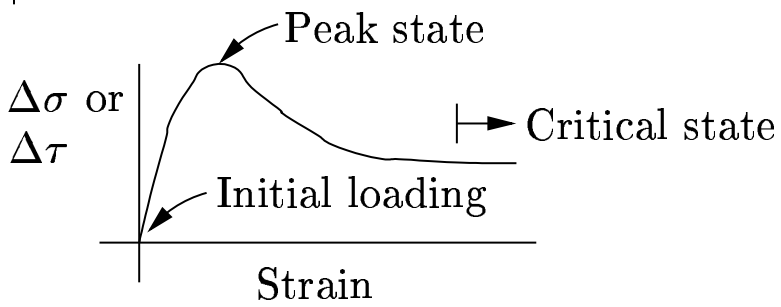
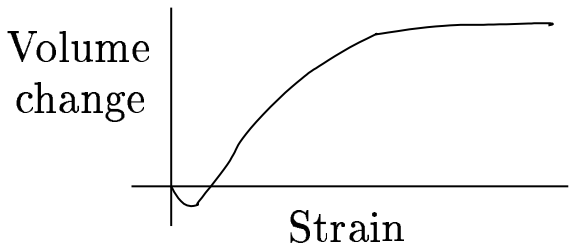
Typical macro-scale behavior (slow loading)









Outline

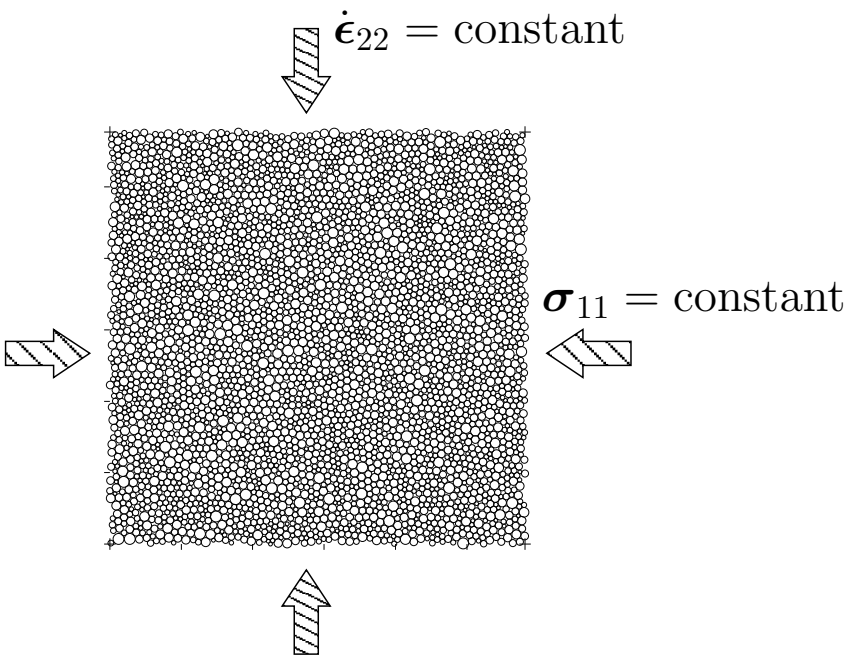
Internal behavior — localization

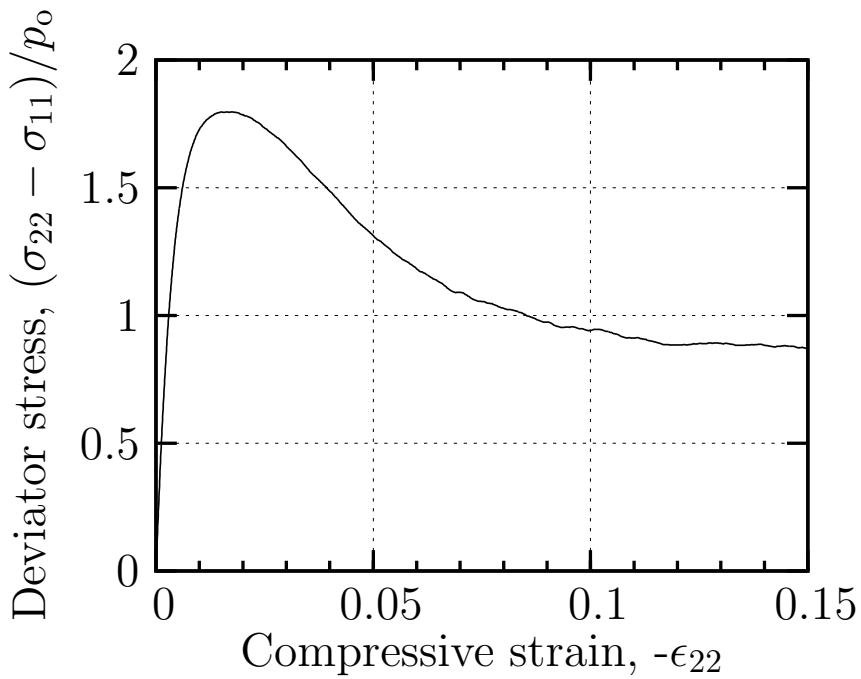
Scale-dependent, localization phenomena

- Force chains
- Circulation cells
- Rotation chains
- Micro-bands
- Dilation clusters
- Size-dependent crushing
- Shear bands and faulting

2D simulations

Biaxial compression of 4096 disks:





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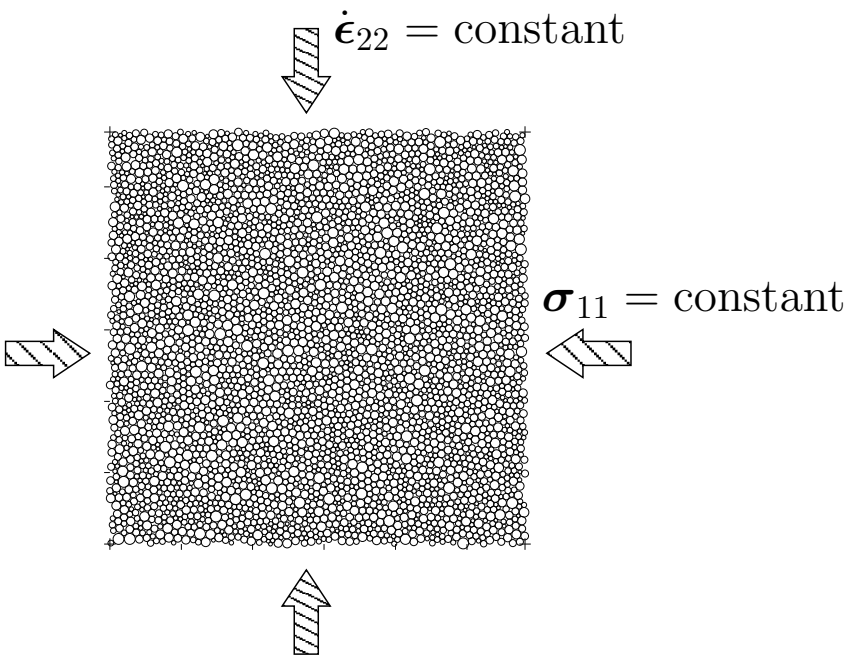
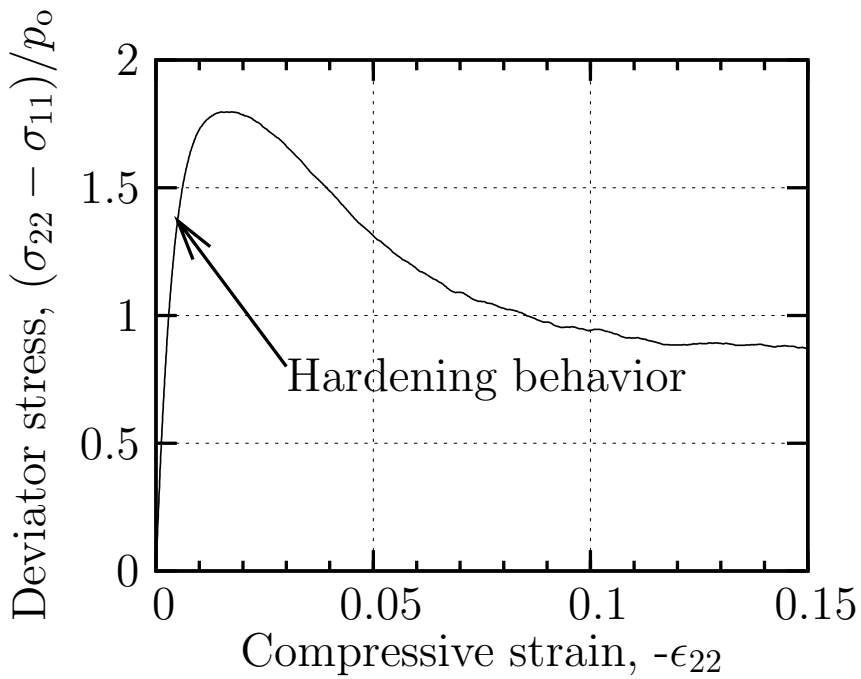
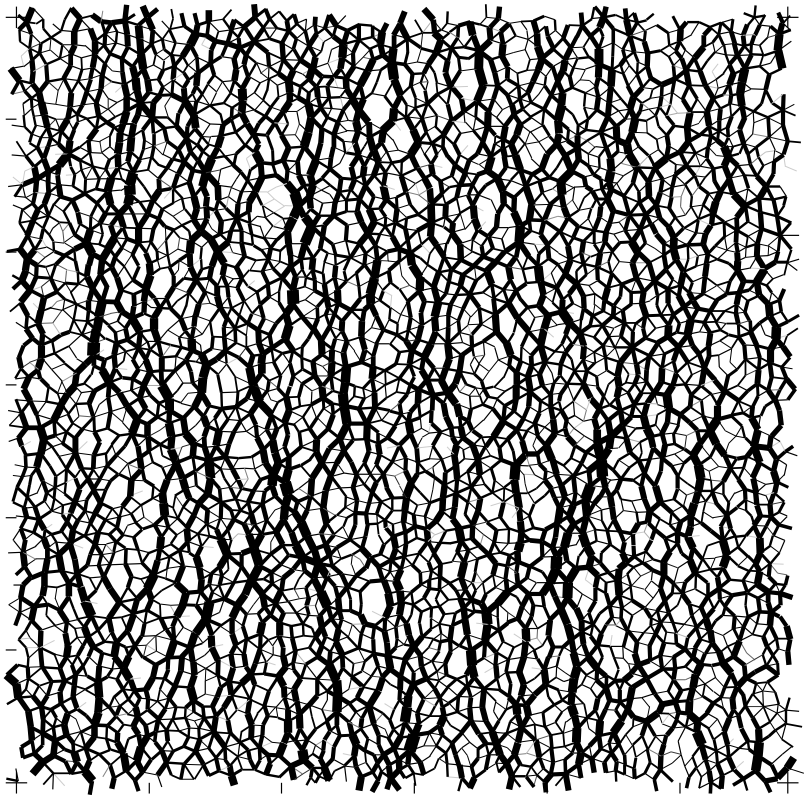
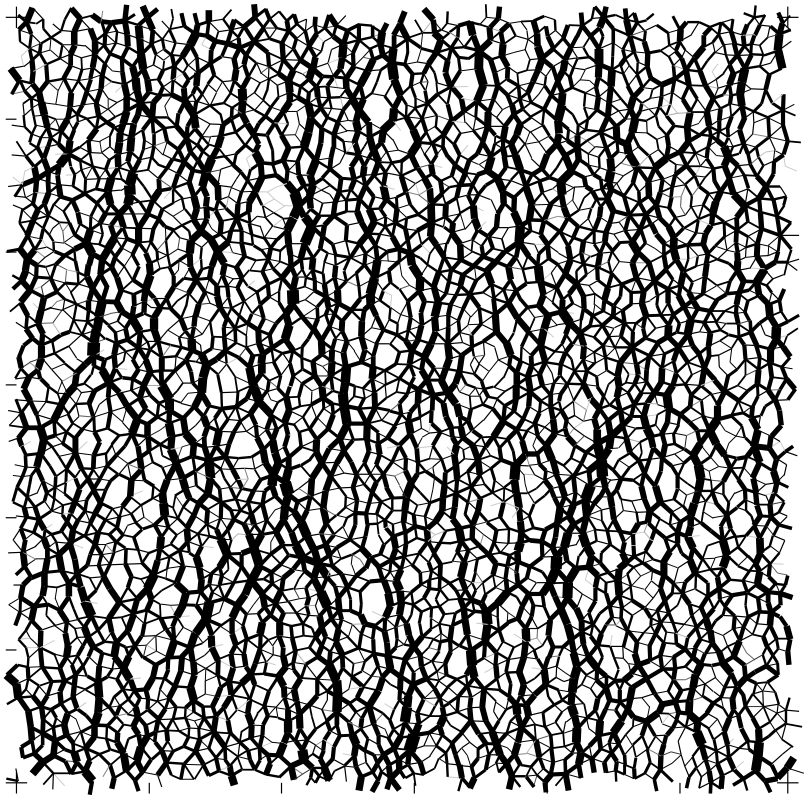


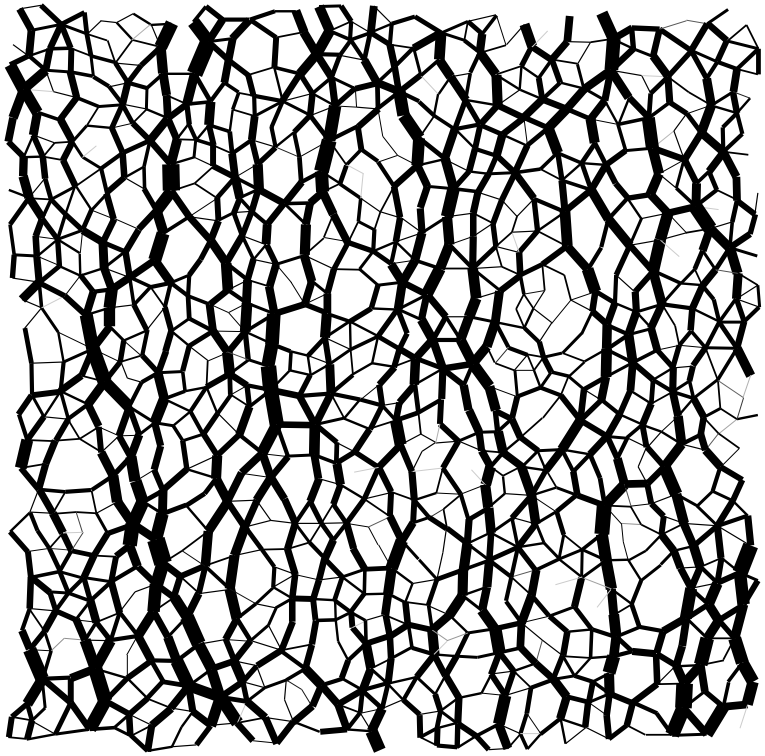


Force chains

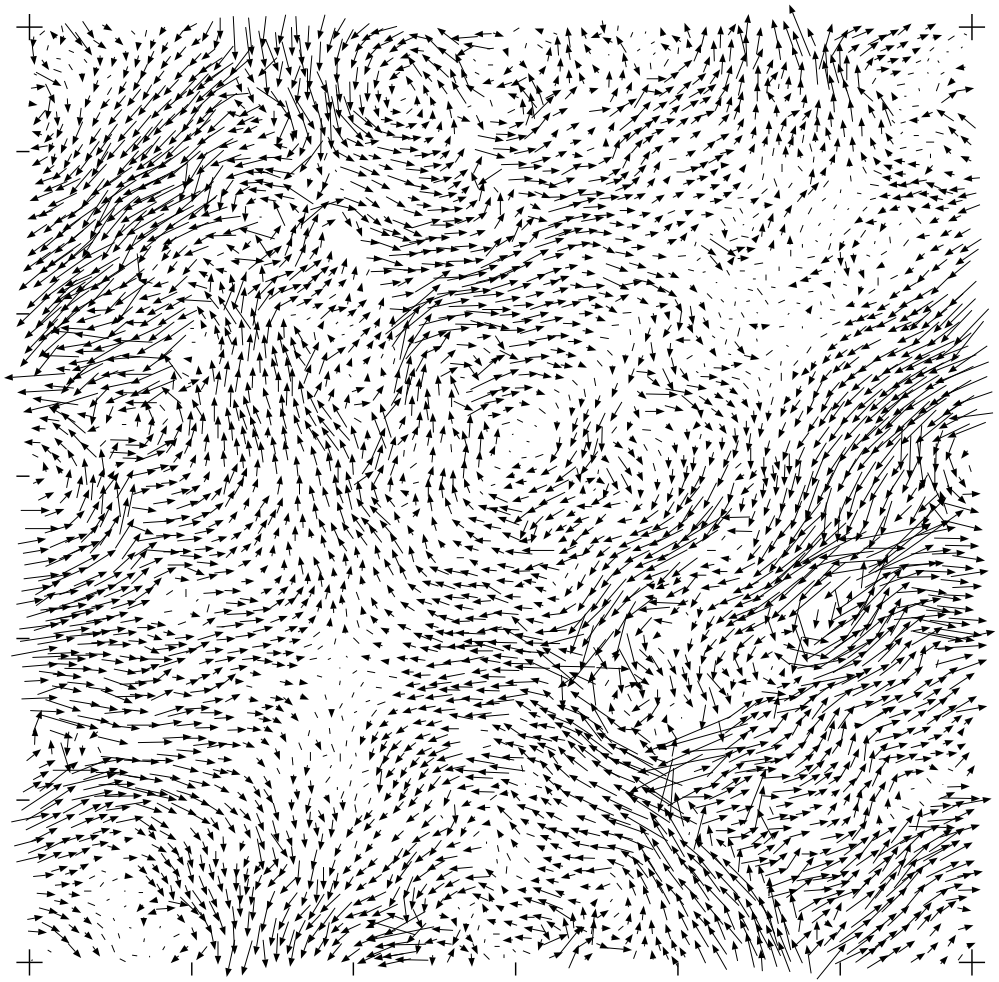


Force chains

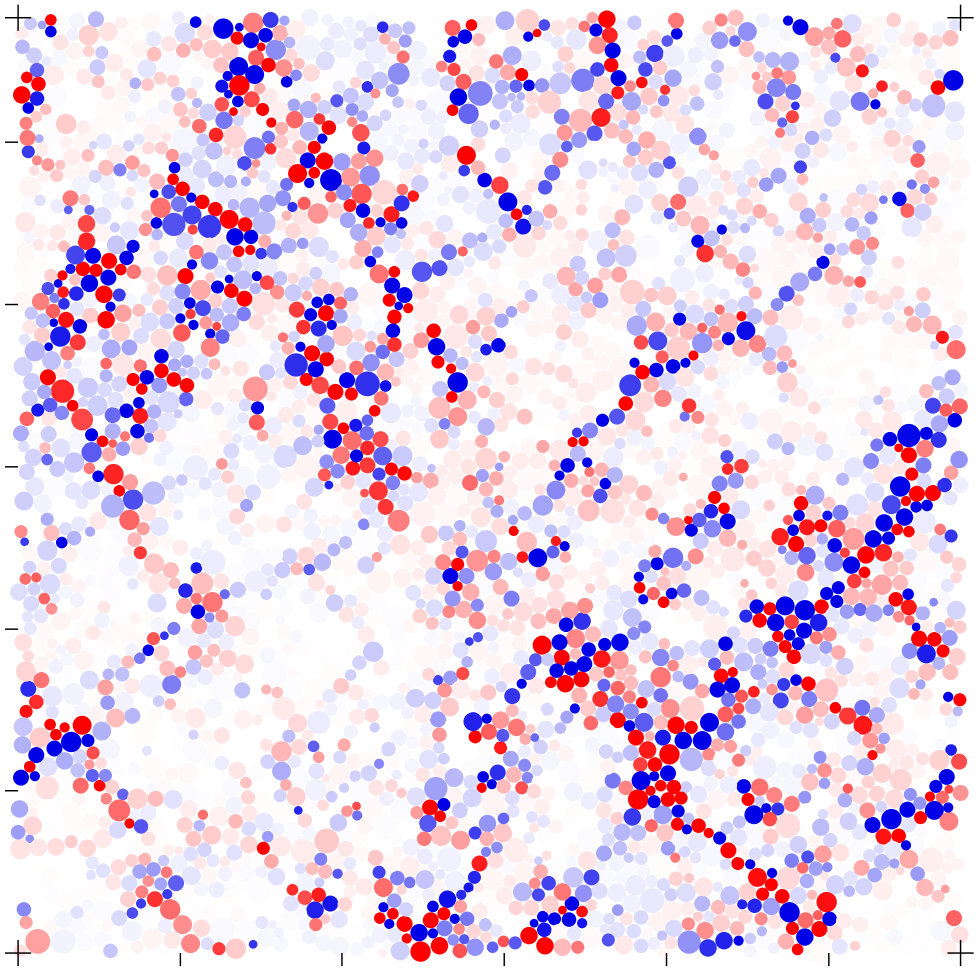




Circulation cells



Rotation chains

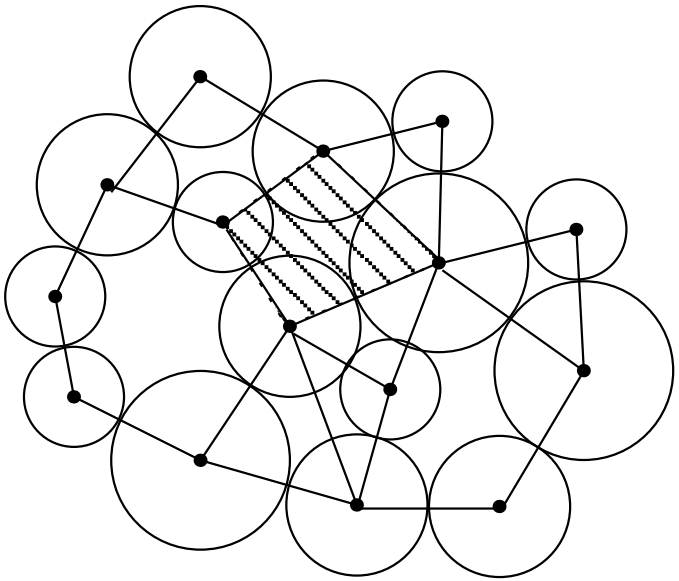


Internal behavior — localization, cont.

Localization phenomena

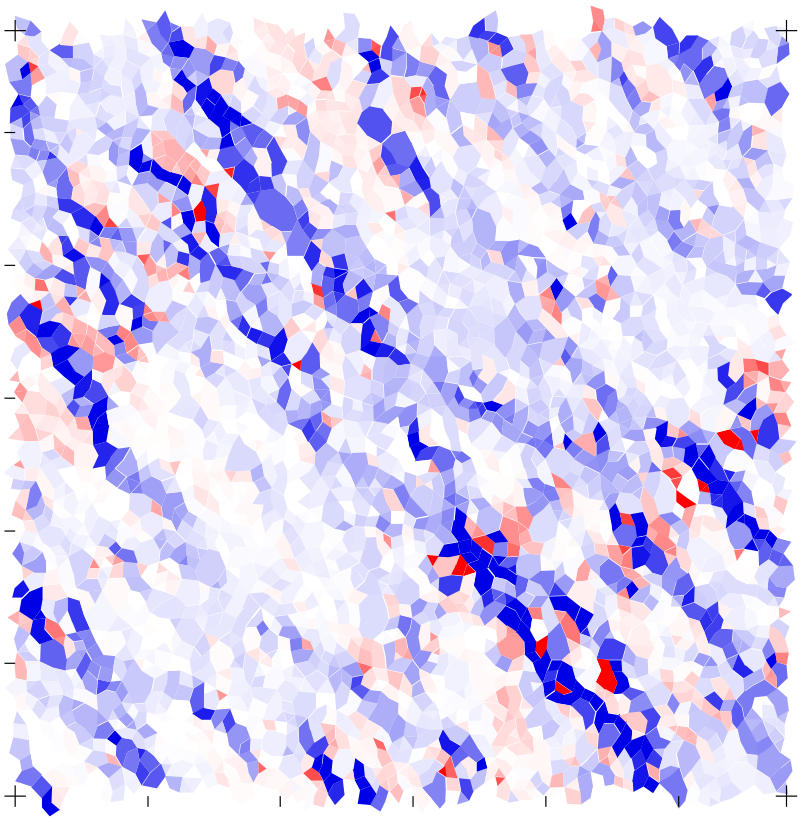
- Force chains
- Circulation cells
- Rotation chains
- Micro-bands
- Dilation clusters
- Size-dependent crushing
- Shear bands and faulting

Particle graph — 2D

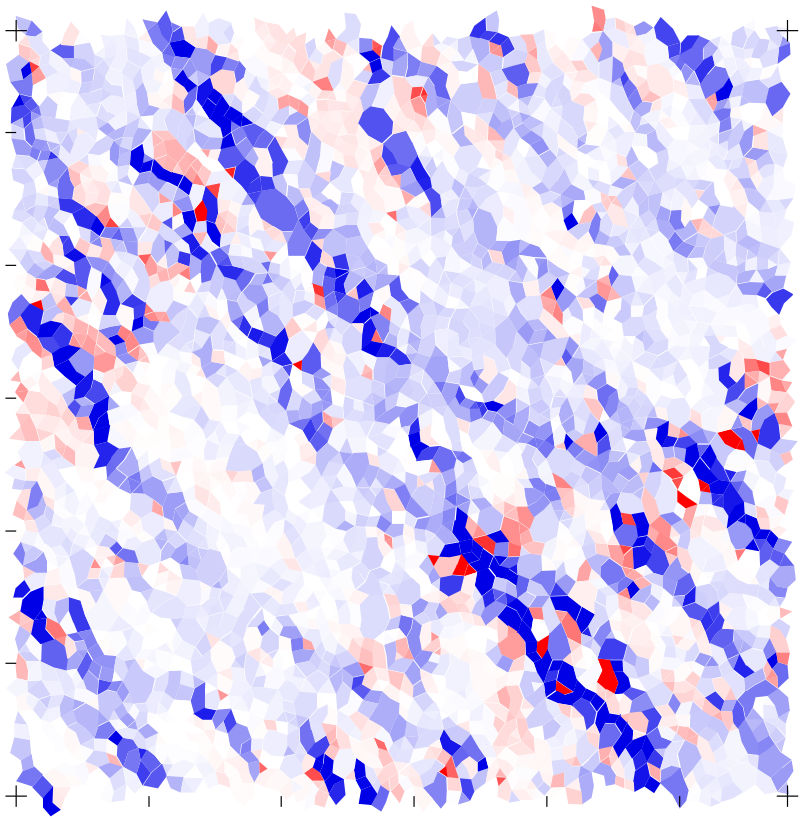


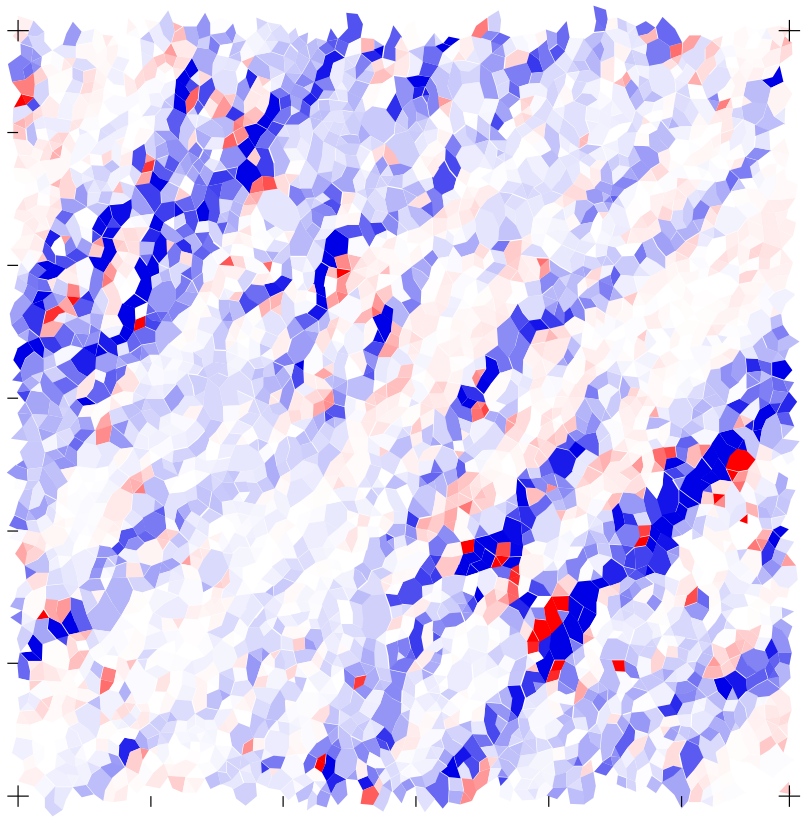
vertices → particles
edges → contacts
faces → void cells

Micro-bands

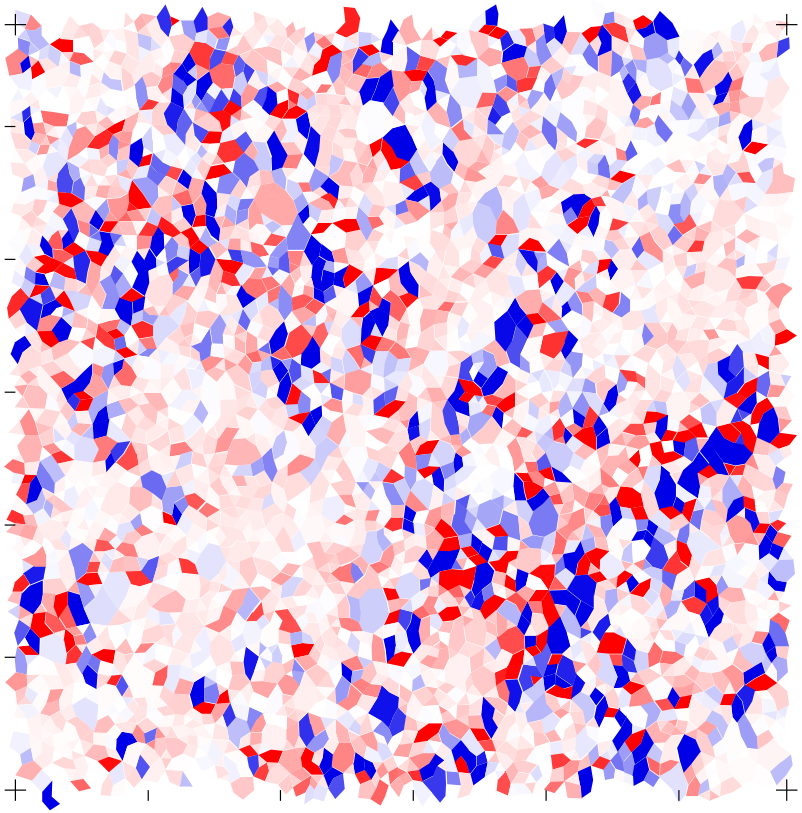


Micro-bands





Dilation clusters



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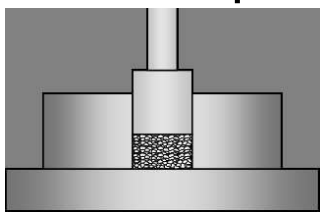
Size-dependent crushing

Uniaxial compression of an embedded agglomerate
1MPa normal pressure



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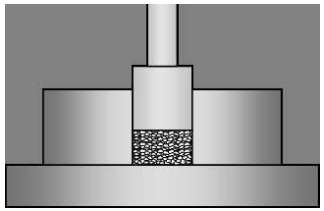
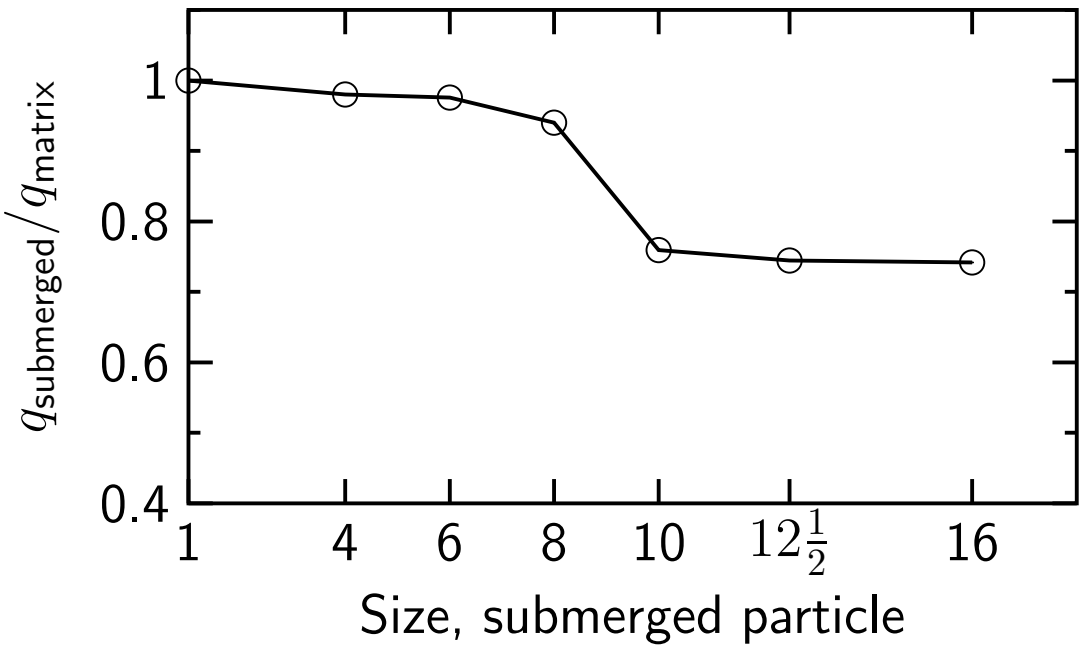


Size ratio ≈ 3.5

Size ratio ≈ 10

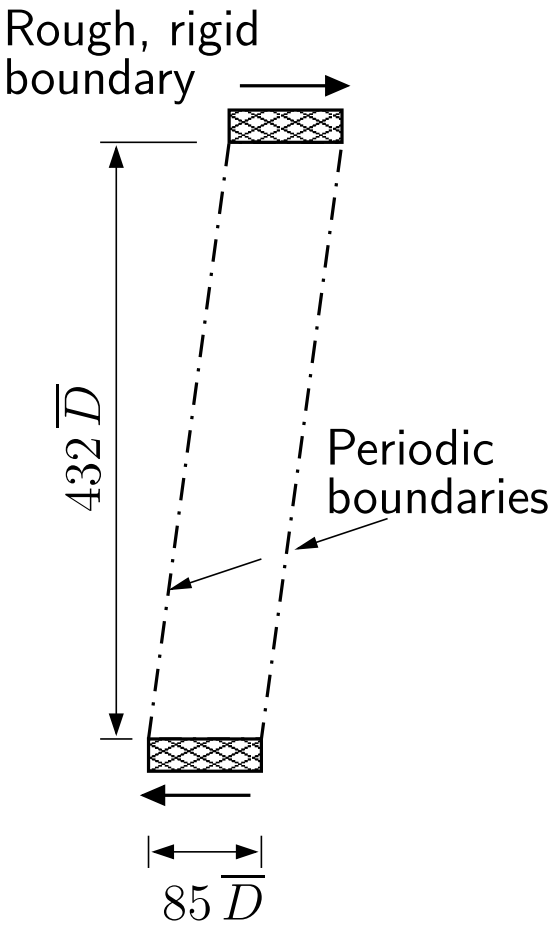
Size-dependent crushing

Crushing tendency vs. Size of embedded particle



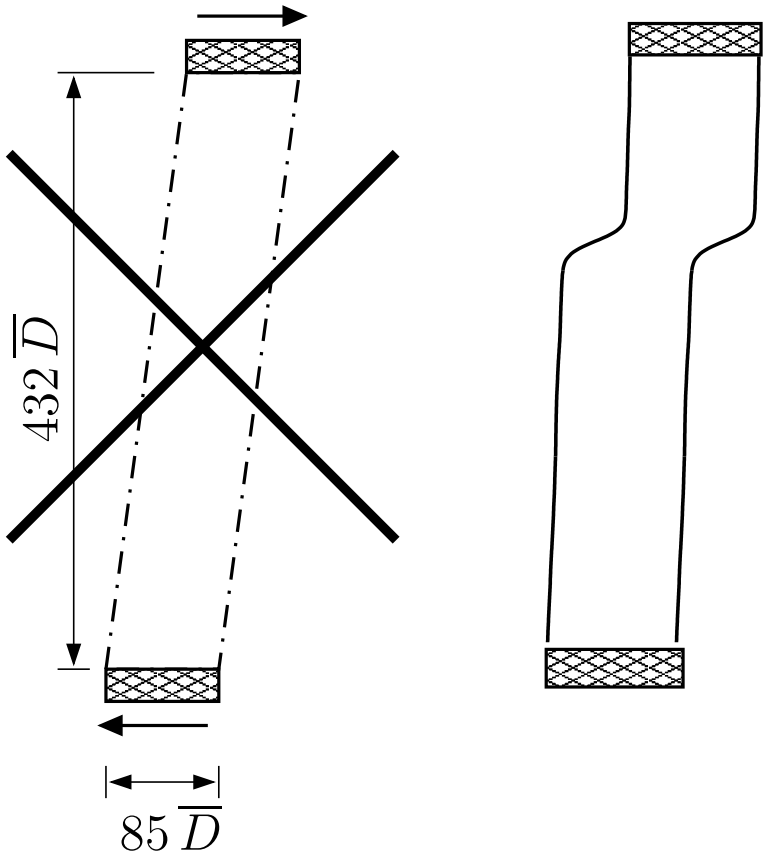
Shear band localization

Localization in 40,500 disks — DEM simulation



Shear band localization

40,500 disks — Localized shearing



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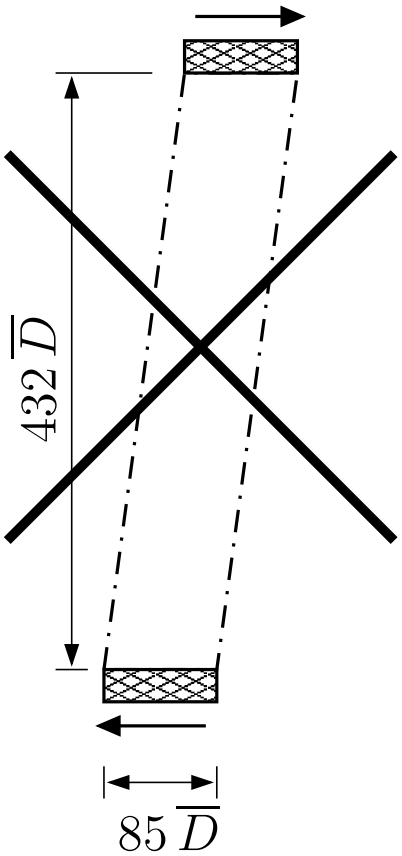
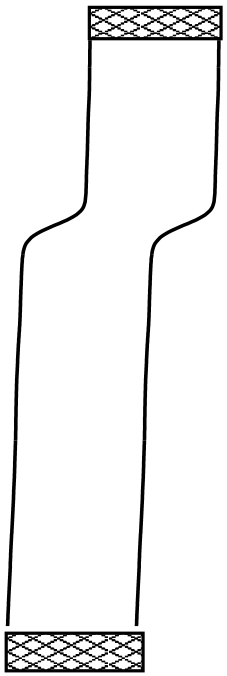
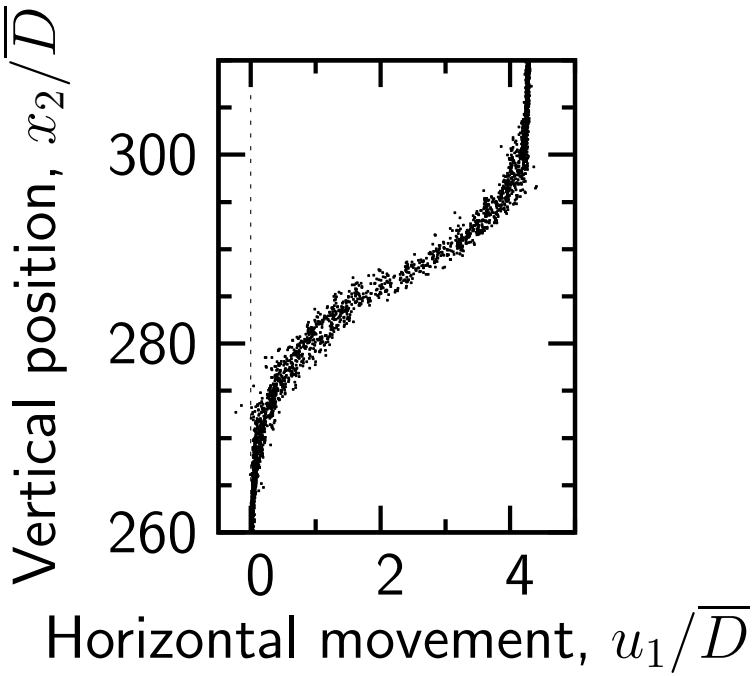
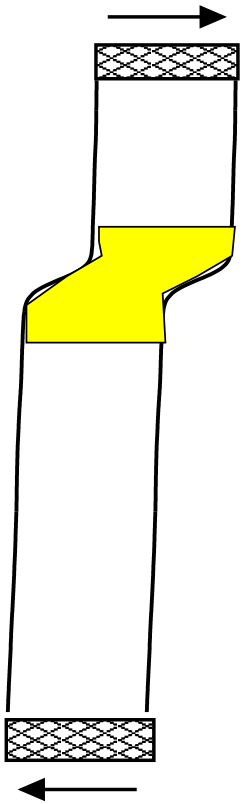


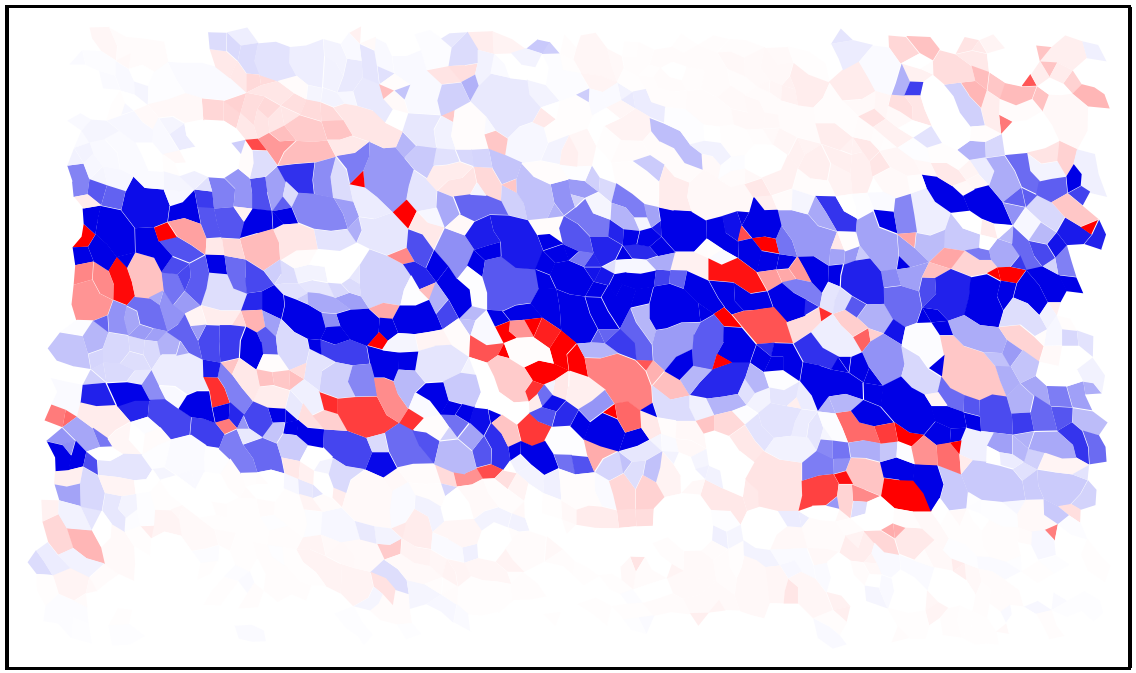




Shear band localization

Localization inside of a shear band





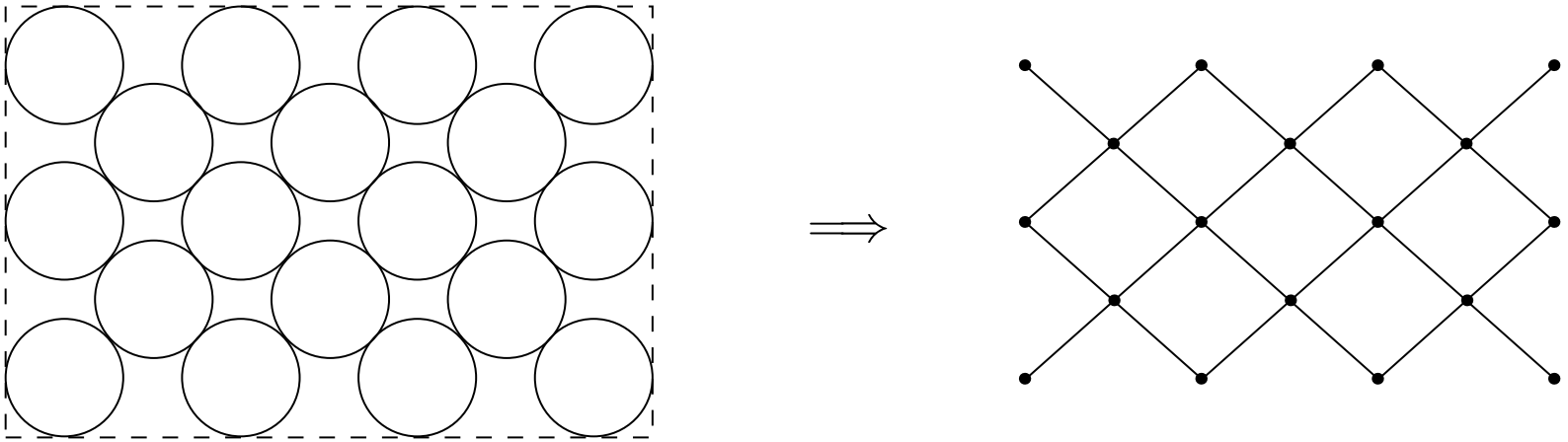
Outline

Multi-scale modeling

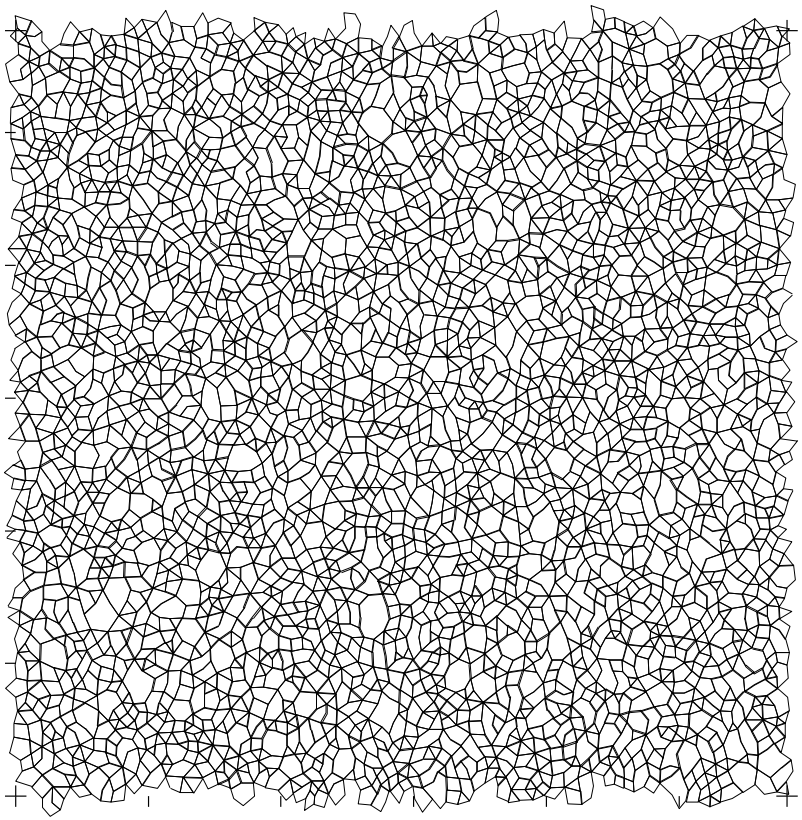
Obstacles to the transition from micro to larger scales:

- Complex micro-topology
- Particle rotations
- Heterogeneous micro-scale deformation and stress
- Strength heterogeneity
- Misconceptions of friction

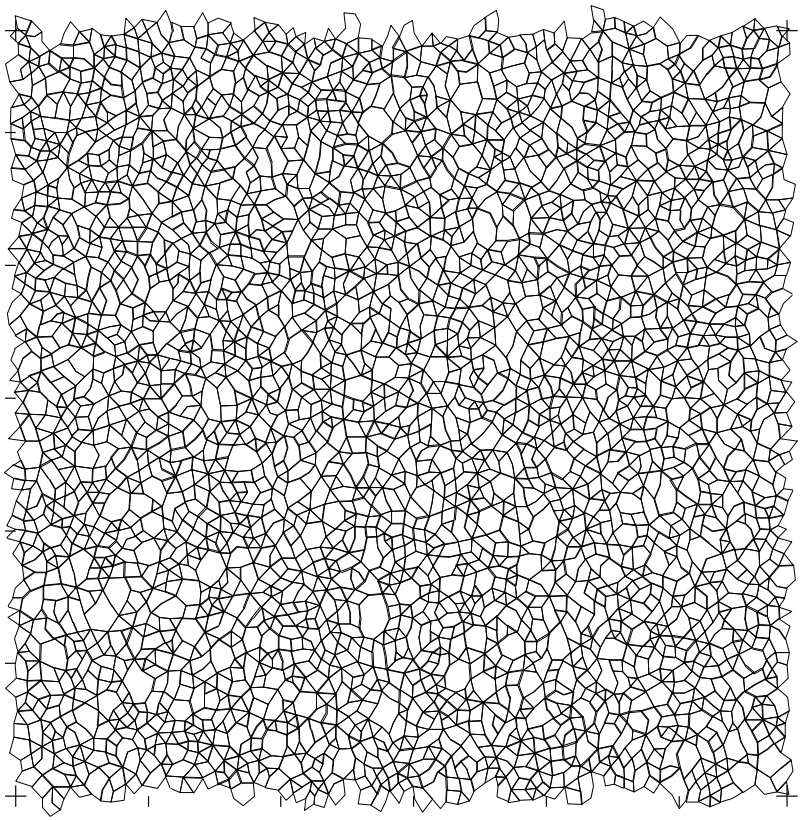
Complex micro-topology

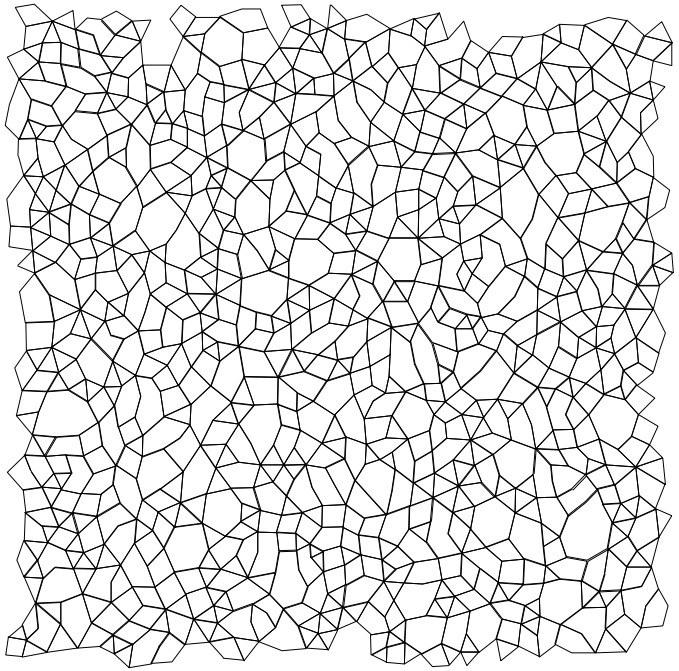


Complex micro-topology



Complex micro-topology





Obstacles to multi-scale modeling

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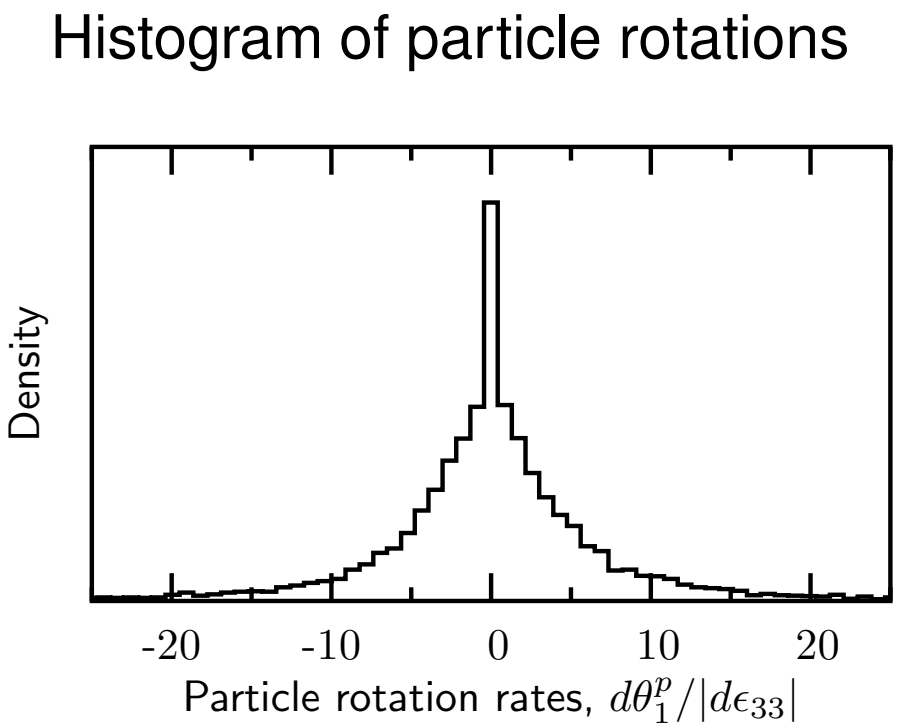
Particle rotation complications

Particle rotations complicate the transition from micro to macro:

1) Particle rotations are large
2) Particle rotations soften/weaken the material response
3) Particle rotations are patterned *and* spatially organized

Particle rotations are large



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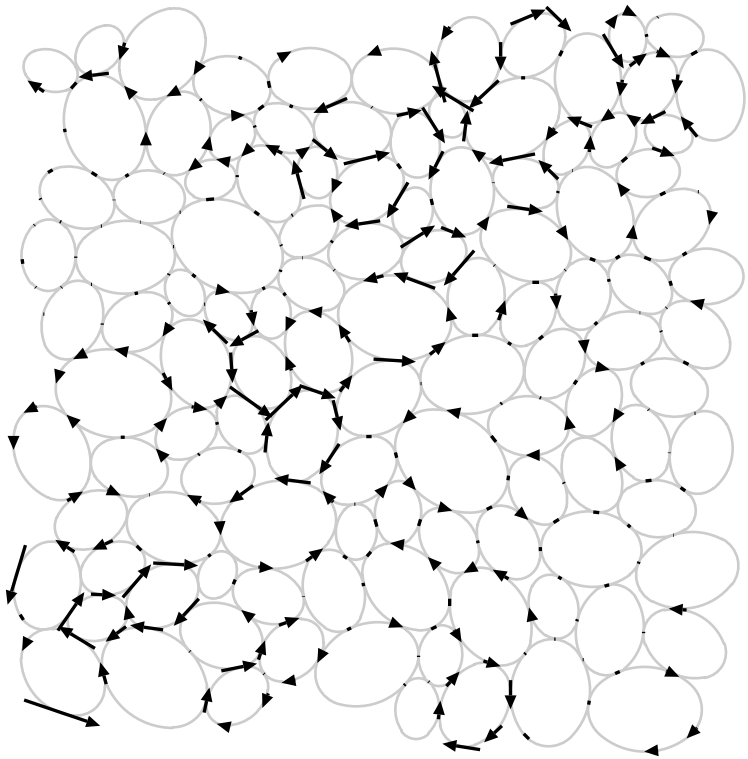
Particle rotations are spatially organized

1) Particles tend to “roll”

2) Particle rolling is spatially organized



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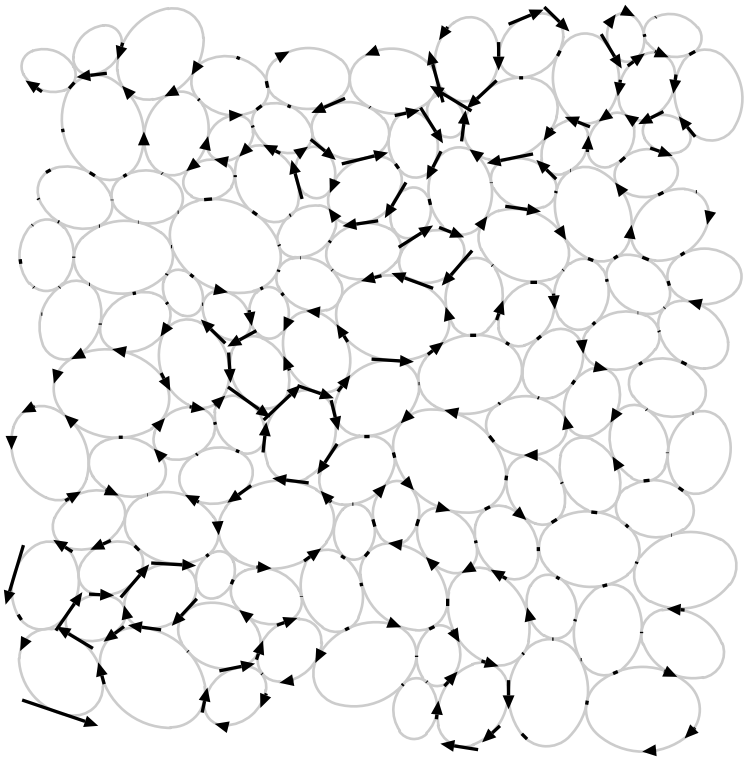

Obstacles to multi-scale modeling

Obstacles to the transition from micro to larger scales:

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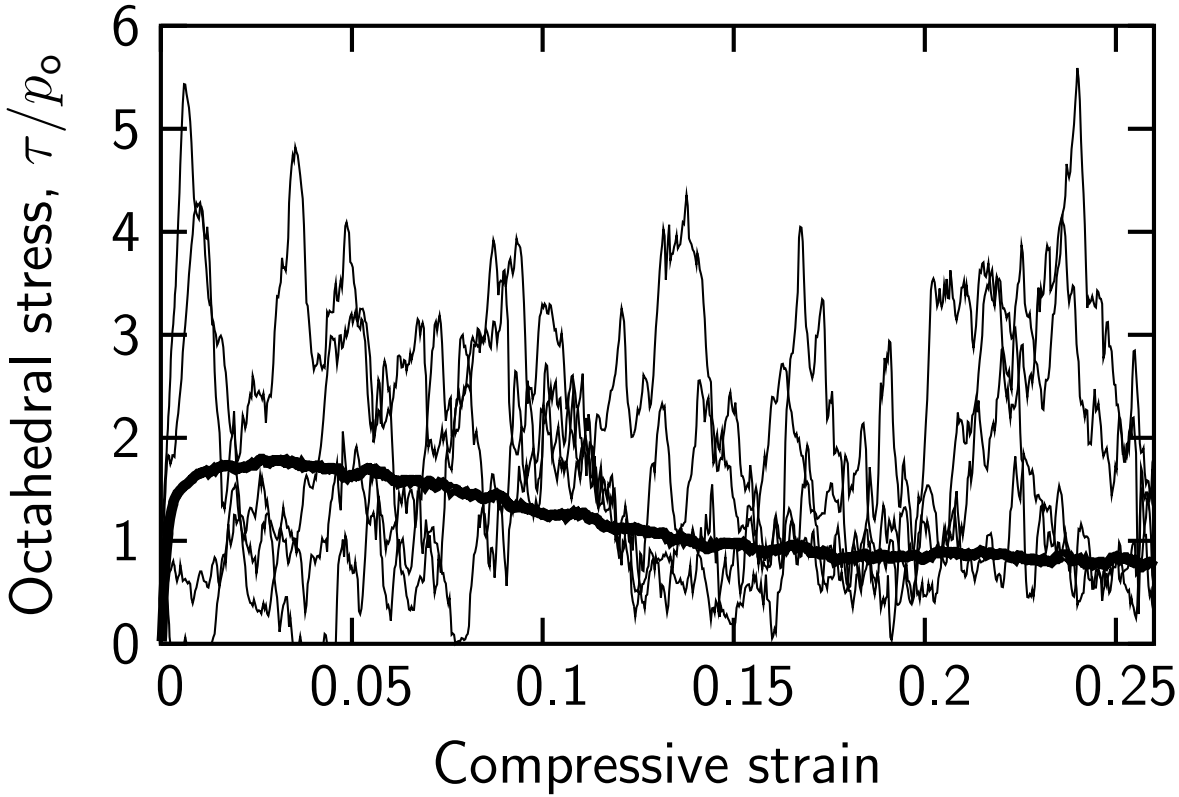
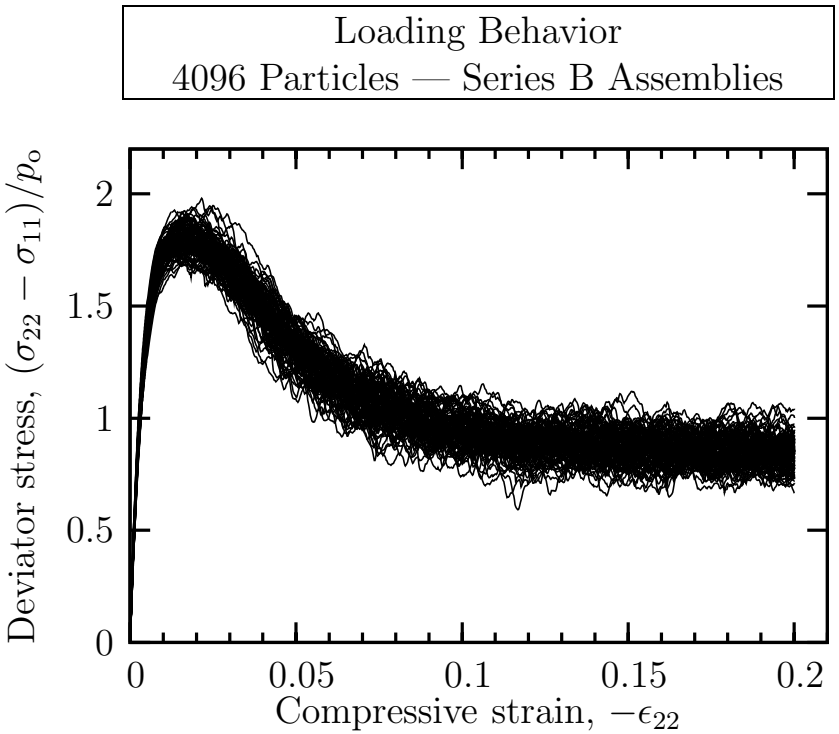
Heterogeneous micro-scale stress

Stresses in four individual particles
&
Average stress among 4096 particles



Strength heterogeneity

Tests on 100 different assemblies:



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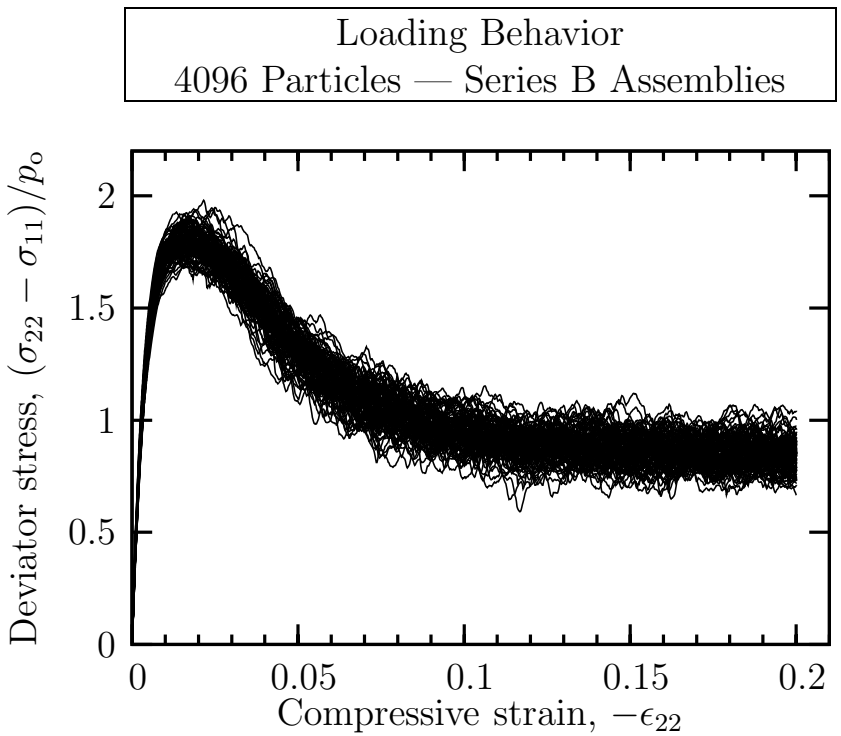

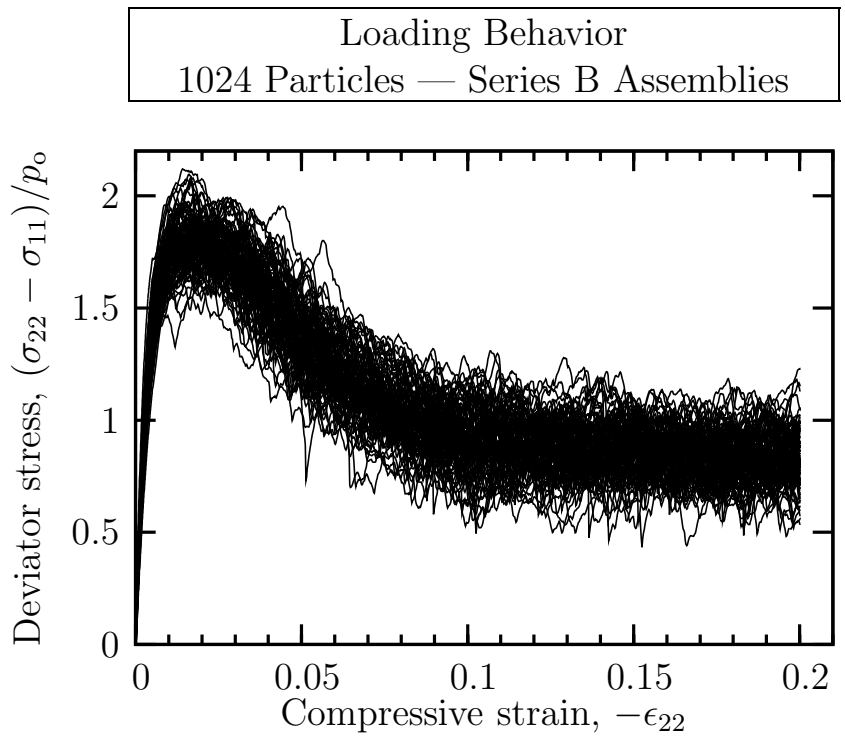

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Friction

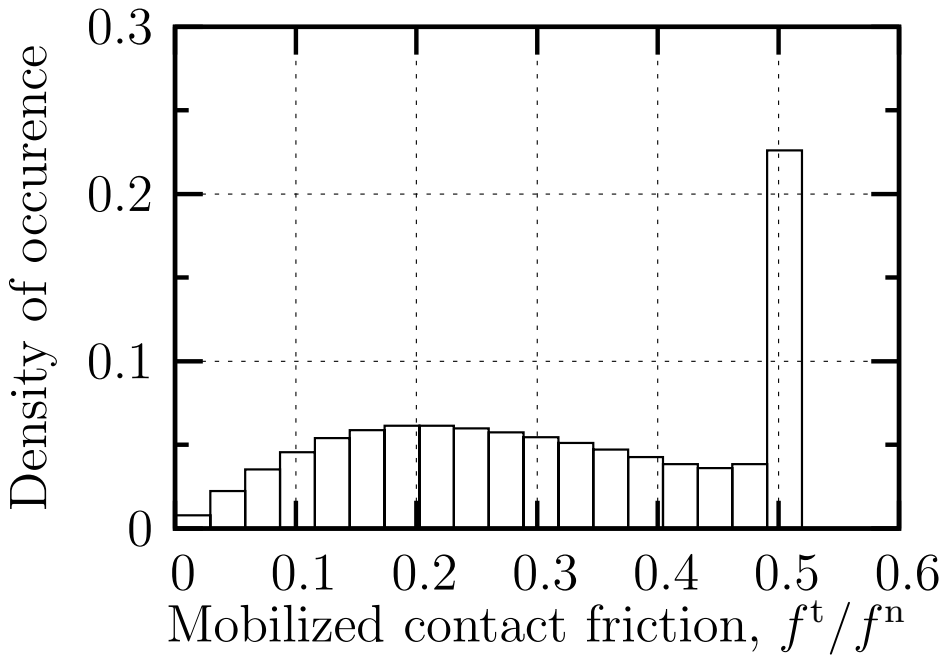
Misconceptions of friction

- Only a small minority of particle contacts are “slipping”
- Slipping contacts are spatially dispersed
- Micro- and macro-scale friction are poorly correlated

Friction

Histogram of contact forces:

$$\frac{\text{Tangential contact force}}{\text{Normal contact force}}$$



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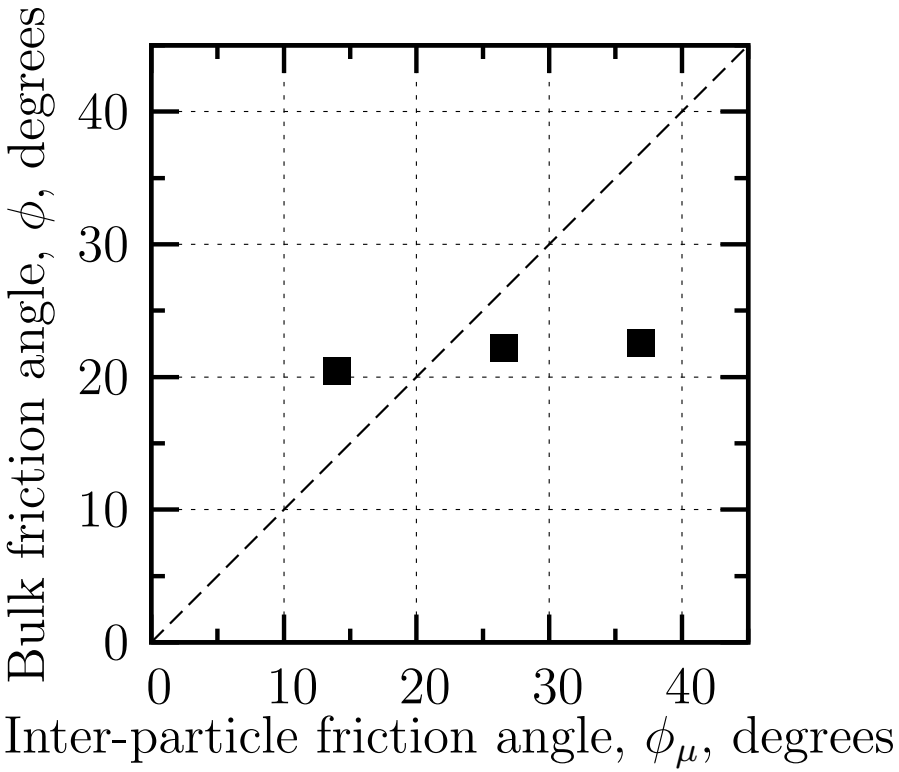
Friction

Macro-scale vs. contact friction



Outline

Strain gradient dependent models

Simple materials:

$$\boldsymbol{\sigma} = f(\boldsymbol{\epsilon},\ \text{material parameters})$$

Gradient-dependent materials:

$$\boldsymbol{\sigma} = f(\epsilon,\ \partial\epsilon/\partial\mathbf{x},\ \partial^2\epsilon/\partial\mathbf{x}^2,\ \ldots,\ \text{material parameters})$$

Are granular materials gradient-dependent?

Strain gradient dependent models

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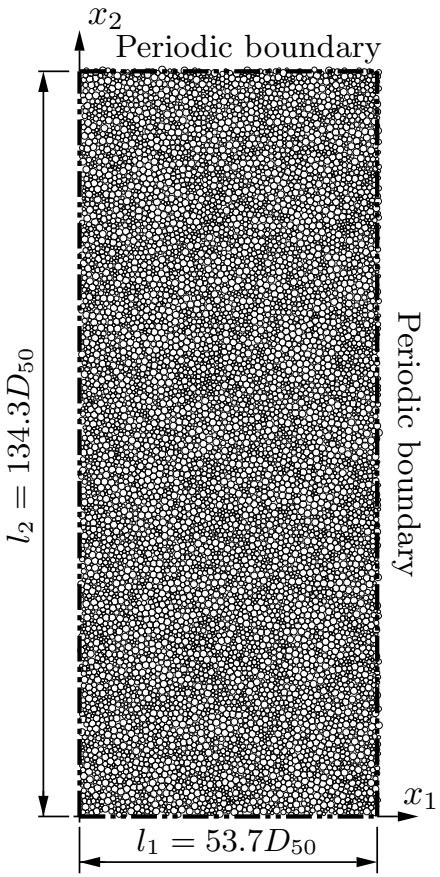
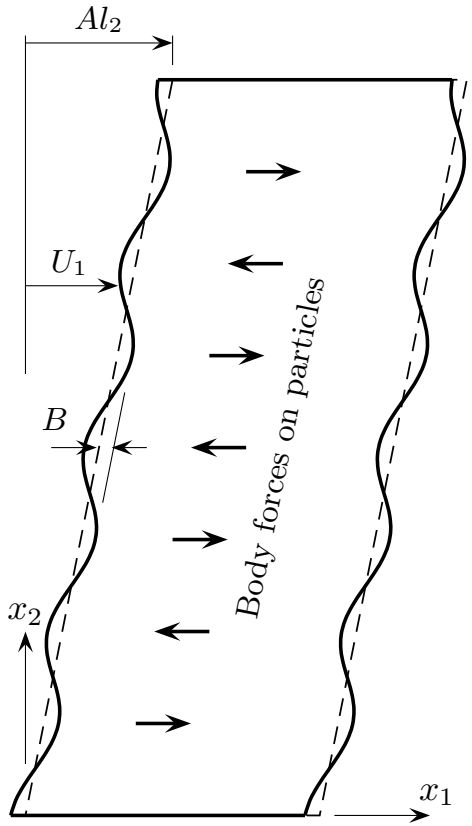
Are granular materials gradient-dependent?

Strain gradient dependent models

Experiments to determine strain gradient-dependence



Undeformed assembly

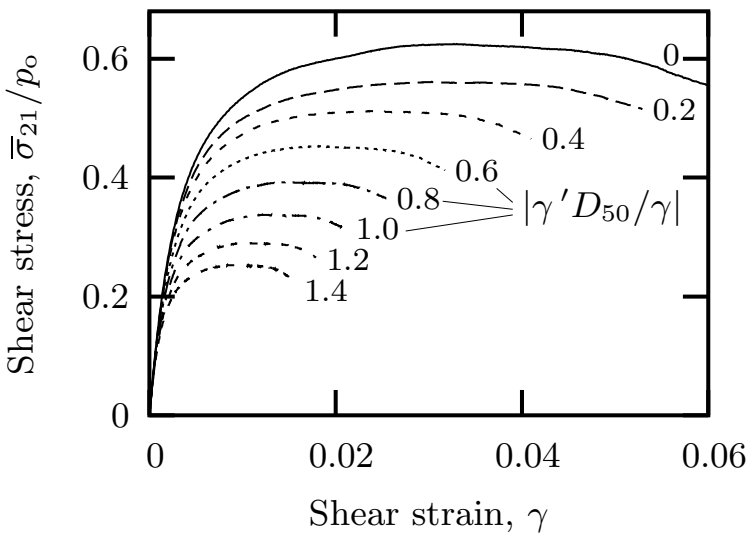


Boundary displacements

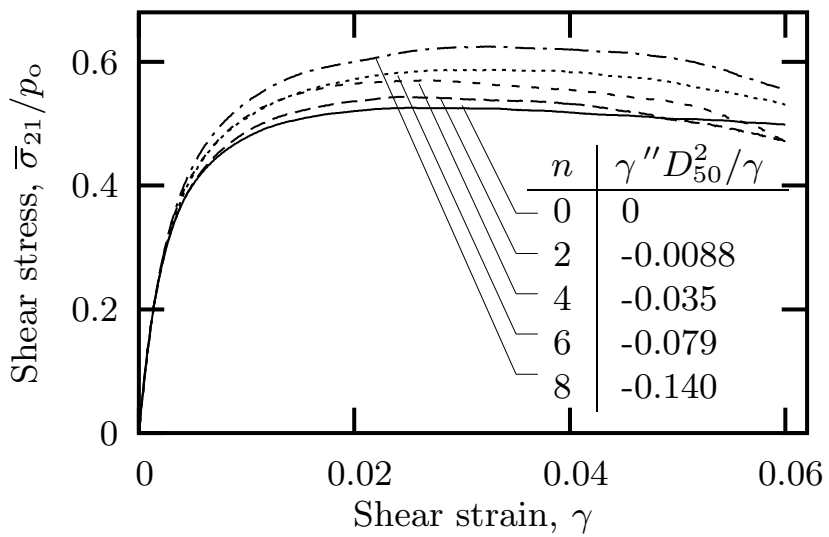
Strain gradient dependent models

Experimental results:

Effect of the 1st gradient, $d\epsilon_{12}/dx_2$



Effect of the 2nd gradient, $d^2\epsilon_{12}/dx_2^2$



Non-local models

In non-local models, stress at a point depends on an averaged strain $\bar{\epsilon}$ within a region $\mathcal{B}$ around the point:

$$\boldsymbol{\sigma} = f(\bar{\epsilon},\ \text{material parameters})$$

where

$$\bar{\epsilon} = \int_{\mathcal{B}} \boldsymbol{\Phi}(\mathbf{x} - \mathbf{x}')\epsilon(\mathbf{x}')\, dV$$

The weighting kernel $\boldsymbol{\Phi}$ is usually a norm $\longrightarrow |\mathbf{x} - \mathbf{x}'|$.

Non-local models

An example non-local model:

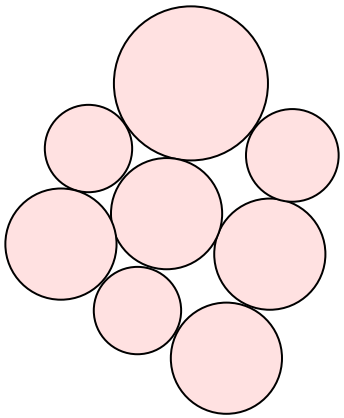
$$\overline{\epsilon} = \int_{\mathcal{B}} \left(\frac{1}{\ell\sqrt{\pi}} e^{\left(\frac{|\mathbf{x}-\mathbf{x}'|}{\ell}\right)^2} \right) \epsilon(\mathbf{x}')\, dV$$

Unfortunate deficiencies:

1) When applied to experimental results, the length scale ℓ is abnormally small: less than 2 particle diameters.
2) Experimental results suggest that the kernel Φ must be asymmetric.

Discrete stiffness models

Incremental stiffness of a particle assembly:



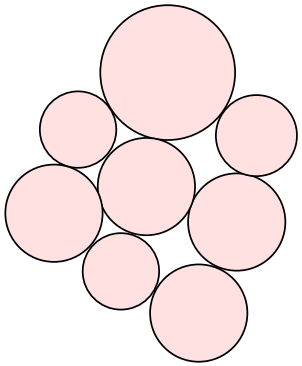
Particle movements

$$\left[\frac{d\mathbf{u}}{d\theta}\right] \Rightarrow \left[\frac{d\mathbf{f}}{d\mathbf{m}}\right]$$

External forces & moments

Discrete stiffness models

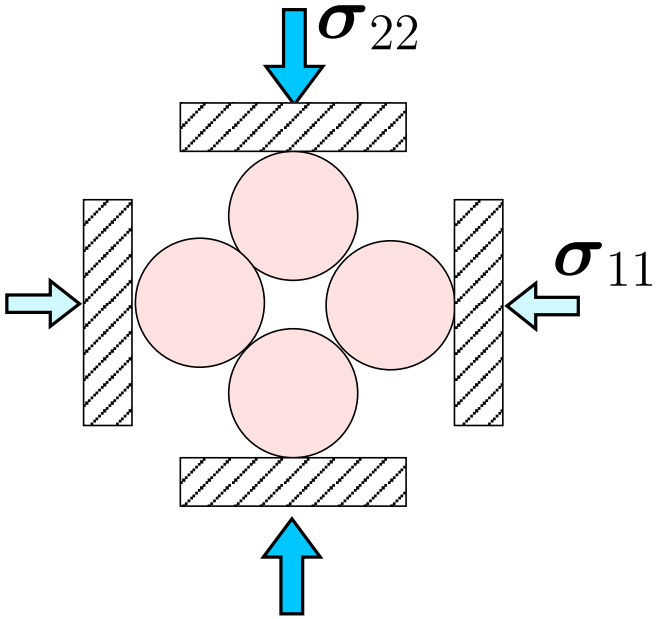
Incremental stiffness matrix:



$$\begin{bmatrix} \\ \mathbf{K} \\ \\ \end{bmatrix} \begin{bmatrix} d\mathbf{u} \\ \hline\hline d\theta \end{bmatrix} = \begin{bmatrix} d\mathbf{f} \\ \hline\hline d\mathbf{m} \end{bmatrix}$$

Discrete stiffness models — example

Instability of 4 particles?

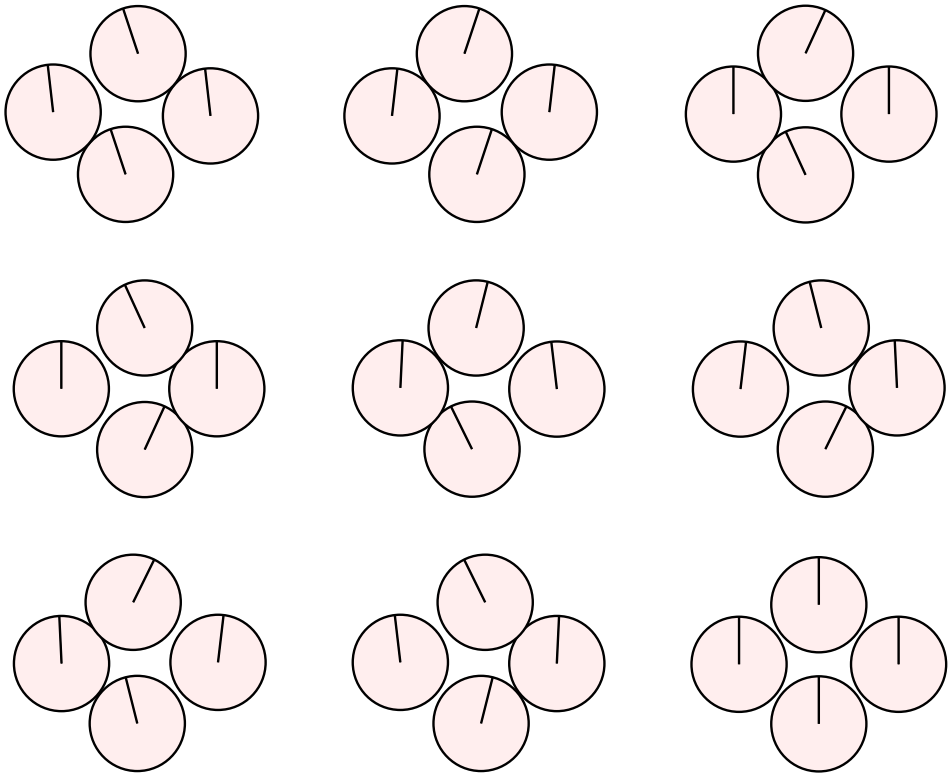


Search for unstable eigenmodes:

$$\lambda < 0 \quad \Rightarrow \quad \delta^2 W < 0$$

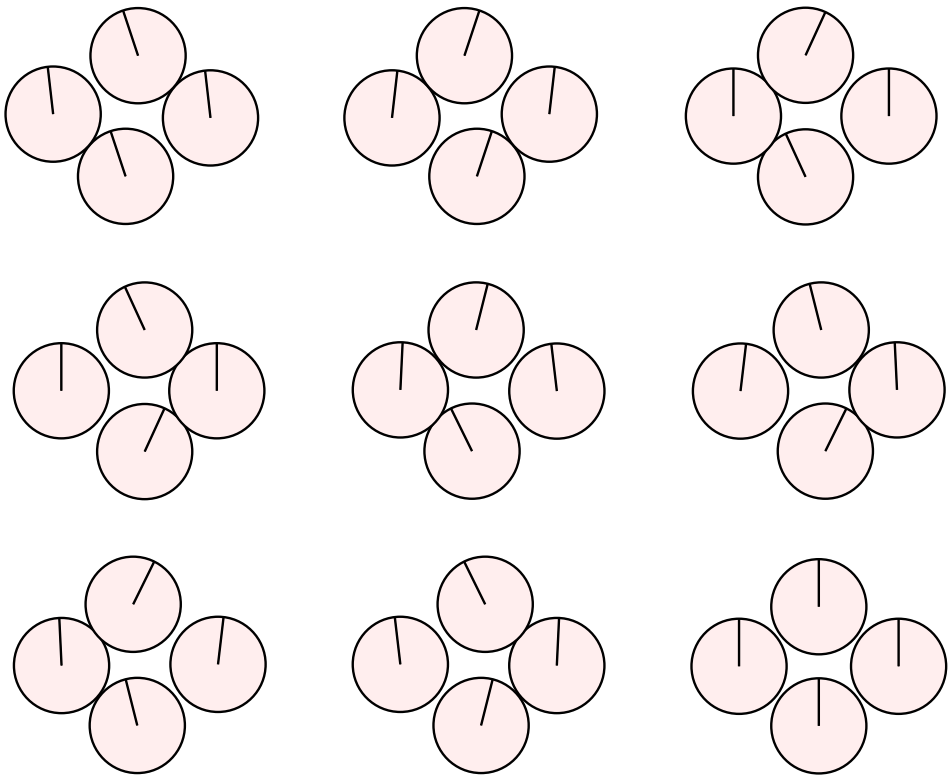
Discrete stiffness models — example

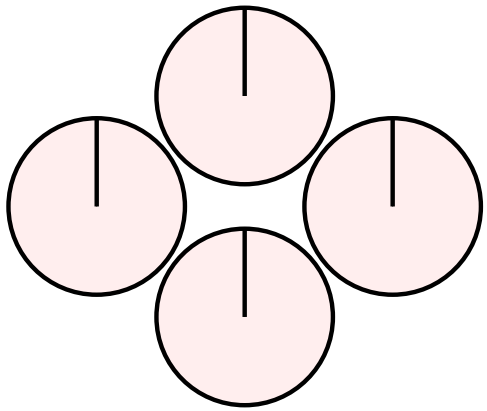
9 Unstable eigenmodes, with $\lambda < 0$:



Discrete stiffness models — example

9 Unstable eigenmodes, with $\lambda < 0$:





Questions?