ME 304 – Finite Element Analysis, Fall 2019
Assignment: Matrix Algebra

The purpose of this assignment to help you learn or review basic matrix algebra. Matrix algebra is at the heart of the finite element method. Although matrices in the finite element method are typically larger than can be managed by hand (that's the computer's job), it is important that students have an understanding of what is happening inside "the black box" (the computer). We will keep to small matrices which demonstrate the principles.

Estimated time: about 2-3 hours

Regarding format: as always, you must follow the standard homework format. However, for pure mathematical work (such as this), there will generally be no assumptions. You need only include the given matrices once (for example, you don't need to restate what the A matrix is for each problem it is used in). Bottom line, the following should ***always*** be clear:

- what you are doing (what question was asked),
- what information you are using,
- what you are doing to answer the question (if anything), SHOW ALL your work
- and the answer

For the following questions, use the matrices as defined here:

$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 11 \\ 6 & 7 & 8 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 4 & 2 & 8 \\ 3 & 5 & 11 \\ 3 & 7 & 1 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 8 & 20 & 0 \\ 3 & 6 & 5 \\ 4 & 2 & 12 \end{bmatrix}$ $\mathbf{D} = \begin{bmatrix} 5 & 11 \\ 7 & 8 \end{bmatrix}$ $\mathbf{E} = \begin{bmatrix} 4 & 11 \\ 6 & 8 \end{bmatrix}$ $\mathbf{F} = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

$\mathbf{G} = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 3 & 6 \end{bmatrix}$ $\mathbf{H} = \begin{bmatrix} 10 & 7 & 9 \\ 8 & 3 & 4 \end{bmatrix}$ $\mathbf{J} = \begin{bmatrix} 8 & 4 \\ 6 & 9 \\ 5 & 6 \end{bmatrix}$ $\mathbf{K} = [\,2 \;\; 3 \;\; 8\,]$ $\mathbf{L} = \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$

(Note, the **G** through **L** matrices look distorted due to Lulay and Equation Editor problems. **L** is a 3X1 matrix). You may use software to ***check*** your answers (using MatLab, https://matrixcalc.org/en/, or similar), but do show all of your work.

1. What is the size (order) of:
   a. **A**
   b. **D**
   c. **G**
   d. **J**
   e. **K**
2. What are the following terms in regard to the **A** matrix: a_{11}, a_{31}, a_{22}, a_{12}
3. Add the following matrices. If it is not possible to do so, very briefly explain why not.
   a. **A+B**
   b. **D+E**
   c. **A+J**
   d. **K+L**

4. Determine the order (size) of the product of the following – do not do the multiplication, yet. If the matrices cannot be multiplied, briefly explain why not.
    a. **AB**
    b. **AG**
    c. **GA**
    d. **KL**
    e. **LK**
5. Multiply the following matrices. If it is not possible to do so, very briefly explain why.
    a. **AB**
    b. **BA**
    c. **DE**
    d. **ED**
    e. **LA**
    f. **AL**
    g. **KL**
    h. **LK**
6. From the previous questions, is **AB=BA**? Is **DE**=**ED**?
7. Show that **DE+FE=(D+F)E**

   Given: **DE** = $\begin{bmatrix} 86 & 143 \\ 76 & 141 \end{bmatrix}$ and **FE** = $\begin{bmatrix} 46 & 84 \\ 66 & 122 \end{bmatrix}$
8. Determine the following. If it is not possible to do so, very briefly explain why.
    a. $\mathbf{A}^T$
    b. $\mathbf{D}^T$
    c. $\mathbf{G}^T$
    d. $\mathbf{K}^T$
    e. $\mathbf{L}^T$
9. Determine the following. If it is not possible to do so, very briefly explain why.
    a. det **D**
    b. det **E**
    c. det **F**
    d. det **A**
    e. det **G**
10. Determine the following. If it is not possible to do so, very briefly explain why.
    a. $\mathbf{D}^{-1}$
    b. $\mathbf{E}^{-1}$
    c. $\mathbf{A}^{-1}$
11. To check that you correctly determined $\mathbf{A}^{-1}$, show that $\mathbf{AA}^{-1}$ = the identity matrix ($\mathbf{I}$).