

CHAPTER 9

PHASE DIAGRAMS

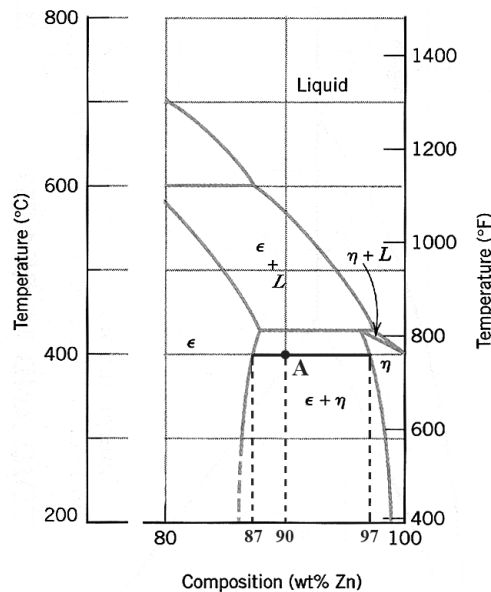
PROBLEM SOLUTIONS

9.14 Determine the relative amounts (in terms of mass fractions) of the phases for the alloys and temperatures given in Problem 9.8.

Solution

This problem asks that we determine the phase mass fractions for the alloys and temperatures in Problem 9.8.

(a) From Problem 9.8a, ϵ and η phases are present for a 90 wt% Zn-10 wt% Cu alloy at 400°C, as represented in the portion of the Cu-Zn phase diagram shown below (at point A).



Furthermore, the compositions of the phases, as determined from the tie line are

$$C_{\epsilon} = 87 \text{ wt\% Zn-13 wt\% Cu}$$

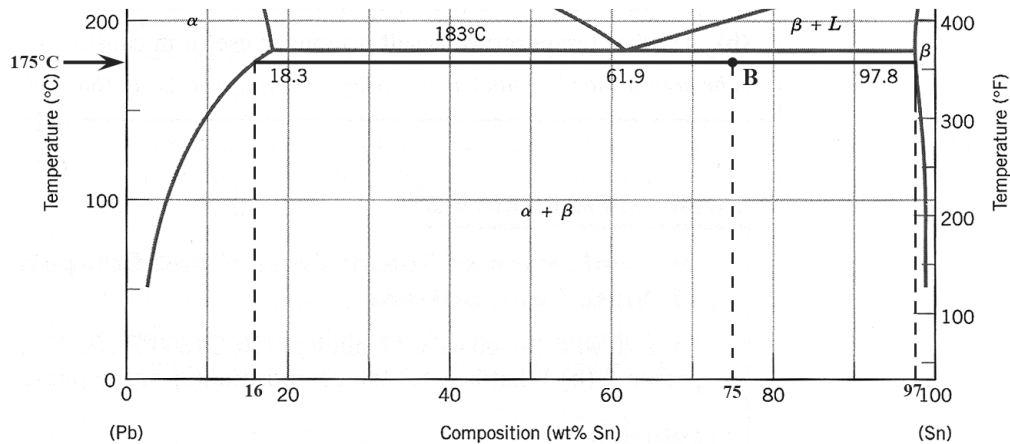
$$C_{\eta} = 97 \text{ wt\% Zn-3 wt\% Cu}$$

Inasmuch as the composition of the alloy $C_0 = 90 \text{ wt\% Zn}$, application of the appropriate lever rule expressions (for compositions in weight percent zinc) leads to

$$W_{\epsilon} = \frac{C_{\eta} - C_0}{C_{\eta} - C_{\epsilon}} = \frac{97 - 90}{97 - 87} = 0.70$$

$$W_{\eta} = \frac{C_0 - C_{\epsilon}}{C_{\eta} - C_{\epsilon}} = \frac{90 - 87}{97 - 87} = 0.30$$

(b) From Problem 9.8b, α and β phases are present for a 75 wt% Sn-25 wt% Pb alloy at 175°C, as represented in the portion of the Pb-Sn phase diagram shown below (at point B).



Furthermore, the compositions of the phases, as determined from the tie line are

$$C_{\alpha} = 16 \text{ wt\% Sn-84 wt\% Pb}$$

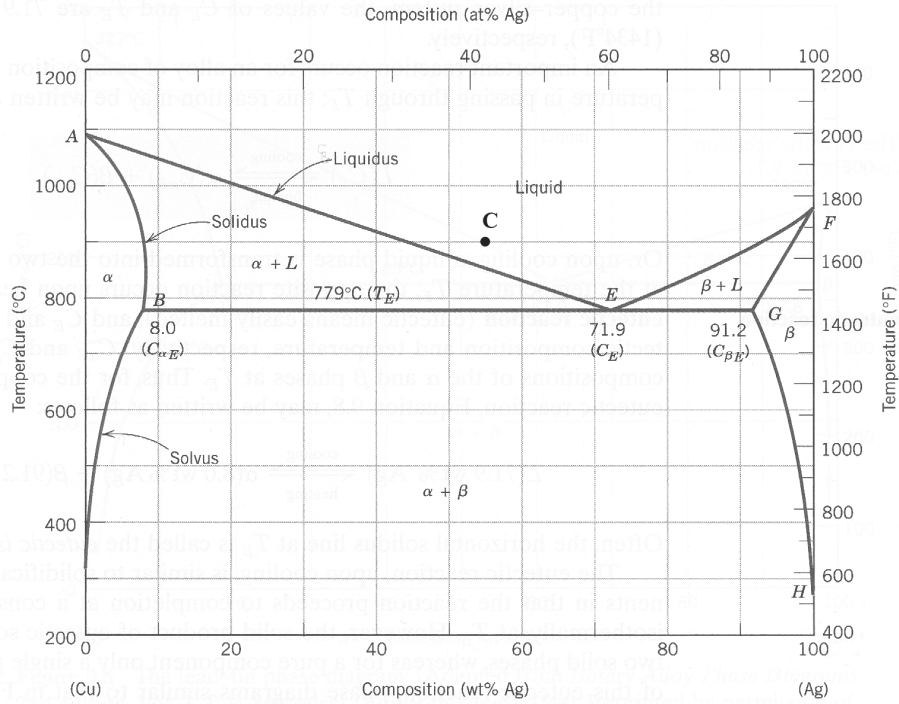
$$C_{\beta} = 97 \text{ wt\% Sn-3 wt\% Pb}$$

Inasmuch as the composition of the alloy $C_0 = 75 \text{ wt\% Sn}$, application of the appropriate lever rule expressions (for compositions in weight percent tin) leads to

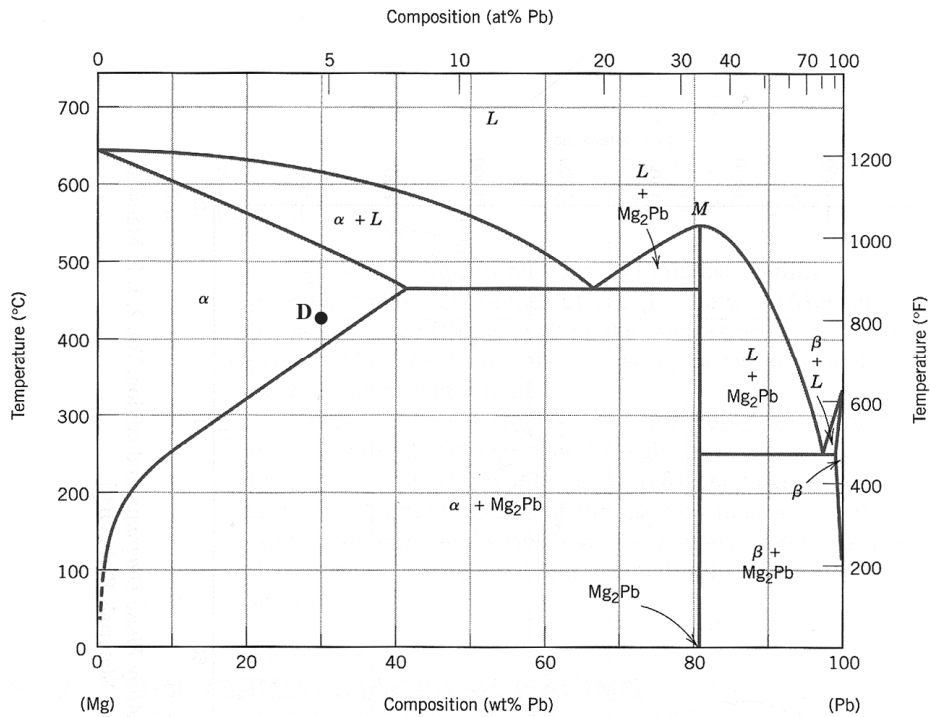
$$W_{\alpha} = \frac{C_{\beta} - C_0}{C_{\beta} - C_{\alpha}} = \frac{97 - 75}{97 - 16} = 0.27$$

$$W_{\beta} = \frac{C_0 - C_{\alpha}}{C_{\beta} - C_{\alpha}} = \frac{75 - 16}{97 - 16} = 0.73$$

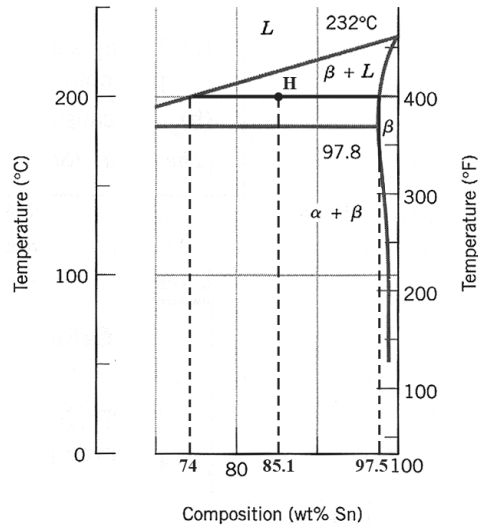
(c) From Problem 9.8c, just the liquid phase is present for a 55 wt% Ag-45 wt% Cu alloy at 900°C, as may be noted in the Ag-Cu phase diagram shown below (at point C)—i.e., $W_L = 1.0$



(d) From Problem 9.8d, just the α phase is present for a 30 wt% Pb-70 wt% Mg alloy at 425°C, as may be noted in the Mg-Pb phase diagram shown below (at point D)—i.e., $W_{\alpha} = 1.0$



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Furthermore, the compositions of the phases, as determined from the tie line are

$$C_{\beta} = 97.5 \text{ wt\% Sn} - 2.5 \text{ wt\% Pb}$$

$$C_L = 74 \text{ wt\% Sn} - 26 \text{ wt\% Pb}$$

Inasmuch as the composition of the alloy $C_0 = 85.1 \text{ wt\% Sn}$, application of the appropriate lever rule expressions (for compositions in weight percent lead) leads to

$$W_{\beta} = \frac{C_0 - C_L}{C_{\beta} - C_L} = \frac{85.1 - 74}{97.5 - 74} = 0.47$$

$$W_L = \frac{C_{\beta} - C_0}{C_{\beta} - C_L} = \frac{97.5 - 85.1}{97.5 - 74} = 0.53$$

9.18 A 30 wt% Sn-70 wt% Pb alloy is heated to a temperature within the α + liquid phase region. If the mass fraction of each phase is 0.5, estimate:

(a) The temperature of the alloy

(b) The compositions of the two phases

Solution

(a) We are given that the mass fractions of α and liquid phases are both 0.5 for a 30 wt% Sn-70 wt% Pb alloy and asked to estimate the temperature of the alloy. Using the appropriate phase diagram, Figure 9.8, by trial and error with a ruler, a tie line within the α + L phase region that is divided in half for an alloy of this composition exists at about 230°C.

(b) We are now asked to determine the compositions of the two phases. This is accomplished by noting the intersections of this tie line with both the solidus and liquidus lines. From these intersections, $C_{\alpha} = 15$ wt% Sn, and $C_L = 43$ wt% Sn.

9.23 Derive Equations 9.6a and 9.7a, which may be used to convert mass fraction to volume fraction, and vice versa.

Solution

This portion of the problem asks that we derive Equation 9.6a, which is used to convert from phase weight fraction to phase volume fraction. Volume fraction of phase α , V_{α} , is defined by Equation 9.5 as

$$V_{\alpha} = \frac{v_{\alpha}}{v_{\alpha} + v_{\beta}} \quad (9.S1)$$

where v_{α} and v_{β} are the volumes of the respective phases in the alloy. Furthermore, the density of each phase is equal to the ratio of its mass and volume, or upon rearrangement

$$v_{\alpha} = \frac{m_{\alpha}}{\rho_{\alpha}} \quad (9.S2a)$$

$$v_{\beta} = \frac{m_{\beta}}{\rho_{\beta}} \quad (9.S2b)$$

Substitution of these expressions into Equation 9.S1 leads to

$$V_{\alpha} = \frac{\frac{m_{\alpha}}{\rho_{\alpha}}}{\frac{m_{\alpha}}{\rho_{\alpha}} + \frac{m_{\beta}}{\rho_{\beta}}} \quad (9.S3)$$

in which m 's and ρ 's denote masses and densities, respectively. Now, the mass fractions of the α and β phases (i.e., W_{α} and W_{β}) are defined in terms of the phase masses as

$$W_{\alpha} = \frac{m_{\alpha}}{m_{\alpha} + m_{\beta}} \quad (9.S4a)$$

$$W_{\beta} = \frac{m_{\beta}}{m_{\alpha} + m_{\beta}} \quad (9.S4b)$$

Which, upon rearrangement yield

$$m_{\alpha} = W_{\alpha} (m_{\alpha} + m_{\beta}) \quad (9.S5a)$$

$$m_{\beta} = W_{\beta} (m_{\alpha} + m_{\beta}) \quad (9.S5b)$$

Incorporation of these relationships into Equation 9.S3 leads to

$$V_{\alpha} = \frac{\frac{W_{\alpha} (m_{\alpha} + m_{\beta})}{\rho_{\alpha}}}{\frac{W_{\alpha} (m_{\alpha} + m_{\beta})}{\rho_{\alpha}} + \frac{W_{\beta} (m_{\alpha} + m_{\beta})}{\rho_{\beta}}}$$

$$V_{\alpha} = \frac{\frac{W_{\alpha}}{\rho_{\alpha}}}{\frac{W_{\alpha}}{\rho_{\alpha}} + \frac{W_{\beta}}{\rho_{\beta}}} \quad (9.S6)$$

which is the desired equation.

Development of Microstructure in Isomorphous Alloys

9.25 (a) *Briefly describe the phenomenon of coring and why it occurs.*

(b) *Cite one undesirable consequence of coring.*

Solution

(a) Coring is the phenomenon whereby concentration gradients exist across grains in polycrystalline alloys, with higher concentrations of the component having the lower melting temperature at the grain boundaries. It occurs, during solidification, as a consequence of cooling rates that are too rapid to allow for the maintenance of the equilibrium composition of the solid phase.

(b) One undesirable consequence of a cored structure is that, upon heating, the grain boundary regions will melt first and at a temperature below the equilibrium phase boundary from the phase diagram; this melting results in a loss in mechanical integrity of the alloy.

Development of Microstructure in Eutectic Alloys

9.28 *Briefly explain why, upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases.*

Solution

Upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases because during the solidification atomic diffusion must occur, and with this layered configuration the diffusion path length for the atoms is a minimum.

9.29 *What is the difference between a phase and a microconstituent?*

Solution

A “phase” is a homogeneous portion of the system having uniform physical and chemical characteristics, whereas a “microconstituent” is an identifiable element of the microstructure (that may consist of more than one phase).

9.33 The microstructure of a lead-tin alloy at 180 °C (355 °F) consists of primary β and eutectic structures. If the mass fractions of these two microconstituents are 0.57 and 0.43, respectively, determine the composition of the alloy.

Solution

Since there is a primary β microconstituent present, then we know that the alloy composition, C_0 is between 61.9 and 97.8 wt% Sn (Figure 9.8). Furthermore, this figure also indicates that $C_\beta = 97.8$ wt% Sn and $C_{\text{eutectic}} = 61.9$ wt% Sn. Applying the appropriate lever rule expression for W_β ,

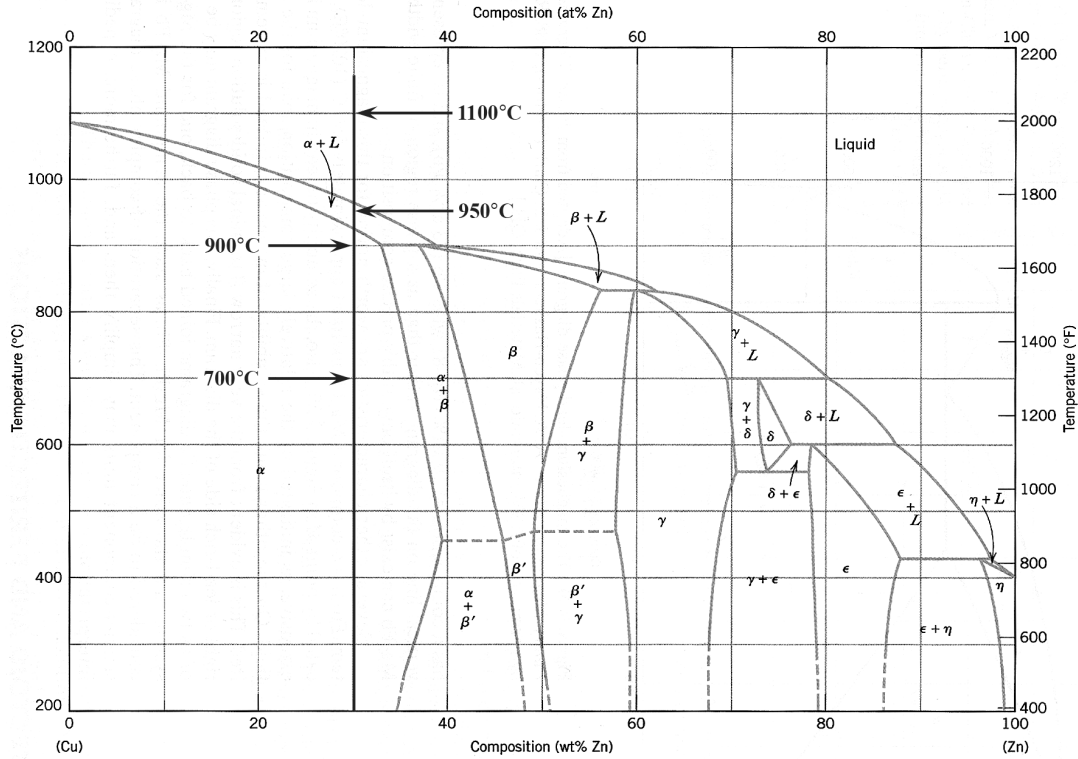
$$W_\beta = \frac{C_0 - C_{\text{eutectic}}}{C_\beta - C_{\text{eutectic}}} = \frac{C_0 - 61.9}{97.8 - 61.9} = 0.57$$

and solving for C_0 yields $C_0 = 82.4$ wt% Sn.

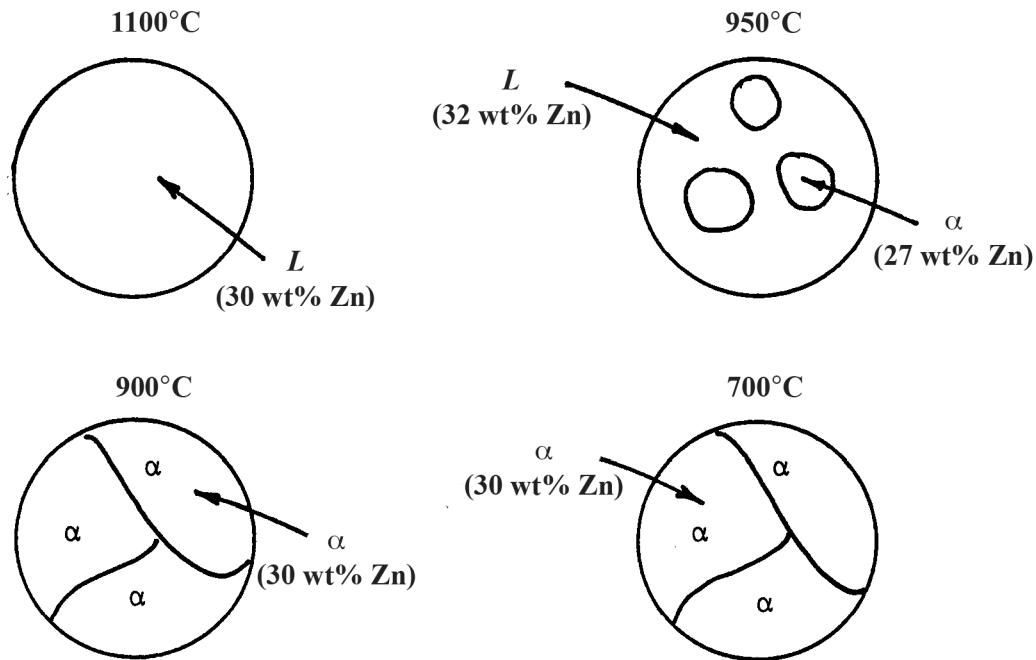
9.37 For a 30 wt% Zn-70 wt% Cu alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1100 °C (2010 °F), 950 °C (1740 °F), 900 °C (1650 °F), and 700 °C (1290 °F). Label all phases and indicate their approximate compositions.

Solution

The illustration below is the Cu-Zn phase diagram (Figure 9.19). A vertical line at a composition of 30 wt% Zn-70 wt% Cu has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 1100°C, 950°C, 900°C, and 700°C).



On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:



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Congruent Phase Transformations Eutectoid and Peritectic Reactions

9.41 *What is the principal difference between congruent and incongruent phase transformations?*

Solution

The principal difference between congruent and incongruent phase transformations is that for congruent no compositional changes occur with any of the phases that are involved in the transformation. For incongruent there will be compositional alterations of the phases.

The Iron-Iron Carbide (Fe-Fe₃C) Phase Diagram Development of Microstructure in Iron-Carbon Alloys