### CHAPTER 9

# PHASE DIAGRAMS

#### PROBLEM SOLUTIONS

## **Solubility Limit**

9.1 Consider the sugar-water phase diagram of Figure 9.1.

(a) How much sugar will dissolve in 1500 g water at 90  $^{\circ}$ C (194  $^{\circ}$ F)?

(b) If the saturated liquid solution in part (a) is cooled to  $20 \,^{\circ}C$  (68 F), some of the sugar will precipitate out as a solid. What will be the composition of the saturated liquid solution (in wt% sugar) at  $20 \,^{\circ}C$ ?

(c) How much of the solid sugar will come out of solution upon cooling to  $20 \,$ °C?

### Solution

(a) We are asked to determine how much sugar will dissolve in 1000 g of water at 90°C. From the solubility limit curve in Figure 9.1, at 90°C the maximum concentration of sugar in the syrup is about 77 wt%. It is now possible to calculate the mass of sugar using Equation 4.3 as

$$C_{\text{sugar}}(\text{wt\%}) = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + m_{\text{water}}} \times 100$$

$$77 \text{ wt\%} = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + 1500 \text{ g}} \times 100$$

Solving for  $m_{sugar}$  yields  $m_{sugar} = 5022$  g

(b) Again using this same plot, at 20°C the solubility limit (or the concentration of the saturated solution) is about 64 wt% sugar.

(c) The mass of sugar in this saturated solution at 20°C ( $m'_{sugar}$ ) may also be calculated using Equation

4.3 as follows:

$$64 \text{ wt\%} = \frac{m'_{\text{sugar}}}{m'_{\text{sugar}} + 1500 \text{ g}} \times 100$$

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which yields a value for  $m'_{sugar}$  of 2667 g. Subtracting the latter from the former of these sugar concentrations yields the amount of sugar that precipitated out of the solution upon cooling  $m''_{sugar}$ ; that is

 $m''_{\text{sugar}} = m_{\text{sugar}} - m\tilde{Q}_{\text{ugar}} = 5022 \text{ g} - 2667 \text{ g} = 2355 \text{ g}$ 

9.2 At 500  $^{\circ}$ C (930  $^{\circ}$ F), what is the maximum solubility (a) of Cu in Ag? (b) Of Ag in Cu?

### Solution

(a) From Figure 9.7, the maximum solubility of Cu in Ag at 500°C corresponds to the position of the  $\beta$ -( $\alpha$  +  $\beta$ ) phase boundary at this temperature, or to about 2 wt% Cu.

(b) From this same figure, the maximum solubility of Ag in Cu corresponds to the position of the  $\alpha$ -( $\alpha$  +  $\beta$ ) phase boundary at this temperature, or about 1.5 wt% Ag.

9.8 Cite the phases that are present and the phase compositions for the following alloys:

- (a) 90 wt% Zn-10 wt% Cu at 400 °C (750 °F)
- (b) 75 wt% Sn-25 wt% Pb at 175 °C (345 °F)
- (c) 55 wt% Ag-45 wt% Cu at 900 °C (1650 °F)
- (d) 30 wt% Pb-70 wt% Mg at 425 °C (795 °F)

### Solution

This problem asks that we cite the phase or phases present for several alloys at specified temperatures.

(a) That portion of the Cu-Zn phase diagram (Figure 9.19) that pertains to this problem is shown below; the point labeled "A" represents the 90 wt% Zn-10 wt% Cu composition at 400°C.



As may be noted, point A lies within the  $\varepsilon$  and  $\eta$  phase field. A tie line has been constructed at 400°C; its intersection with the  $\varepsilon - \varepsilon + \eta$  phase boundary is at 87 wt% Zn, which corresponds to the composition of the  $\varepsilon$  phase. Similarly, the tie-line intersection with the  $\varepsilon + \eta - \eta$  phase boundary occurs at 97 wt% Zn, which is the composition of the  $\eta$  phase. Thus, the phase compositions are as follows:

$$C_{\varepsilon} = 87 \text{ wt\% Zn-13 wt\% Cu}$$
  
 $C_{n} = 97 \text{ wt\% Zn-3 wt\% Cu}$ 

(b) That portion of the Pb-Sn phase diagram (Figure 9.8) that pertains to this problem is shown below; the point labeled "B" represents the 75 wt% Sn-25 wt% Pb composition at 175°C.



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As may be noted, point B lies within the  $\alpha + \beta$  phase field. A tie line has been constructed at 175°C; its intersection with the  $\alpha - \alpha + \beta$  phase boundary is at 16 wt% Sn, which corresponds to the composition of the  $\alpha$  phase. Similarly, the tie-line intersection with the  $\alpha + \beta - \beta$  phase boundary occurs at 97 wt% Sn, which is the composition of the  $\beta$  phase. Thus, the phase compositions are as follows:

$$C_{\alpha} = 16 \text{ wt\% Sn-84 wt\% Pb}$$
  
 $C_{\beta} = 97 \text{ wt\% Sn-3 wt\% Pb}$ 

(c) The Ag-Cu phase diagram (Figure 9.7) is shown below; the point labeled "C" represents the 55 wt% Ag-45 wt% Cu composition at 900°C.



As may be noted, point C lies within the Liquid phase field. Therefore, only the liquid phase is present; its composition is 55 wt% Ag-45 wt% Cu.

(d) The Mg-Pb phase diagram (Figure 9.20) is shown below; the point labeled "D" represents the 30 wt% Pb-70 wt% Mg composition at 425°C.

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As may be noted, point D lies within the  $\alpha$  phase field. Therefore, only the  $\alpha$  phase is present; its composition is 30 wt% Pb-70 wt% Mg.