6.4 *A cylindrical specimen of a titanium alloy having an elastic modulus of 107 GPa (15.5 $\times 10^6$ psi) and an original diameter of 3.8 mm (0.15 in.) will experience only elastic deformation when a tensile load of 2000 N (450 lb_f) is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm (0.0165 in.).*

Solution

We are asked to compute the maximum length of a cylindrical titanium alloy specimen (before deformation) that is deformed elastically in tension. For a cylindrical specimen

$$A_0 = \pi\left(\frac{d_0}{2}\right)^2$$

where d_0 is the original diameter. Combining Equations 6.1, 6.2, and 6.5 and solving for l_0 leads to

$$l_0 = \frac{\Delta l}{\varepsilon} = \frac{\Delta l}{\dfrac{\sigma}{E}} = \frac{\Delta l\, E}{\dfrac{F}{A_0}} = \frac{\Delta l\, E \pi \left(\dfrac{d_0}{2}\right)^2}{F} = \frac{\Delta l\, E \pi\, d_0^2}{4F}$$

$$= \frac{(0.42 \times 10^{-3}\,\mathrm{m})(107 \times 10^9\,\mathrm{N/m^2})\,(\pi)\,(3.8 \times 10^{-3}\,\mathrm{m})^2}{(4)(2000\ \mathrm{N})}$$

$$= 0.255\ \mathrm{m} = 255\ \mathrm{mm}\ (10.0\ \mathrm{in.})$$

6.8 *A cylindrical rod of copper (E = 110 GPa, 16 $\times 10^6$ psi) having a yield strength of 240 MPa (35,000 psi) is to be subjected to a load of 6660 N (1500 lb_f). If the length of the rod is 380 mm (15.0 in.), what must be the diameter to allow an elongation of 0.50 mm (0.020 in.)?*

Solution

This problem asks us to compute the diameter of a cylindrical specimen of copper in order to allow an elongation of 0.50 mm. Employing Equations 6.1, 6.2, and 6.5, assuming that deformation is entirely elastic

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for d_0

$$d_0 = \sqrt{\frac{4\,l_0 F}{\pi\,E\,\Delta l}}$$

$$= \sqrt{\frac{(4)\left(380 \times 10^{-3}\,\mathrm{m}\right)(6660\ \mathrm{N})}{(\pi)\left(110 \times 10^9\ \mathrm{N/m^2}\right)\left(0.50 \times 10^{-3}\,\mathrm{m}\right)}}$$

$$= 7.65 \times 10^{-3}\ \mathrm{m} = 7.65\ \mathrm{mm}\ \ (0.30\ \mathrm{in.})$$

6.19 *Consider a cylindrical specimen of some hypothetical metal alloy that has a diameter of 8.0 mm (0.31 in.). A tensile force of 1000 N (225 lb_f) produces an elastic reduction in diameter of 2.8×10^{-4} mm (1.10×10^{-5} in.). Compute the modulus of elasticity for this alloy, given that Poisson's ratio is 0.30.*

Solution

This problem asks that we calculate the modulus of elasticity of a metal that is stressed in tension. Combining Equations 6.5 and 6.1 leads to

$$E = \frac{\sigma}{\varepsilon_z} = \frac{F}{A_0 \varepsilon_z} = \frac{F}{\varepsilon_z \pi \left(\frac{d_0}{2}\right)^2} = \frac{4F}{\varepsilon_z \pi d_0^2}$$

From the definition of Poisson's ratio, (Equation 6.8) and realizing that for the transverse strain, $\varepsilon_x = \frac{\Delta d}{d_0}$

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\Delta d}{d_0 \nu}$$

Therefore, substitution of this expression for ε_z into the above equation yields

$$E = \frac{4F}{\varepsilon_z \pi d_0^2} = \frac{4F\nu}{\pi d_0 \Delta d}$$

$$= \frac{(4)(1000\text{ N})(0.30)}{\pi \left(8 \times 10^{-3}\text{ m}\right)\left(2.8 \times 10^{-7}\text{ m}\right)} = 1.705 \times 10^{11}\text{ Pa} = 170.5\text{ GPa} \ \ (24.7 \times 10^6\text{ psi})$$

6.28 *A bar of a steel alloy that exhibits the stress-strain behavior shown in Figure 6.21 is subjected to a tensile load; the specimen is 300 mm (12 in.) long, and of square cross section 4.5 mm (0.175 in.) on a side.*

(a) Compute the magnitude of the load necessary to produce an elongation of 0.45 mm (0.018 in.).

(b) What will be the deformation after the load has been released?

Solution

(a) We are asked to compute the magnitude of the load necessary to produce an elongation of 0.45 mm for the steel displaying the stress-strain behavior shown in Figure 6.21. First, calculate the strain, and then the corresponding stress from the plot.

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.45 \text{ mm}}{300 \text{ mm}} = 1.5 \times 10^{-3}$$

This is near the end of the elastic region; from the inset of Figure 6.21, this corresponds to a stress of about 300 MPa (43,500 psi). Now, from Equation 6.1

$$F = \sigma A_0 = \sigma b^2$$

in which b is the cross-section side length. Thus,

$$F = \left(300 \times 10^6 \text{ N/m}^2\right)\left(4.5 \times 10^{-3} \text{ m}\right)^2 = 6075 \text{ N} \quad (1366 \text{ lb}_\text{f})$$

(b) After the load is released there will be no deformation since the material was strained only elastically.

6.50 *A steel alloy specimen having a rectangular cross section of dimensions 12.7 mm × 6.4 mm (0.5 in. × 0.25 in.) has the stress–strain behavior shown in Figure 6.21. If this specimen is subjected to a tensile force of 38,000 N (8540 lb$_f$) then*

(a) Determine the elastic and plastic strain values.

(b) If its original length is 460 mm (18.0 in.), what will be its final length after the load in part (a) is applied and then released?

Solution

(a) We are asked to determine both the elastic and plastic strain values when a tensile force of 38,000 N (8540 lb$_f$) is applied to the steel specimen and then released. First it becomes necessary to determine the applied stress using Equation 6.1; thus

$$\sigma = \frac{F}{A_0} = \frac{F}{b_0 d_0}$$

where b_0 and d_0 are cross-sectional width and depth (12.7 mm and 6.4 mm, respectively). Thus

$$\sigma = \frac{38{,}000 \text{ N}}{(12.7 \times 10^{-3}\,\text{m})(6.4 \times 10^{-3}\,\text{m})} = 468 \times 10^6 \text{ N/m}^2 = 468 \text{ MPa} \ \ (68{,}300 \text{ psi})$$

From Figure 6.21, this point is in the plastic region so the specimen will be both elastic and plastic strains. The total strain at this point, ε_t, is about 0.010. We are able to estimate the amount of permanent strain recovery ε_e from Hooke's law, Equation 6.5 as

$$\varepsilon_e = \frac{\sigma}{E}$$

And, since E = 207 GPa for steel (Table 6.1)

$$\varepsilon_e = \frac{468 \text{ MPa}}{207 \times 10^3\,\text{MPa}} = 0.00226$$

The value of the plastic strain, ε_p is just the difference between the total and elastic strains; that is

$$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.010 - 0.00226 = 0.00774$$

(b) If the initial length is 460 mm (18.0 in.) then the final specimen length l_i may be determined from a rearranged form of Equation 6.2 using the plastic strain value as

$$l_i = l_0(1 + \varepsilon_p) = (460 \text{ mm})(1 + 0.00774) = 463.6 \text{ mm } (18.14 \text{ in.})$$