# FRACTURE MECHANICS EXAMPLES

Determine: residual strength for a plate with a center-crack. Tip-to-tip crack length is 3 inches ($2a = 3''$).
Width of plate $= W = 9''$
$K_{Ic} = 45 \text{ ksi}\sqrt{\text{in}}$ $\sigma_{ys} = 80 \text{ ksi}$

Assume: LEFM is valid ($\sigma < 0.8\sigma_{ys}$)
Plane strain dominates at the crack tip $\left(B > 2.5\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2\right)$

Note plate thickness ($B$) is not given so validity of plane strain cannot be validated.

Sol'n: Failure (fracture → critical (rapid) crack growth) occurs when $K_I \geq K_{Ic}$

$K_I$: $K_I = \sigma\sqrt{\pi a}\sqrt{\sec\frac{\pi a}{W}}$ (back of CH 7)

$2a = 3'' \Rightarrow a = 1.5''$ $W = 9''$

at critical crack growth $K_I = K_{Ic}$

$$K_{Ic} = \sigma_{crit}\sqrt{\pi a_{crit}}\left(\sec\frac{\pi a_{crit}}{w}\right)^{1/2}$$

$$\Rightarrow \quad \sigma_{cr} = \frac{K_{Ic}}{\sqrt{\pi a_{cr}}\left(\sec\frac{\pi a_{cr}}{w}\right)^{1/2}}$$

$$= \frac{45\,ksi\sqrt{in}}{\left(\pi(1.5in)\sec\frac{1.5\pi}{9''}\right)^{1/2}}$$

$$= 19.29\,ksi = \underline{19\,ksi}$$

Now check validity of LEFM assumption

$\sigma_{cr} < 0.8\sigma_{ys}$ ?

$19\,ksi < (0.8)(80\,ksi)$ TRUE

$\therefore$ LEFM valid

NOTE: $\sec(\theta) = \frac{1}{\cos(\theta)}$

- FOR THE PREVIOUS EXAMPLE, WHAT IS THE MINIMUM THICKNESS (B) THAT ALLOWS VALID ASSUMPTION OF PLANE STRAIN?

SOL'N: $B > 2.5\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2 = 2.5\left(\frac{45\ ksi\sqrt{in}}{80\ ksi}\right)^2$

$= 0.79\ in \simeq \underline{0.8\ in}$

∴ For $B > 0.8$" plane strain to be valid

If $B < 0.8$ " " is not dominant

- If $B = 2\ in$, what is the residual strength in the previous example. Express the answer is both force & stress.

If $B > 0.8$", then the above analysis is valid.

resid. strength is $\underline{19\ ksi}$

force: $F = \sigma \cdot A = (19\ ksi)(B \cdot w)$

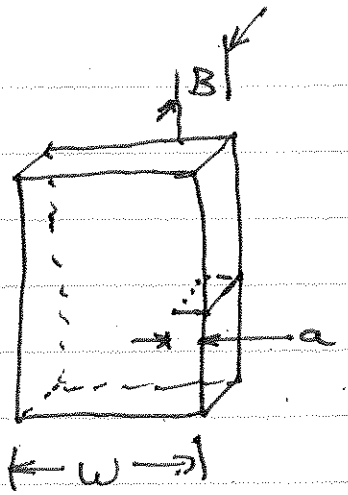
$= (19\ ksi)(2\ in)(9\ in) = \underline{342\ kip}$

- If $B = 0.5\ in$, what is the res. strength in the previous ex? $B < 0.8$" ∴ not pl. strain

∴ residual strength is greater than $\underline{19\ ksi}$

Force $> \sigma \cdot A = (19\ ksi)(0.5")(9") = \underline{85.5\ kip}$

$\underline{F_{res.} > 85.5\ kip}$




Ex. Given: edge crack in plate

$a = 0.2"$

$K_{Ic} = 50 \text{ ksi}\sqrt{\text{in}}$

$\sigma_{ys} = 60 \text{ ksi}$

$B = 2" \quad w = 8"$

Find: Force req'd to fracture (critical crack growth)

Assume: LEFM $\left(\sigma < 0.8\sigma_{ys}\right)$

plane strain $\quad B > 2.5\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2$

check plane strain:

$$2.5\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2 = 2.5\left(\frac{50 \text{ ksi}\sqrt{\text{in}}}{60 \text{ ksi}}\right)^2$$

$$= 1.74 \text{ in} < B$$

$\therefore$ plane strain dominates

If $\frac{a}{w}$ is "small": $K_I = 1.99\sigma\sqrt{a}$

$\frac{a}{w} = \frac{0.2}{8} = 0.025 \sim$ small

At critical crack growth: $K_I = K_{Ic} \quad \sigma = \sigma_{cr} \quad a = a_{cr}$

$$\sigma_{cr} = \frac{K_{Ic}}{1.99\sqrt{a_{cr}}} = \frac{50 \text{ ksi}\sqrt{\text{in}}}{1.99(0.2 \text{ in})^{1/2}} = 56.2 \text{ ksi}$$

$F_{cr} = \sigma_{cr} A = (54.2 \text{ ksi})(8" \cdot 2") = \underline{900 \text{ k}}$

Check LEFM:

is $\sigma_{cr} < (0.8)\sigma_{ys}$

$54.2 \text{ ksi} \; ?< \; (0.8)(60 \text{ ksi}) = 48 \text{ ksi}$

FALSE, LEFM is not valid.

Therefore, see pg 7-16 to determine resid. strength if LEFM is not valid.

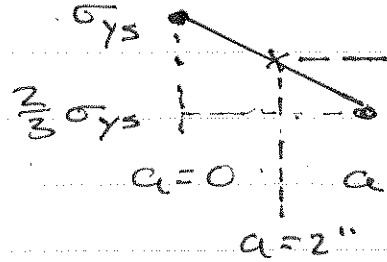
- Find $a_{cr}$ if $\sigma_{cr} = \frac{2}{3}\sigma_{ys} = (\frac{2}{3})(60 \text{ ksi}) = 40 \text{ ksi}$

$$a_{cr} = \left(\frac{K_{IC}}{1.99\sigma_{cr}}\right)^2 = \left(\frac{50 \text{ ksi}\sqrt{\text{in}}}{1.99(40 \text{ ksi})}\right)^2$$

$a_{cr} = 0.4 \text{ in}$

Actual crack length in this problem is 0.2 in

Linear interpolate:




$\sigma_{res\,strength}$ = (half way in this problem between $\sigma_{ys}$ & $\frac{2}{3}\sigma_{ys}$

$\therefore \sigma_{res\,strength} = \underline{50 \text{ ksi}}$

#11
Brock

Calculate the max permissible crack size in a center cracked panel subjected to a uniform stress of 20ksi.

Given $K_{Ic} = 50 ksi\sqrt{in}$ $\sigma_{ys} = 60 ksi$ $W = 12 in$

$$K_I = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

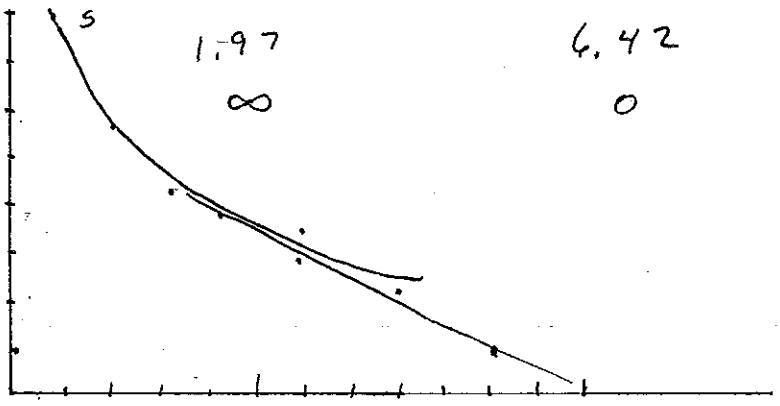
~~Most iterate or use calculator a/g?~~

Create a diagram w/ $\sigma$ vs $a$

Assume LEFM ($\sigma < 0.8\sigma_{ys}$), Plane Strain

| $2a$ | $a$ | $\beta = \sqrt{\sec\frac{\pi a}{W}}$ | $\sigma_c = \frac{K_{Ic}}{\beta\sqrt{\pi a}}$ |
|---|---|---|---|
| "0" | "0" | 1 | — |
| 1" | 0.5" | 1.004 | 39.73 ksi |
| 2 | 1 | 1.017 | 27.73 |
| 3 | 1.5 | 1.040 | 22.14 |
| 4 | 2 | 1.075 | 18.56 |
| 6 | 3 | 1.189 | 13.70 |
| 8 | 4 | 1.414 | 9.98 |
| 10 | 5 | 1.97 | 6.42 |
| 12 | | $\infty$ | 0 |

$2a \approx 3.5$
$a = 1.75 in$



$2a$