1. Given $S_{max} = 20\,ksi$; $S_{min} = 0$

Edge crack w/ "wide" plate

$C = 1 \times 10^{-8}$ $n = 4$ (Paris Law)

Assume Paris Law is valid, even though it isn't

Find, using numerical methods

a) N for $a_0 = 0.1"$ to $a_1 = 0.35"$ & $a_2 = 1.0"$

b) N " $a_0 = 0.25$ to $a_1 = 0.50"$ & $a_3 = 1.0"$

c) Repeat a & b using integration

Soln

$$\frac{\Delta a}{\Delta N} = C(\Delta K)^n$$

$$\Delta K = 1.99(\Delta S)\sqrt{a}$$

$$\Delta S = 20\,ksi - 0\,ksi = 20\,ksi$$

$$\Delta N = \frac{\Delta a}{C(\Delta K)^n}$$ – use excel

letting $\Delta a = 0.01"$

let $a_0 = 0.01"$

Main Data Pts

a (in)	N_{TOT} (from $a_0 = 0.01"$)
0.1	2190
0.25	2414
0.35	2458
0.50	2491
1.00	2531

1 cont Soln 2/

a) $a_0 = 0.1$ to 0.35 $N = 2458 - 2190 = \underline{268}$

a') $a_0 = 0.1$ to 1.00 $N = 2531 - 2190 = \underline{341}$

b) $a_0 = 0.25$ to 0.50 $N = 2491 - 2414 = \underline{77}$

$a_0 = 0.25$ to 1.00 $N = 2531 - 2414 = \underline{117}$

∴ the larger crack (0.25) grows much faster than the smaller crack

c)

$$N = \int_{a_0}^{a_1} \frac{da}{C(\Delta K)^n} = \int \frac{da}{C\left(1.99\left(a^{1/2}\right)20\right)^4}$$

$$= \int \frac{da}{0.025\, a^2} = 40 \int \frac{da}{a^2} = -40\left(\frac{1}{a}\right)\Bigg|_{a_0}^{a_1}$$

a) $a_0 = 0.1$ $a_1 = 0.35$ $N = \underline{285}$

a' $a_0 = 0.1$ $a_1 = 1$ $N = \underline{360}$

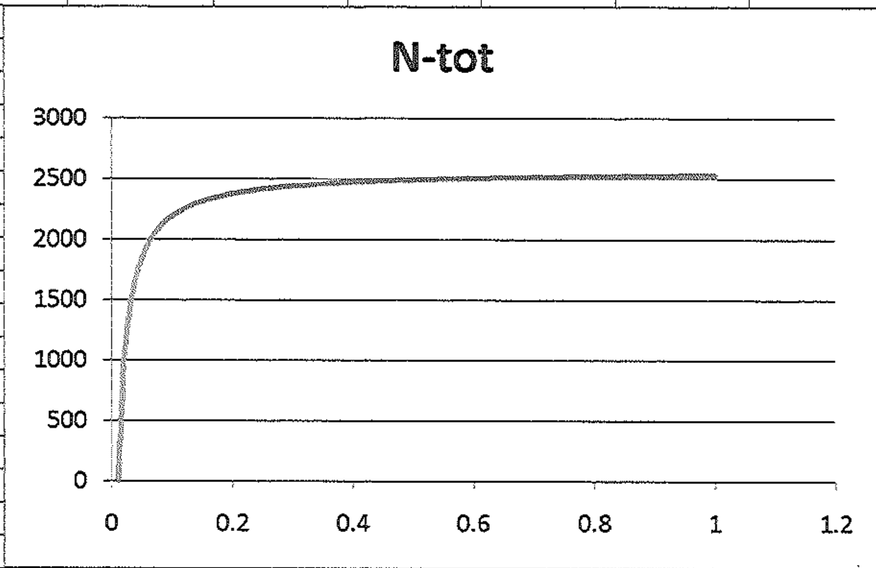
b) $a_0 = 0.25$ $a_1 = 0.5$ $N = \underline{80}$

b' $a_0 = 0.25$ $a_1 = 1.0$ $N = \underline{120}$

Compare: they are similar w/ numerical integration being slightly less

Prob 1

Given:		
Smax	20	ksi
Smin	0	
n	4	
C	1.00E-08	
edge crack, small a/W		
Find: number of cycles to grow 0.25" more, and number of cycles to reach a-crit		
part a)	a-orig	0.01
part b)	a-orig	0.25



a	delta a	delta K	C(dK)^n	delta N	N-tot
0.01					0
0.02	0.01	6	1.00E-05	9.96E+02	996
0.03	0.01	7	2.26E-05	4.43E+02	1439
0.04	0.01	8	4.01E-05	2.49E+02	1688
0.05	0.01	9	6.27E-05	1.59E+02	1848
0.06	0.01	10	9.03E-05	1.11E+02	1958
0.07	0.01	11	1.23E-04	8.13E+01	2040
0.08	0.01	11	1.61E-04	6.23E+01	2102
0.09	0.01	12	2.03E-04	4.92E+01	2151
0.1	0.01	13	2.51E-04	3.99E+01	2191
0.11	0.01	13	3.04E-04	3.29E+01	2224
0.12	0.01	14	3.61E-04	2.77E+01	2252
0.13	0.01	14	4.24E-04	2.36E+01	2275
0.14	0.01	15	4.92E-04	2.03E+01	2296
0.15	0.01	15	5.65E-04	1.77E+01	2313
0.16	0.01	16	6.42E-04	1.56E+01	2329
0.17	0.01	16	7.25E-04	1.38E+01	2343
0.18	0.01	17	8.13E-04	1.23E+01	2355
0.19	0.01	17	9.06E-04	1.10E+01	2366
0.2	0.01	18	1.00E-03	9.96E+00	2376
0.21	0.01	18	1.11E-03	9.04E+00	2385
0.22	0.01	19	1.21E-03	8.23E+00	2393
0.23	0.01	19	1.33E-03	7.53E+00	2401
0.24	0.01	19	1.45E-03	6.92E+00	2408
0.25	0.01	20	1.57E-03	6.38E+00	2414
0.26	0.01	20	1.70E-03	5.90E+00	2420
0.27	0.01	21	1.83E-03	5.47E+00	2425
0.28	0.01	21	1.97E-03	5.08E+00	2430
0.29	0.01	21	2.11E-03	4.74E+00	2435
0.3	0.01	22	2.26E-03	4.43E+00	2440
0.31	0.01	22	2.41E-03	4.15E+00	2444
0.32	0.01	23	2.57E-03	3.89E+00	2448
0.33	0.01	23	2.73E-03	3.66E+00	2451
0.34	0.01	23	2.90E-03	3.45E+00	2455
0.35	0.01	24	3.07E-03	3.25E+00	2458
0.36	0.01	24	3.25E-03	3.08E+00	2461
0.37	0.01	24	3.44E-03	2.91E+00	2464
0.38	0.01	25	3.62E-03	2.76E+00	2467
0.39	0.01	25	3.82E-03	2.62E+00	2469

2. Given: $S_{max} = 15$ ksi $S_{min} = 3$ ksi

center crack

$W = 6$ $B = 0.1$ $\sigma_{yo} = 48$ ksi

$K_{Ic} = 80 \text{ ksi}\sqrt{in}$

a (in)	N
0.05	0
0.20	24,000
0.40	54,000
0.70	68,000
1.00	74,000
2.00	77,000

Determine Paris Law Parameters C & n

$$\Delta K = (\Delta\sigma)\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

$$\Delta\sigma = S_{max} - S_{min} = 15 \text{ ksi} - 3 \text{ ksi}$$

$$= 12 \text{ ksi}$$

$$\Delta K = \frac{12\sqrt{\pi a_{ave}}}{\left(\cos\frac{\pi a_{ave}}{W}\right)^{1/2}}$$

Excel Example:

a	N	Δa	ΔN	a_{ave} etc
0.05	0			
0.20	2400	0.2 - 0.05	2400 - 1	$\frac{1}{2}(0.2 + 0.05)$

2 cont

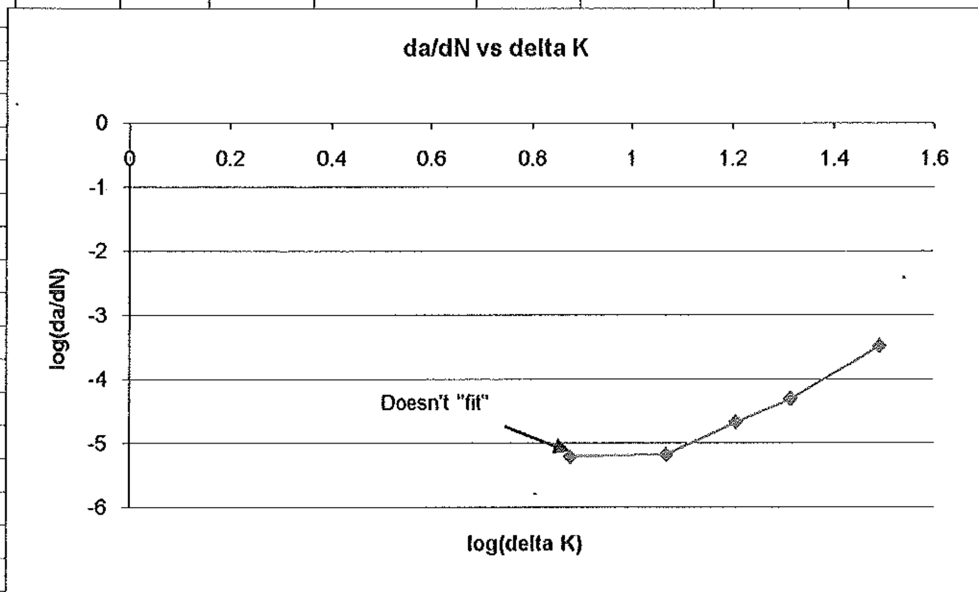
Plotting all 5 sets of $\log \frac{\Delta a}{\Delta N}$ vs $\log(\Delta K)$ show the first set does not fit on a straight line. Then plot other 4 sets, fit straight line

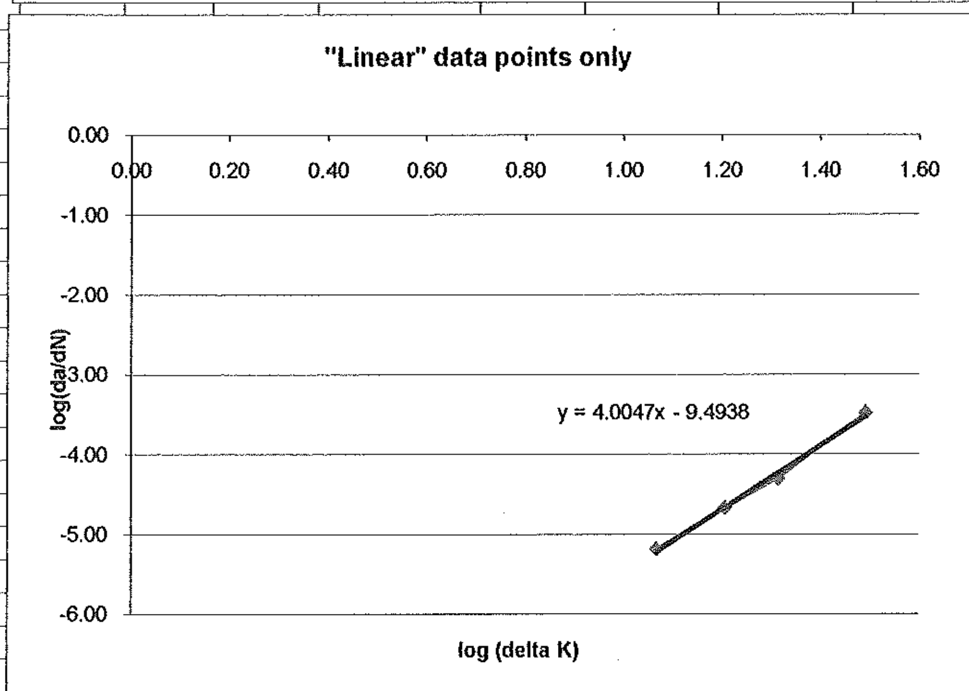
$$y = 4.0x - 9.494$$

$$n = 4.0 \qquad \log(C) = (-9.494)$$

$$C = 3.21 \times 10^{-10}$$

a	N	da	dN	da/dN	ave a	dK	log(da/dN)	lg(dK)	log(da/dN)
0.05	0								
0.20	24000	0.15	24000	0.00000625	0.125	7.53	-5.20	0.88	-5.20
0.40	54000	0.2	30000	6.6667E-06	0.3	11.72	-5.18	1.07	-5.18
0.70	68000	0.3	14000	2.1429E-05	0.55	16.11	-4.67	1.21	-4.67
1.00	74000	0.3	6000	0.00005	0.85	20.64	-4.30	1.31	-4.30
2.00	77000	1	3000	0.00033333	1.5	30.98	-3.48	1.49	-3.48





C

3. Given $n = 4$ $C = 3.21 \times 10^{-10}$ Paris Law

$a_0 = 0.5''$ edge crack

$\frac{da}{dN} = 8 \times 10^{-6}$ in/cycl ($a_0 = 0.5$)

$R = \phi$ ($\therefore S_{min} = 0$)

Find S_{max}

Assume: Paris Law is valid

LEFM ($\sigma < 0.8\sigma_{ys}$ – checked ✓)

W is large ($\frac{a}{W}$ is small)

Soln: Find S_{max}

$$K_I = 1.99\,\sigma\sqrt{a}$$

Paris Law $$\frac{da}{dN} = C(\Delta K)^n$$

$$\log\left(\frac{da}{dN}\right) = \log C + n\log(\Delta K)$$

$$\log(8 \times 10^{-6}) = \log(3.21 \times 10^{-10}) + 4\log(\Delta K)$$

$$\log(\Delta K) = 1.099$$

$$\Delta K = 12.56 = 1.99\,\sigma\sqrt{0.5}$$

$$S_{max} = \sigma = \underline{8.9\,ksi}$$

Soln

4. Given Inconel X-750 at Room temp:

$\Delta K = 18\ MPa\sqrt{m}$ $da/dN = 9\times10^{-6}$ mm/cycle

$\Delta K = 50\ MPa\sqrt{m}$ $da/dN = 1\times10^{-3}$ mm/cycle

SEM image provided for a test at 650°C on ~~#~~ with $\Delta K = 35\ MPa\sqrt{m}$

Q: Does fatigue rate increase or decrease at 650°F vs RT ?

Assume: LEFM & Paris Law are valid

Soln:

From SEM, $da/dN = \frac{10\times10^{-3}\ mm}{12\ cycles} = 8.3\times10^{-4}\ \frac{mm}{cyc}$

Plot the following

$\log(da/dN$ vs $\log(\Delta K)$ for

$\Delta K = 18\ MPa\sqrt{m}$, $da/dN = 9\times10^{-6}$ mm/cyc

$\Delta K = 50\ MPa\sqrt{m}$ $da/dN = 1\times10^{-3}$ mm/cy

$\Delta K = 35\ MPa\sqrt{m}$ $da/d = 8.3\times10^{-4}$ mm/cyc

If temp has no effect, then the 650°C data pt will be in-line w/ the other 2 pt.

From attached Excel plot, for $\Delta K = 35\ MPa\sqrt{m}$

$\log(da/dN) = -3.6$ (in-line w/ RT data)

$\frac{da}{dN} = 2.5\times10^{-4}$ mm/cyc $< 8.3\times10^{-4}$ mm/cycle

∴ crack growth is increased at 650°

RT

Delta K	da/dN (mm/cyc)	log(delta K)	log(da/dN)
18	9.00E-06	1.26	-5.05
50	1.00E-03	1.70	-3.00
35	8.30E-04	1.54	-3.08