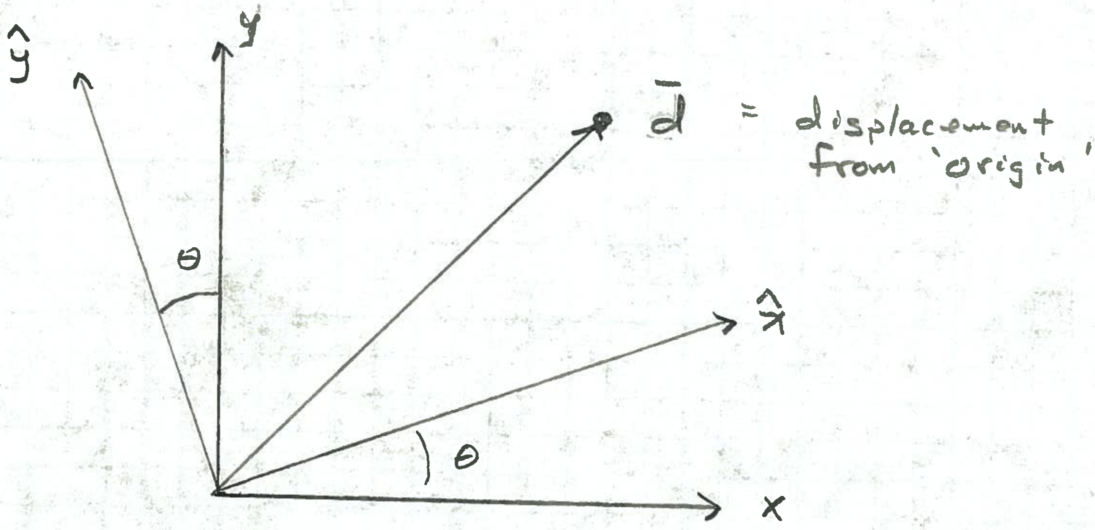
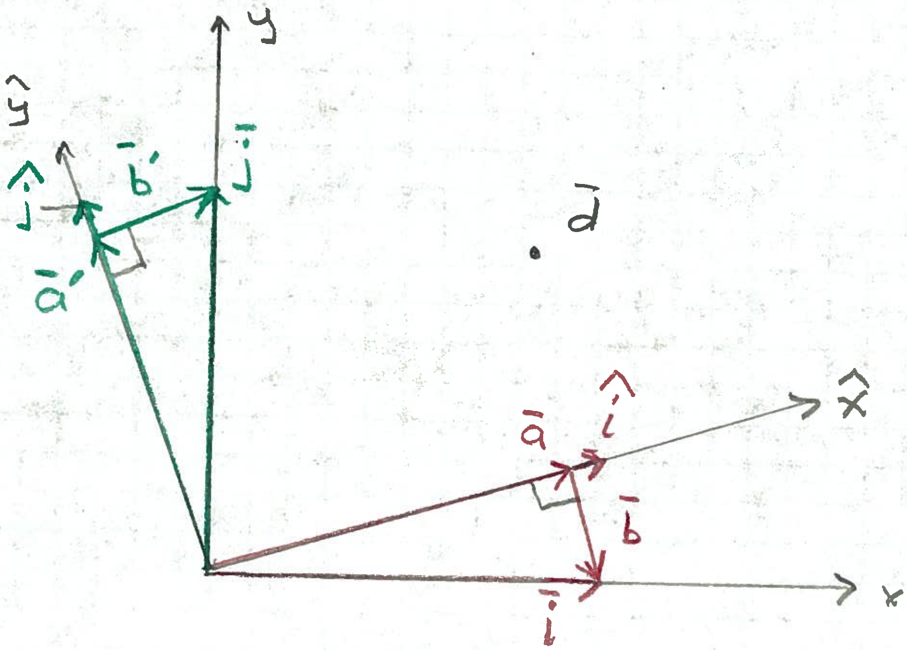
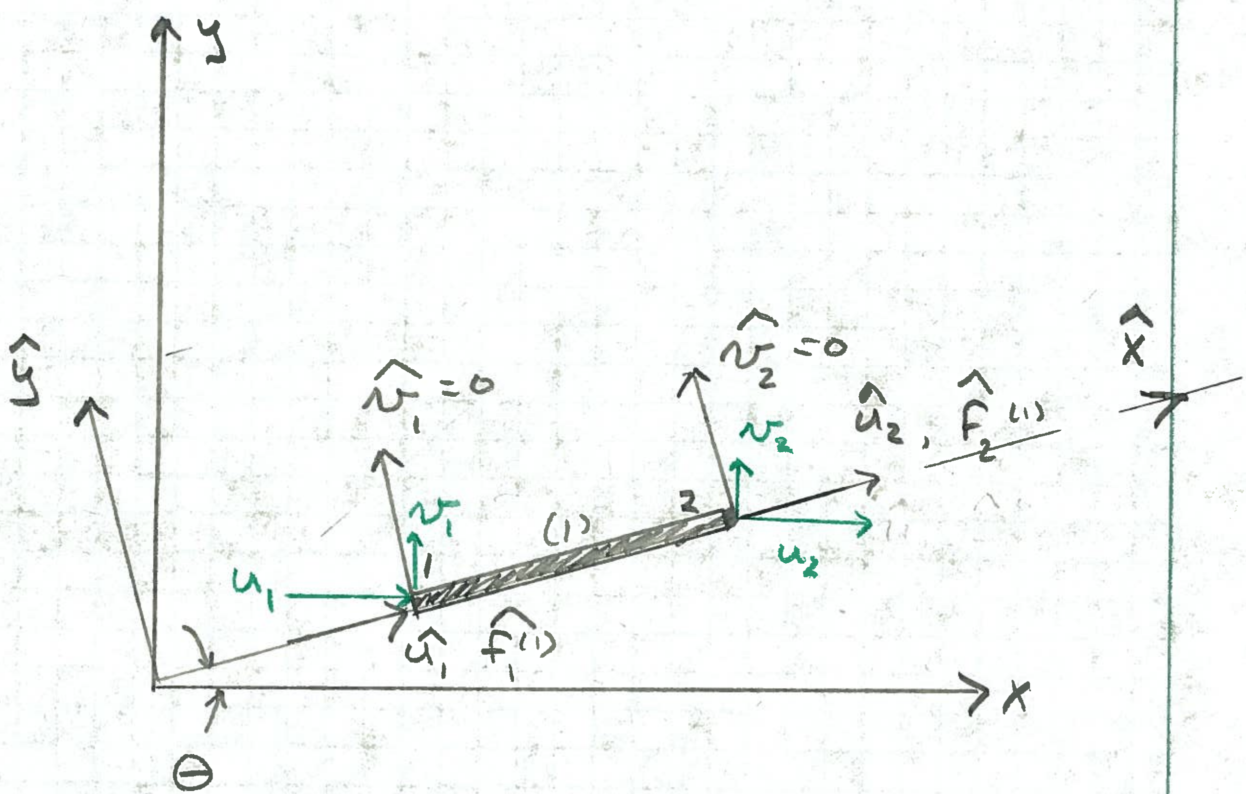



eg 1: $\bar{d} = d_x \bar{i} + d_y \bar{j} = d_{\hat{x}} \hat{i} + d_{\hat{y}} \hat{j}$




| | LOCAL | GLOBAL |
|---|---|---|
| COORD | $\hat{x}$ $\hat{y}$ | $x$ $y$ |
| Displacement wrt $x$ | $\hat{u}_i$ | $u_i$ |
| wrt $y$ | $\hat{v}_i$ | $v_i$ |
| DOF | 1 | 2 |

An element at $\theta \neq 0$




LET:

LOCAL $\hat{x}\ \hat{y}$

$$\hat{u}_1 = d_{\hat{x}1} \qquad \hat{u}_2 = d_{\hat{x}2}$$
$$\hat{v}_1 = d_{\hat{y}1} \qquad \hat{v}_2 = d_{\hat{y}2}$$

GLOBAL $x\ y$

$$u_1 = d_{x1} \qquad u_2 = d_{x2}$$
$$v_1 = d_{y1} \qquad v_2 = d_{y2}$$

$\bar{a} + \bar{b} = \bar{i}$

also, law of cosines:

$|a| = |i| \cos\theta = \cos\theta$

$|b| = \sin\theta$

$\bar{a}$ is in the $\hat{i}$ $(\hat{x})$ direction

$\bar{b}$ is in the $\hat{j}$ $(\hat{y})$ direction

$\therefore \bar{a} = |a|\,\hat{i} = \cos\theta\,\hat{i}$

$\bar{b} = |b|(-\hat{j}) = \sin\theta(-\hat{j})$

EQ 2: $\therefore \bar{i} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j} \quad (= \bar{a} + \bar{b})$

also $\bar{a}' + \bar{b}' = \bar{j}$

$\bar{a}' = \cos\theta\,\hat{j}$

$\bar{b}' = \sin\theta\,\hat{i}$

EQ 3: $\bar{j} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j} \quad (= \bar{b}' + \bar{a}')$

FROM EQN 1, 2, 3:

$d_x\,\bar{i} + d_y\,\bar{j} = d_x(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) + d_y(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$= d_{\hat{x}}\,\hat{i} + d_{\hat{y}}\,\hat{j}$

EQ 4: $\left[\begin{array}{l} d_x\cos\theta + d_y\sin\theta = d_{\hat{x}} \\ -d_x\sin\theta + d_y\cos\theta = d_{\hat{y}} \end{array}\right.$

MATRIX FORM:

$$\begin{Bmatrix} d_{\hat{x}} \\ d_{\hat{y}} \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix}$$

TRANSFORMATION (OR ROTATION)

WHERE: $C = \cos\theta \qquad S = \sin\theta$

We have established (EQ 4)

$$d_{\hat{x}} = d_x \cos\theta + d_y \sin\theta$$

Therefore, for nodes 1 & 2;

$$\hat{u}_1 = u_1 \cos\theta + v_1 \sin\theta$$

$$\hat{u}_2 = u_2 \cos\theta + v_2 \sin\theta$$

Where;

| | | | | | | |
|---|---|---|---|---|---|---|
| local | $\hat{u}_1$ | - displacement | of | node | 1 | in $\hat{x}$ |
| local | $\hat{u}_2$ | - " | " | " | 2 | in $\hat{x}$ |
| Global | $u_1$ | - " | " | " | 1 | in $x$ |
| Global | $v_1$ | - " | " | " | 1 | in $y$ |
| Global | $u_2$ | - " | " | " | 2 | in $x$ |
| Global | $v_2$ | - " | " | " | 2 | in $y$ |

$$\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix} = \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

$$\underline{\hat{u}} = \underline{T}^{*} \underline{u}$$

$$T^{*} = \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix}$$

$C = \cos\theta \qquad S = \sin\theta$

we already know for a single spring element:

$$\begin{Bmatrix} \hat{f}_1^{(1)} \\ \hat{f}_2^{(1)} \end{Bmatrix} = k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix}$$

$$\underline{\hat{f}} = \underline{\hat{k}}\ \underline{\hat{d}} \qquad (\text{local})$$

letting $u$ represent x-displacement & $v$ represent y-dis,
And for linear spring regardless of coordinate orientation:

$$f_{1x}^{(1)} = k_1 d_{1x} = k_1 u_1$$
$$f_{1y}^{(1)} = k_1 d_{1y} = k_1 v_1$$

$$f_{2x}^{(1)} = k_1 d_{2x} = k_1 u_2$$
$$f_{2y}^{(1)} = k_1 d_{2y} = k_1 v_2$$

$$\begin{pmatrix} f_{1x}^{(1)} \\ f_{1y}^{(1)} \\ f_{2x}^{(1)} \\ f_{2y}^{(1)} \end{pmatrix} = \underline{k} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix}$$

$$\underline{f} = \underline{k}\ \underline{u}$$