ME304 Finite Element Analysis, Fall 2019
HW 5: Static Failure Theories

Applying static failure theories properly is a critical talent for stress analysis engineers. Here are some suggestions and commentary that may help:

1) When dealing with failure theories, always draw 3D Mohr's circle even for plane stress. 3D means three circles – they represent the stresses at a point and each circle is associated with one of the three orthogonal planes (the three planes containing the principal stresses). This will remind you that there are always three principal stresses.
2) Draw and read Mohr's circle ***without*** doing any calculations is strongly advised! It is much quicker, less prone to errors, and requires you to understand a process rather than memorize equations. If you are uncomfortable not using calculations it is an indication you don't really understand what Mohr's circle is. It is a graphical way of showing all at once the stress at a point. Neither stress values (σ_x=35ksi, etc.) nor equations show you that. From Mohr's circle, we recognize that the stress element oriented with the principal directions contain no shear stress. From Mohr's circle we can very quickly determine (without equations) maximum shear stress and principal stresses.
3) Understand what static failure theories are telling you, the engineer. You must understand what causes a material to "fail" and what "failure" means. Brittle materials fail due to different mechanisms than ductile materials. Therefore, the failure criteria (what explains failure) will depend upon the material and its failure mechanism.
    a. If failure is caused by "pulling apart the atomic structure", then one would expect tensile stress (aka "pulling") to cause fracture (a type of failure). Think about Mohr's circle for torsion in a shaft. The maximum normal stress (σ_1) is at 90 degrees from τ_{max} on Mohr's circle, therefore, 45 degrees on the real part. Brittle materials fail on planes normal to the direction of σ_1 (maximum normal stress) because brittle fracture is caused by "pulling apart" the material. This why the fracture of a brittle material will be a spiral on a torsion-loaded shaft and it is why it is the fracture is perpendicular to the axis for uniaxial tension loads.
    b. Generally, ductile metals and alloys "fail" at stresses far below those needed to cause fracture. Generally, yielding (plastic deformation) is considered failure for ductile materials – a bent paperclip won't hold the paper together very well, bent wings don't fly well, bent hip implants are painful. From materials science, you know that plastic deformation is caused by dislocation slip – and slip is caused by shear stresses. In uniaxial tension test, we apply a normal stress (σ =F/A), but it is actually the shear stress that causes material to yield. According to the maximum shear stress theory, the amount of shear stress required to cause dislocations to slip in a ductile metal or alloy is ½ S_{ys} (draw Mohr's circle to help you visualize that).
    c. von Mises (aka "distortion energy") failure theory is a more complex version of the Maximum Shear Stress (aka "Tresca") failure theory. Although both give very similar results for predicting plastic deformation, von Mises tends to be match experimental

results slightly better. It is also more commonly used with FEA. You are free to choose either one for determining onset of yielding. However, without a calculator, von Mises is a bit more computationally challenging and without notes, it is more difficult to remember. None the less, you should understand how to apply it – it is probable that a question about von Mises to show up at some point in your future.

A better question than "*what do I need to know for the exam*" is "*how well do I need to know these topics for the exam?*" Especially for the topic of static failure theories, my answer is: *know it well enough so that you don't even have to think much while working these problems*. Once you reach that level of understanding, you will look back on recent days and wonder "why did I not understand this – it's so easy…." Really, really, truly! So how to get there? The same way an athlete or musician does ---- practice. If you need some "coaching" – ask.

Consider the stress conditions given in the table below. Assume quasi-static overload failure is defined by "yielding" (aka, plastic deformation) and fracture (which ever comes with the lowest loading). Note, that materials with ductility less than about 2 or 3%EL are generally considered to be brittle. Those with greater ductility are considered to be ductile.

For each of the problems below, answer these two questions:

a) The ductility of a material is given as 10%EL. Determine the required strength (yield strength or tensile strength) for a factor of safety of 5 for a quasi-static overload failure.
b) The ductility of a material is given as 1%EL. Determine the required strength (yield strength or tensile strength) for a factor of safety of 5 for a quasi-static overload.

NOMENCLATURE: in more advanced studies of stress, all stresses (normal and shear) are indicated with the letter sigma (σ) and with 2 subscripts. Examples how to relate what you have learned to this "new" nomenclature:

- the normal stress in the x-direction: $\sigma_{xx} = \sigma_x$
- the shear stress in the x-y plane: $\sigma_{xy} = \tau_{xy}$

Problem #	σ_{xx}	σ_{yy}	σ_{zz}	σ_{yz}	σ_{xz}	σ_{xy}
1	10	-20	0	0	0	10
2	50	50	0	0	0	20
3	50	-10	0	0	0	20
4	-50	-30	0	0	0	10
5	0	50	30	-10	0	0
6	0	100	0	0	0	0
7	0	0	0	0	0	50
8	10	20	40	0	0	0
9	-10	-20	-40	0	0	0
10	10	20	-40	0	0	0

All units are ksi or MPa, you pick.

You play like you practice; therefore, no calculator but **DO** USE a template or compass to draw nice scalable circles. And no, you are not expected to read a graph to 3 significant figures.

Be sure to include assumption about failure mode (yielding, brittle fracture) for the following.

11) For a material with S_{ys} = 100ksi, determine the maximum shear stress in a circular shaft loaded with torsion, T, only (τ = Tr/J).
12) For a material with S_{UT} = 100ksi (no yield strength published), determine the maximum shear stress in a circular shaft loaded with torsion, T, only (τ = Tr/J).
13) For a material with S_{ys} = 100ksi, determine the maximum normal stress for a uniaxial loaded bar (σ = F/A).
14) For a material with S_{UT} = 100ksi (no yield strength published), determine the maximum normal stress for a uniaxial loaded bar (σ = F/A).
15) Determine the factor of safety against yielding based on both the Maximum Shear Stress and Distortion Energy theories for the following stress condition: σ_x = 100MPa, σ_y = -50MPa, τ_{xy} =0MPa. Let S_{ys} = 250MPa.

Factor of Safety, FOS = n = "designed ability" / "actual anticipated load." FOS should be greater than 1.

For further explanation, see the course web page for link title "Factor of Safety"