Chapter 2

Fundamentals of the Mechanical Behavior of Materials

Questions

2.1 Can you calculate the percent elongation of materials based only on the information given in Fig. 2.6? Explain.

Recall that the percent elongation is defined by Eq. (2.6) on p. 33 and depends on the original gage length (l_o) of the specimen. From Fig. 2.6 on p. 37 only the necking strain (true and engineering) and true fracture strain can be determined. Thus, we cannot calculate the percent elongation of the specimen; also, note that the elongation is a function of gage length and increases with gage length.

2.2 Explain if it is possible for the curves in Fig. 2.4 to reach 0% elongation as the gage length is increased further.

The percent elongation of the specimen is a function of the initial and final gage lengths. When the specimen is being pulled, regardless of the original gage length, it will elongate uniformly (and permanently) until necking begins. Therefore, the specimen will always have a certain finite elongation. However, note that as the specimen's gage length is increased, the contribution of localized elongation (that is, necking) will decrease, but the total elongation will not approach zero.

2.3 Explain why the difference between engineering strain and true strain becomes larger as strain increases. Is this phenomenon true for both tensile and compressive strains? Explain.

The difference between the engineering and true strains becomes larger because of the way the strains are defined, respectively, as can be seen by inspecting Eqs. (2.1) on p. 30 and (2.9) on p. 35. This is true for both tensile and compressive strains.

2.4 Using the same scale for stress, we note that the tensile true-stress-true-strain curve is higher than the engineering stress-strain curve. Explain whether this condition also holds for a compression test.

During a compression test, the cross-sectional area of the specimen increases as the specimen height decreases (because of volume constancy) as the load is increased. Since true stress is defined as ratio of the load to the instantaneous cross-sectional area of the specimen, the true stress in compression will be lower than the engineering stress for a given load, assuming that friction between the platens and the specimen is negligible.

2.5 Which of the two tests, tension or compression, requires a higher capacity testing machine than the other? Explain.

The compression test requires a higher capacity machine because the cross-sectional area of the

specimen increases during the test, which is the opposite of a tension test. The increase in area requires a load higher than that for the tension test to achieve the same stress level. Furthermore, note that compression-test specimens generally have a larger original cross-sectional area than those for tension tests, thus requiring higher forces.

2.6 Explain how the modulus of resilience of a material changes, if at all, as it is strained: (1) for an elastic, perfectly plastic material, and (2) for an elastic, linearly strain-hardening material.

2.7 If you pull and break a tension-test specimen rapidly, where would the temperature be the highest? Explain why.

Since temperature rise is due to the work input, the temperature will be highest in the necked region because that is where the strain, hence the energy dissipated per unit volume in plastic deformation, is highest.

2.8 Comment on the temperature distribution if the specimen in Question 2.7 is pulled very slowly.

If the specimen is pulled very slowly, the temperature generated will be dissipated throughout the specimen and to the environment. Thus, there will be no appreciable temperature rise anywhere, particularly with materials with high thermal conductivity.

2.9 In a tension test, the area under the true-stress-true-strain curve is the work done per unit volume (the specific work). We also know that the area under the load-elongation curve represents the work done on the specimen. If you divide this latter work by the volume of the specimen between the gage marks, you will determine the work done per unit volume (assuming that all deformation is confined between the gage marks). Will this specific work be the same as the area under the true-stress-true-strain curve? Explain. Will your answer be the same for any value of strain? Explain.

If we divide the work done by the total volume of the specimen between the gage lengths, we obtain the average specific work throughout the specimen. However, the area under the true

stress-true strain curve represents the specific work done at the necked (and fractured) region in the specimen where the strain is a maximum. Thus, the answers will be different. However, up to the onset of necking (instability), the specific work calculated will be the same. This is because the strain is uniform throughout the specimen until necking begins.

2.10 The note at the bottom of Table 2.5 states that as temperature increases, C decreases and m increases. Explain why.

The value of C in Table 2.5 on p. 43 decreases with temperature because it is a measure of the strength of the material. The value of m increases with temperature because the material becomes more strain-rate sensitive, due to the fact that the higher the strain rate, the less time the material has to recover and recrystallize, hence its strength increases.

2.11 You are given the K and n values of two different materials. Is this information sufficient to determine which material is tougher? If not, what additional information do you need, and why?

Although the K and n values may give a good estimate of toughness, the true fracture stress and the true strain at fracture are required for accurate calculation of toughness. The modulus of elasticity and yield stress would provide information about the area under the elastic region; however, this region is very small and is thus usually negligible with respect to the rest of the stress-strain curve.

2.12 Modify the curves in Fig. 2.7 to indicate the effects of temperature. Explain the reasons for your changes.

These modifications can be made by lowering the slope of the elastic region and lowering the general height of the curves. See, for example, Fig. 2.10 on p. 42.

2.13 Using a specific example, show why the deformation rate, say in m/s, and the true strain rate are not the same.

The deformation rate is the quantity v in Eqs. (2.14), (2.15), (2.17), and (2.18) on pp. 41-46. Thus, when v is held constant during de-

formation (hence a constant deformation rate), the true strain rate will vary, whereas the engineering strain rate will remain constant. Hence, the two quantities are not the same.

2.14 It has been stated that the higher the value of m, the more diffuse the neck is, and likewise, the lower the value of m, the more localized the neck is. Explain the reason for this behavior.

As discussed in Section 2.2.7 starting on p. 41, with high m values, the material stretches to a greater length before it fails; this behavior is an indication that necking is delayed with increasing m. When necking is about to begin, the necking region's strength with respect to the rest of the specimen increases, due to strain hardening. However, the strain rate in the necking region is also higher than in the rest of the specimen, because the material is elongating faster there. Since the material in the necked region becomes stronger as it is strained at a higher rate, the region exhibits a greater resistance to necking. The increase in resistance to necking thus depends on the magnitude of m. As the tension test progresses, necking becomes more *diffuse*, and the specimen becomes longer before fracture; hence, total elongation increases with increasing values of m (Fig. 2.13 on p. 45). As expected, the elongation after necking (*postuniform elongation*) also increases with increasing m. It has been observed that the value of m decreases with metals of increasing strength.

2.15 Explain why materials with high m values (such as hot glass and silly putty) when stretched slowly, undergo large elongations before failure. Consider events taking place in the necked region of the specimen.

The answer is similar to Answer 2.14 above.

2.16 Assume that you are running four-point bending tests on a number of identical specimens of the same length and cross-section, but with increasing distance between the upper points of loading (see Fig. 2.21b). What changes, if any, would you expect in the test results? Explain.

As the distance between the upper points of loading in Fig. 2.21b on p. 51 increases, the magnitude of the bending moment decreases.

However, the volume of material subjected to the maximum bending moment (hence to maximum stress) increases. Thus, the probability of failure in the four-point test increases as this distance increases.

2.17 Would Eq. (2.10) hold true in the elastic range? Explain.

Note that this equation is based on volume constancy, i.e., $A_o l_o = Al$. We know, however, that because the Poisson's ratio ν is less than 0.5 in the elastic range, the volume is not constant in a tension test; see Eq. (2.47) on p. 69. Therefore, the expression is not valid in the elastic range.

2.18 Why have different types of hardness tests been developed? How would you measure the hardness of a very large object?

There are several basic reasons: (a) The overall hardness range of the materials; (b) the hardness of their constituents; see Chapter 3; (c) the thickness of the specimen, such as bulk versus foil; (d) the size of the specimen with respect to that of the indenter; and (e) the surface finish of the part being tested.

2.19 Which hardness tests and scales would you use for very thin strips of material, such as aluminum foil? Why?

Because aluminum foil is very thin, the indentations on the surface must be very small so as not to affect test results. Suitable tests would be a microhardness test such as Knoop or Vickers under very light loads (see Fig. 2.22 on p. 52). The accuracy of the test can be validated by observing any changes in the surface appearance opposite to the indented side.

2.20 List and explain the factors that you would consider in selecting an appropriate hardness test and scale for a particular application.

Hardness tests mainly have three differences:

(a) type of indenter,

(b) applied load, and

(c) method of indentation measurement (depth or surface area of indentation, or rebound of indenter).

Problems

2.46 A strip of metal is originally 1.5 m long. It is stretched in three steps: first to a length of 1.75 m, then to 2.0 m, and finally to 3.0 m. Show that the total true strain is the sum of the true strains in each step, that is, that the strains are additive. Show that, using engineering strains, the strain for each step cannot be added to obtain the total strain.

The true strain is given by Eq. (2.9) on p. 35 as

$$\epsilon = \ln\left(\frac{l}{l_o}\right)$$

Therefore, the true strains for the three steps are:

$$\epsilon_1 = \ln\left(\frac{1.75}{1.5}\right) = 0.1541$$

$$\epsilon_2 = \ln\left(\frac{2.0}{1.75}\right) = 0.1335$$

$$\epsilon_3 = \ln\left(\frac{3.0}{2.0}\right) = 0.4055$$

The sum of these true strains is $\epsilon = 0.1541 + 0.1335 + 0.4055 = 0.6931$. The true strain from step 1 to 3 is

$$\epsilon = \ln\left(\frac{3}{1.5}\right) = 0.6931$$

Therefore the true strains are additive. Using the same approach for engineering strain as defined by Eq. (2.1), we obtain $e_1 = 0.1667$, $e_2 = 0.1429$, and $e_3 = 0.5$. The sum of these strains is $e_1 + e_2 + e_3 = 0.8096$. The engineering strain from step 1 to 3 is

$$e = \frac{l - l_o}{l_o} = \frac{3 - 1.5}{1.5} = \frac{1.5}{1.5} = 1$$

Note that this is not equal to the sum of the engineering strains for the individual steps.

2.47 A paper clip is made of wire 1.20-mm in diameter. If the original material from which the wire is made is a rod 15-mm in diameter, calculate the longitudinal and diametrical engineering and true strains that the wire has undergone.

Assuming volume constancy, we may write

$$\frac{l_f}{l_o} = \left(\frac{d_o}{d_f}\right)^2 = \left(\frac{15}{1.20}\right)^2 = 156.25 \approx 156$$

Letting l_0 be unity, the longitudinal engineering strain is $e_1 = (156 - 1)/1 = 155$. The diametral engineering strain is calculated as

$$e_d = \frac{1.2 - 15}{15} = -0.92$$

The longitudinal true strain is given by Eq. (2.9) on p. 35 as

$$\epsilon = \ln\left(\frac{l}{l_o}\right) = \ln(155) = 5.043$$

The diametral true strain is

$$\epsilon_d = \ln\left(\frac{1.20}{15}\right) = -2.526$$

Note the large difference between the engineering and true strains, even though both describe the same phenomenon. Note also that the sum of the true strains (recognizing that the radial strain is $\epsilon_r = \ln\left(\frac{0.60}{7.5}\right) = -2.526$) in the three principal directions is zero, indicating volume constancy in plastic deformation.

2.48 A material has the following properties: UTS = 50,000 psi and $n = 0.25$ Calculate its strength coefficient K.

Let us first note that the true UTS of this material is given by $\text{UTS}_{\text{true}} = Kn^n$ (because at necking $\epsilon = n$). We can then determine the value of this stress from the UTS by following a procedure similar to Example 2.1. Since $n = 0.25$, we can write

$$\begin{aligned}\text{UTS}_{\text{true}} &= \text{UTS}\left(\frac{A_o}{A_{\text{neck}}}\right) = \text{UTS}\left(e^{0.25}\right) \\ &= (50,000)(1.28) = 64,200 \text{ psi}\end{aligned}$$

Therefore, since $\text{UTS}_{\text{true}} = Kn^n$,

$$K = \frac{\text{UTS}_{\text{true}}}{n^n} = \frac{64,200}{0.25^{0.25}} = 90,800 \text{ psi}$$

Thus

$$\text{UTS} = \frac{110,000}{1.1} = 100,000 \text{ psi}$$

Hence the maximum load is

$$F = (\text{UTS})(A_o) = (100,000)(0.196)$$

or $F = 19,600$ lb.

2.53 Using the data given in Table 2.1, calculate the values of the shear modulus G for the metals listed in the table.

The important equation is Eq. (2.24) on p. 49 which gives the shear modulus as

$$G = \frac{E}{2(1+\nu)}$$

The following values can be calculated (mid-range values of ν are taken as appropriate):

Material	E (GPa)	ν	G (GPa)
Al & alloys	69–79	0.32	26–30
Cu & alloys	105–150	0.34	39–56
Pb & alloys	14	0.43	4.9
Mg & alloys	41–45	0.32	15.5–17.0
Mo & alloys	330–360	0.32	125–136
Ni & alloys	180–214	0.31	69–82
Steels	190–200	0.30	73–77
Stainless steels	190–200	0.29	74–77
Ti & alloys	80–130	0.32	30–49
W & alloys	350–400	0.27	138–157
Ceramics	70–1000	0.2	29–417
Glass	70–80	0.24	28–32
Rubbers	0.01–0.1	0.5	0.0033–0.033
Thermoplastics	1.4–3.4	0.36	0.51–1.25
Thermosets	3.5–17	0.34	1.3–6.34

2.54 Derive an expression for the toughness of a material whose behavior is represented by the equation $\sigma = K(\epsilon + 0.2)^n$ and whose fracture strain is denoted as ϵ_f.

Recall that toughness is the area under the stress-strain curve, hence the toughness for this material would be given by

$$\begin{aligned}\text{Toughness} &= \int_0^{\epsilon_f} \sigma \, d\epsilon \\ &= \int_0^{\epsilon_f} K(\epsilon + 0.2)^n \, d\epsilon \\ &= \frac{K}{n+1}\left[(\epsilon_f + 0.2)^{n+1} - 0.2^{n+1}\right]\end{aligned}$$

2.55 A cylindrical specimen made of a brittle material 1 in. high and with a diameter of 1 in. is subjected to a compressive force along its axis. It is found that fracture takes place at an angle of $45°$ under a load of 30,000 lb. Calculate the shear stress and the normal stress acting on the fracture surface.

Assuming that compression takes place without friction, note that two of the principal stresses will be zero. The third principal stress acting on this specimen is normal to the specimen and its magnitude is

$$\sigma_3 = \frac{30,000}{\pi(0.5)^2} = 38,200 \text{ psi}$$

The Mohr's circle for this situation is shown below.
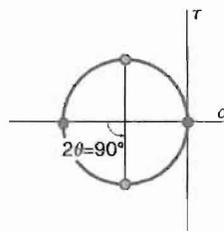
The fracture plane is oriented at an angle of $45°$, corresponding to a rotation of $90°$ on the Mohr's circle. This corresponds to a stress state on the fracture plane of $\sigma = -19,100$ psi and $\tau = 19,100$ psi.

2.56 What is the modulus of resilience of a highly cold-worked piece of steel with a hardness of 300 HB? Of a piece of highly cold-worked copper with a hardness of 150 HB?

Referring to Fig. 2.24 on p. 55, the value of c in Eq. (2.29) on p. 54 is approximately 3.2 for highly cold-worked steels and around 3.4 for cold-worked aluminum. Therefore, we can approximate $c = 3.3$ for cold-worked copper. However, since the Brinell hardness is in units of kg/mm², from Eq. (2.29) we can write

$$T_{\text{steel}} = \frac{H}{3.2} = \frac{300}{3.2} = 93.75 \text{ kg/mm}^2 = 133 \text{ ksi}$$

$$T_{\text{Cu}} = \frac{H}{3.3} = \frac{150}{3.3} = 45.5 \text{ kg/mm}^2 = 64.6 \text{ ksi}$$

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From Table 2.1, $E_{\text{steel}} = 30 \times 10^6$ psi and $E_{\text{Cu}} = 15 \times 10^6$ psi. The modulus of resilience is calculated from Eq. (2.5). For steel:

$$\text{Modulus of Resilience} = \frac{Y^2}{2E} = \frac{(133,000)^2}{2(30 \times 10^6)}$$

or a modulus of resilience for steel of 295 in-lb/in^3. For copper,

$$\text{Modulus of Resilience} = \frac{Y^2}{2E} = \frac{(62,200)^2}{2(15 \times 10^6)}$$

or a modulus of resilience for copper of 129 in-lb/in^3.

Note that these values are slightly different than the values given in the text; this is due to the fact that (a) highly cold-worked metals such as these have a much higher yield stress than the annealed materials described in the text, and (b) arbitrary property values are given in the statement of the problem.

2.57 Calculate the work done in frictionless compression of a solid cylinder 40 mm high and 15 mm in diameter to a reduction in height of 75% for the following materials: (1) 1100-O aluminum, (2) annealed copper, (3) annealed 304 stainless steel, and (4) 70-30 brass, annealed.

The work done is calculated from Eq. (2.62) on p. 71 where the specific energy, u, is obtained from Eq. (2.60). Since the reduction in height is 75%, the final height is 10 mm and the absolute value of the true strain is

$$\epsilon = \ln\left(\frac{40}{10}\right) = 1.386$$

K and n are obtained from Table 2.3 as follows:

Material	K (MPa)	n
1100-O Al	180	0.20
Cu, annealed	315	0.54
304 Stainless, annealed	1300	0.30
70-30 brass, annealed	895	0.49

The u values are then calculated from Eq. (2.60). For example, for 1100-O aluminum, where K is 180 MPa and n is 0.20, u is calculated as

$$u = \frac{K\epsilon^{n+1}}{n+1} = \frac{(180)(1.386)^{1.2}}{1.2} = 222 \text{ MN/m}^3$$

The volume is calculated as $V = \pi r^2 l = \pi(0.0075)^2(0.04) = 7.069 \times 10^{-6}$ m^3. The work done is the product of the specific work, u, and the volume, V. Therefore, the results can be tabulated as follows.

Material	u (MN/m^3)	Work (Nm)
1100-O Al	222	1562
Cu, annealed	338	2391
304 Stainless, annealed	1529	10,808
70-30 brass, annealed	977	6908

2.58 A material has a strength coefficient $K = 100,000$ psi Assuming that a tensile-test specimen made from this material begins to neck at a true strain of 0.17, show that the ultimate tensile strength of this material is 62,400 psi.

The approach is the same as in Example 2.1. Since the necking strain corresponds to the maximum load and the necking strain for this material is given as $\epsilon = n = 0.17$, we have, as the true ultimate tensile strength:

$$\text{UTS}_{\text{true}} = (100,000)(0.17)^{0.17} = 74,000 \text{ psi.}$$

The cross-sectional area at the onset of necking is obtained from

$$\ln\left(\frac{A_o}{A_{\text{neck}}}\right) = n = 0.17.$$

Consequently,

$$A_{\text{neck}} = A_o e^{-0.17}$$

and the maximum load, P, is

$$P = \sigma A = (\text{UTS}_{\text{true}})A_o e^{-0.17}$$
$$= (74,000)(0.844)(A_o) = 62,400A_o \text{ lb.}$$

Since UTS$= P/A_o$, we have UTS $= 62,400$ psi.

2.59 A tensile-test specimen is made of a material represented by the equation $\sigma = K(\epsilon + n)^n$. (a) Determine the true strain at which necking will begin. (b) Show that it is possible for an engineering material to exhibit this behavior.