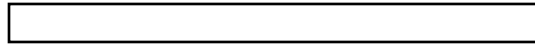


In FEA, each element has an “ideal” shape. For example, a 2D element’s ideal shape is typically a square. The Jacobian Matrix “maps” the actual element locally into the global stiffness matrix and part of that transformation involves mapping non-square elements into a ‘square.’ The more the element deviates from being square (the larger the aspect ratio, and the more an internal angle deviates from 90 degrees) the less precise the transformation.

Poor transformation:

High aspect ratio:



Poor ‘shape’ (internal angle approaching 180 deg):



The following is from a discussion group thread that explains Jacobian about as well as can be explained in one page:

<https://www.eng-tips.com/viewthread.cfm?qid=310852>

*In the Finite Element Method, an element's Jacobian Matrix relates the quantities wrote in the natural coordinate space and the real space. The bigger the element is distorted in comparison with a ideal shape element, the worse will be the transformation*

*of the quantities from the natural space to the real space. In fact, it means that the distortion on the element shape will introduce error in the mathematical trasformation from natural space to the real space.*

*In a FE Software, the Jacobian (also called Jacobian Ratio) is a measure of the deviation of a given element from an ideally shaped element. The jacobian value ranges from -1.0 to 1.0, where 1.0 represents a perfectly shaped element. The ideal shape for an element depends on the element type. The check is performed by mapping an ideal element in parametric coordinates onto the actual element defined in global coordinates. For example, the coordinates of the corners of an ideal quad element in parametric coordinates are (-1,-1), (1,-1), (1,1), and (-1,1). The determinant of the jacobian relates the local stretching of the parametric space required to fit it onto global coordinate space. The picture attatched ilustrates an real QUAD4 element, frequently found in a mesh, and the ideal QUAD4 element.*

*An element's Jacobian Matrix is a square matrix which have dimension of 1x1 for 1D elements, 2x2 2D elements and 3x3 for 3D elements. The terms inside this matrix are functions that depends on the parametric coordinates "r", "s" and "t" in most cases. Therefore, the determinant of an element's Jacobian Matrix will depend on wich points the FE package will use in the calculation. Many softwares (HyperMesh, for exemple) evaluates the determinant of the Jacobian Matrix at each of the element's integration points (also called Gauss points). For example, one element which has 4 integration points will have 4 values of determinant. The Jacobian in the Finite Element vocabulary is defined as the ratio*

*between the smallest and the largest value of the Jacobian Matrix determinant. Therefore, the Jacobian is always between 0 and 1.*

*Different solver codes use different patterns of integration points, and the same solver may use different patterns for different formulations of the same element configuration. Some solvers use the element nodes coordinates to calculate the Jacobian, some use the integration points coordinates and some use the former for some elements and the later for others. It's also possible to choose which kind of points (integration points or nodes/corners) the FE software should use.*

*Both first and second order solid and plate elements are included in this check; however, first order tria and tetra elements always have a jacobian value of 1.0 because the terms inside Jacobian Matrix are constants.*

*COMMENT: There are FE softwares that calculate the Jacobian as the ratio between the largest and the smallest value of the Jacobian Matrix determinant. This is the case of ANSYS and COSMOS/SolidWork. In these softwares help, there are recommendations of the maximum allowable Jacobian value. These recommendations were created after exhaustive correlation tests made by the FEA specialists of the software company.*