Given: $k_1 = 10\ N/mm$ $u_1 = 2mm$ $u_2 = 0mm$

find $f_1^{(1)}$ & $f_2^{(1)}$

$$\begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} = \frac{N}{mm}\begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}\begin{Bmatrix} 2mm \\ 0mm \end{Bmatrix}$$

$$f_1^{(1)} = 10\frac{N}{mm} \cdot 2mm - 10\frac{N}{mm}(0) = 20N$$

$$f_2^{(1)} = \left(-10\frac{N}{mm}\right)(2mm) + 10\frac{N}{mm}(0) = -20N$$

$$f_2^{(1)} = -f_1^{(1)}$$

Given $k_1 = 10\ N/mm$ $f_2^{(1)} = 40N$

find u_1 & u_2

$$\underline{f} = \underline{k}\,\underline{u}$$

$$\underline{k}^{-1}\underline{f} = \underline{k}^{-1}\underline{k}\,\underline{u} \qquad \underline{k}^{-1}\underline{k} = \underline{I}$$

$$\underline{k}^{-1} = \frac{1}{\det[\underline{k}]}[\underline{C}]^T \qquad k = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

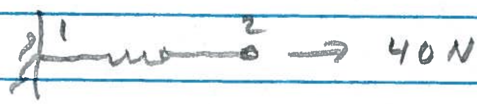
$$\det[\underline{k}] = k^2 - (-k)^2 = 0$$

... oops. $\frac{1}{0}$ – undefined

If all displacements are unknown, this allows for rigid body displacement.
THE OBJECT IS FREE TO MOVE.
<u>NEED</u> to constrain.

TRY AGAIN

Given: $k_1 = 10\ \text{N/mm}$ $f_2^{(1)} = 40\,N$, $u_1^{(1)} = \phi$



$$\begin{Bmatrix} \cancel{f_1^{(1)}} \\ f_2^{(1)} \end{Bmatrix} = \begin{bmatrix} \cancel{k_1} & \cancel{-k_1} \\ \cancel{-k_1} & k_1 \end{bmatrix} \begin{Bmatrix} \cancelto{0}{u_1} \\ u_2 \end{Bmatrix}$$

If $u_1 = 0$, then cross-out
ROW 1 IN $\underline{f}$, $\underline{k}$ & $\underline{u}$
AND COLUMN 1 in $\underline{k}$

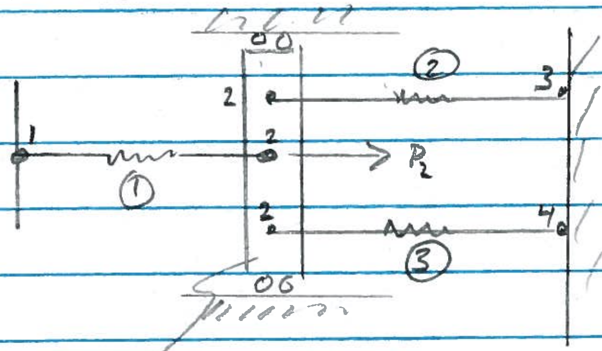
$$f_2^{(1)} = k_1 u_2$$

$$u_2 = k_1^{-1} f_2^{(1)} = \frac{1}{10\ \text{N/mm}}\, 40\,N = \underline{\underline{4\ \text{mm}}}$$

THEN IF NEEDED SOLVE OTHER UNKNOWNS

EX (PG 48, LOGAN 4th Ed)

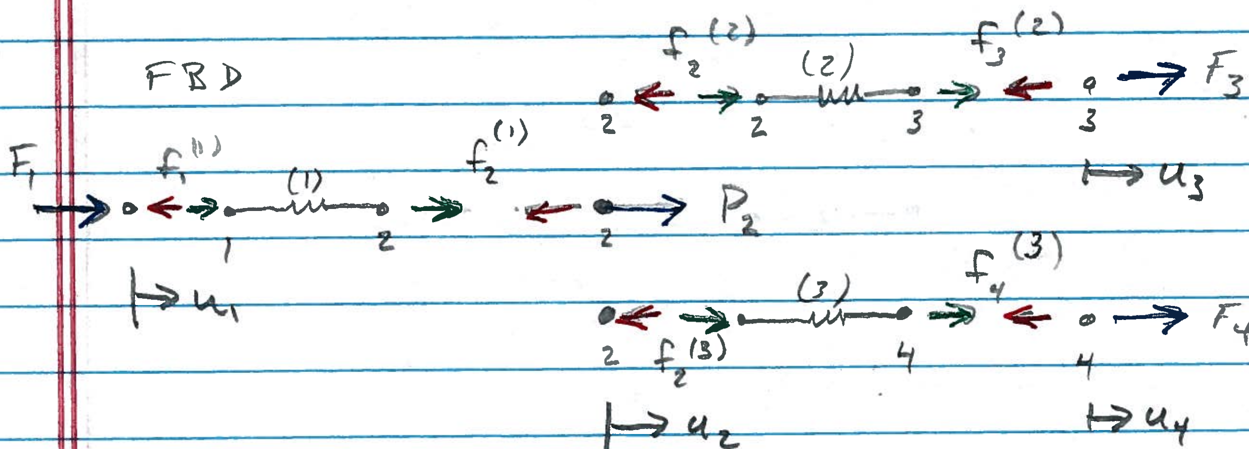
DETERMINE FORCES AT 1, 3, 4 & DISP AT 2



rigid bar, does not rotate

NODE 2 contains 3 "points". Since all 3 pts are attached to a non-rotating rigid bar, they all have same displacement ∴ one NODE

FBD



EQUIL.

NODE 1: ΣF: $F_1 - f_1^{(1)} = 0 \qquad F_1 = f_1^{(1)}$

NODE 2: $P_2 = f_2^{(1)} + f_2^{(2)} + f_2^{(3)}$

NODE 3: $F_3 = f_3^{(2)}$

NODE 4: $F_4 = f_4^{(2)}$

ELEM 1: $f_1^{(1)} = -f_2^{(1)}$

ELEM 2: $f_2^{(2)} = -f_3^{(2)}$

ELEM 3: $f_2^{(3)} = -f_4^{(3)}$

SPRING EQN

$$F_1 = f_1^{(1)} = -k_1\delta_1 = -k_1(u_2 - u_1) = k_1 u_1 - k_1 u_2$$
$$F_3 = f_3^{(2)} = k_2\delta_3 = k_2(u_3 - u_2) = -k_2 u_2 + k_2 u_3$$
$$F_4 = f_4^{(3)} = k_3\delta_4 = k_3(u_4 - u_2) = -k_3 u_2 + k_3 u_4$$

$$P_2 = f_2^{(1)} + f_2^{(2)} + f_2^{(3)} = -f_1^{(1)} - f_3^{(2)} - f_4^{(3)}$$
$$P_2 = -(k_1 u_1 - k_1 u_2) - (-k_2 u_2 + k_2 u_3) - (-k_3 u_2 + k_3 u_4)$$
$$P_2 = -k_1 u_1 + (k_1 + k_2 + k_3) u_2 - (k_2) u_3 - k_3 u_4$$

ASSEMBLE

$$\begin{Bmatrix} F_1 \\ P_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2+k_3 & -k_2 & -k_3 \\ 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \begin{array}{l} \to 0 \\ \\ \to 0 \\ \to 0 \end{array}$$

APPLY BC's $u_1 = u_3 = u_4 = 0$

∴ CROSS OUT ROWS 1, 3 & 4 in $\underline{F}, \underline{K}, \underline{u}$

" " COLUMS 1, 3 & 4 in $\underline{K}$

$$P_2 = (k_1 + k_2 + k_3) u_2$$

$$\Rightarrow \quad u_2 = \frac{P_2}{k_1 + k_2 + k_3}$$

THEN $F_1 = -k_1 u_2 \quad F_3 = -k_2 u_2 \quad F_4 = -k_3 u_2$