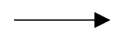
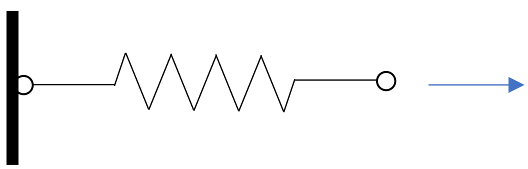
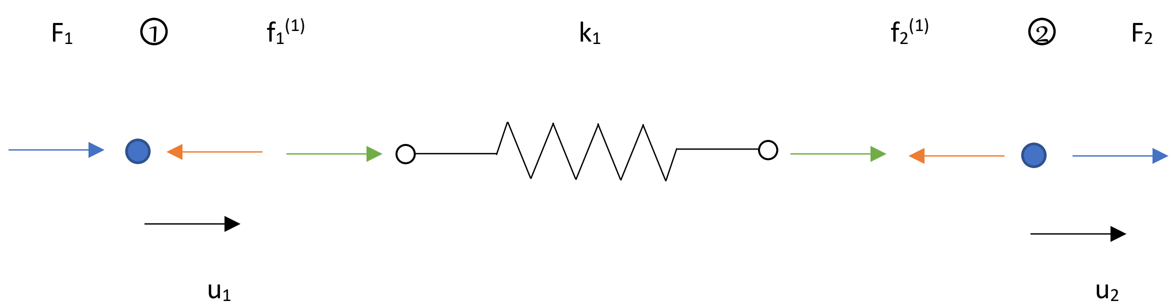
Nomenclature	**Arrow convention**
External forces are applied to nodes only (use capital letters). F_i is the external force applied to node i.	
Internal forces act on nodes or elements (equal and opposite). $f_i^{(j)}$ is the internal force acting on node i with respect to element j. $f_i^{(j)}$ internal force acting on a node $f_i^{(j)}$ internal force acting on an element (equal and opposite to $f_i^{(j)}$ on the node)	 
u_i is the displacement of node i	
k_i is the stiffness of element i.	
δ_i is the elongation (change in length) of the element	

"*element*" and "*spring*" are synonymous for 1-dimensional FEA.

GIVEN: spring attached on one end with a force on the other end.



FBD's of the spring:



Equilibrium:

Node 1: $\Sigma F = F_1 - f_1^{(1)} = 0$, therefore $F_1 = f_1^{(1)}$

Node 2: $F_2 = f_2^{(1)}$

Element 1 (the spring): $\Sigma F = f_1^{(1)} + f_2^{(1)} = 0$, therefore $f_1^{(1)} = -f_2^{(1)}$

$\delta_1 = u_2 - u_1$

Create system of equations:

$f_2^{(1)} = k_1\delta_1 = k_1 (u_2 - u_1) = - k_1u_1 + k_1u_2$

$f_1^{(1)} = - f_2^{(1)} = - k_1\delta_1 = k_1 (u_1 - u_2) = k_1u_1 - k_1u_2$

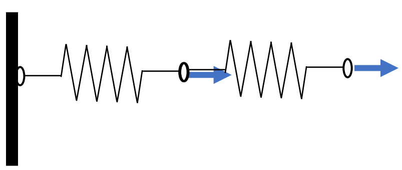
Assemble matrices:

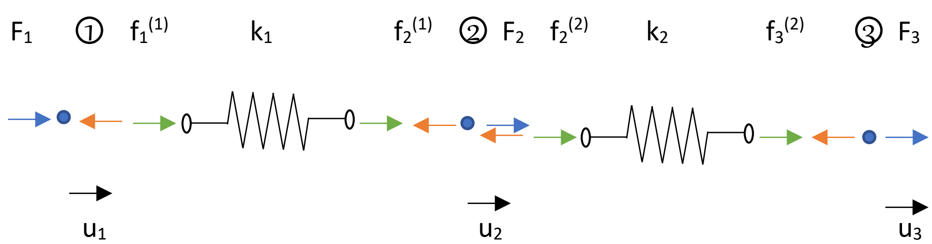
$$\begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

This is the "local" equation (for a single element). **F = ku**

Two springs…I mean, two elements: ready, set go….

Two forces are applied – one at the right end, one in between the two springs, plus the reaction at the wall. So 3 total applied forces:





Equilibrium:

Node 1: $F_1 = f_1^{(1)}$

Node 2: $F_2 = f_2^{(1)} + f_2^{(2)}$

Node 2: $F_3 = f_3^{(2)}$

Element 1: $f_1^{(1)} = -f_2^{(1)}$

Element 2: $f_2^{(2)} = -f_3^{(2)}$

Create system of equations:

$F_1 = f_1^{(1)} = -f_2^{(1)} = -k_1 (u_2 - u_1) = k_1u_1 - k_1u_2$

$F_3 = f_3^{(2)} = k_2 (u_3 - u_2) = - k_2u_2 + k_2u_3$

$F_2 = f_2^{(1)} + f_2^{(2)} = - f_1^{(1)} - f_3^{(2)} = - (k_1u_1 - k_1u_2) - (- k_2u_2 + k_2u_3) = -k_1u_1 + (k_1 + k_2)u_2 - k_2u_3$

Aligning the displacement (u) terms:

	u_1	u_2	u_3
F_1	k_1	$- k_1$	0
F_2	$- k_1$	$k_1 + k_2$	$- k_2$
F_3	0	$- k_2$	k_2

Assembling the three equations:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \mathbf{F} = \mathbf{Ku}$$

F is the Global Force Matrix, **K** is the Global Stiffness Matrix, **u** is the Global Displacement Matrix.

Just for fun, here are the individual elements' equations, which can be added together to create the Global equation above.

Element 1:

$$\begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Element 2:

$$\begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$