2010 - ME421 - Fract. Mech 1

1) Given: Flat plate, ω = unity
thru thickness center crack
Plot K_I vs a ($a \to .01 \to \omega$)
using "short crack" & "general" eqns
When do they differ by >5%?

short: $K_I = \sigma\sqrt{\pi a}$

general $K_I = \sigma\sqrt{\pi a}\left(\frac{1}{\cos\left(\frac{a\pi}{\omega}\right)}\right)^{1/2}$

Let $\sigma = 1$, let $Y = \sqrt{a}$ for short

$Y = \sqrt{\frac{a}{\cos\left(\frac{\pi a}{\omega}\right)}}$ for gen'l

$\therefore K_I = Y\sqrt{a}$

From Excel: at ~~a~~ $\frac{a}{\omega} \approx 0.15$
there is a 5.6 % d.t

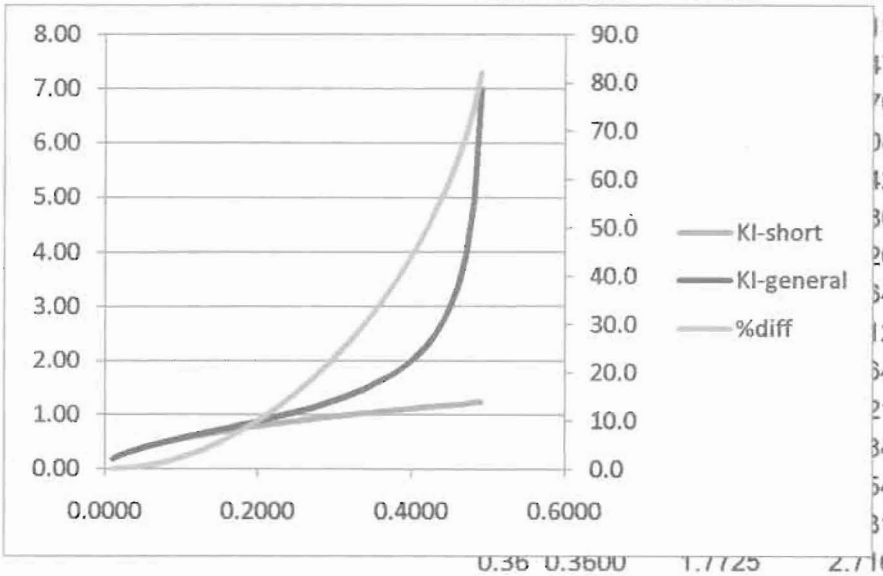
ME421-2010, FM 1, Problem 1

a (inch)	varies
W (inch)	1
stress	1

PROBLEM:

Plot KI for center crack using short crack

	a	a/W	Y-short	Y-general	KI-short	KI-general	%diff
	0.01	0.0100	1.7725	1.773	0.18	0.18	0.0
	0.02	0.0200	1.7725	1.774	0.25	0.25	0.1
	0.03	0.0300	1.7725	1.776	0.31	0.31	0.2
	0.04	0.0400	1.7725	1.779	0.35	0.36	0.4
	0.05	0.0500	1.7725	1.783	0.40	0.40	0.6
	0.06	0.0600	1.7725	1.788	0.43	0.44	0.9
	0.07	0.0700	1.7725	1.794	0.47	0.47	1.2
	0.08	0.0800	1.7725	1.801	0.50	0.51	1.6
	0.09	0.0900	1.7725	1.809	0.53	0.54	2.0
	0.1	0.1000	1.7725	1.817	0.56	0.57	2.5
	0.11	0.1100	1.7725	1.827	0.59	0.61	3.0
	0.12	0.1200	1.7725	1.838	0.61	0.64	3.6
	0.13	0.1300	1.7725	1.850	0.64	0.67	4.2
	0.14	0.1400	1.7725	1.863	0.66	0.70	4.9
>5%	**0.15**	**0.1500**	**1.7725**	**1.878**	**0.69**	**0.73**	**5.6**
	0.16	0.1600	1.7725	1.893	0.71	0.76	6.4
	0.17	0.1700	1.7725	1.910	0.73	0.79	7.2
	0.18	0.1800	1.7725	1.929	0.75	0.82	8.1
	0.19	0.1900	1.7725	1.949	0.77	0.85	9.1
>10%	**0.2**	**0.2000**	**1.7725**	**1.971**	**0.79**	**0.88**	**10.1**
	0.21	0.2100	1.7725	1.994	0.81	0.91	11.1
					9	0.83	0.95	12.2
					47	0.85	0.98	13.4
					76	0.87	1.02	14.6
					08	0.89	1.05	15.9
					42	0.90	1.09	17.3
					80	0.92	1.13	18.7
					20	0.94	1.17	20.2
					64	0.95	1.22	21.7
					12	0.97	1.27	23.3
					64	0.99	1.32	25.0
					21	1.00	1.37	26.8
					84	1.02	1.43	28.7
					54	1.03	1.49	30.6
					31	1.05	1.56	32.6
	0.36	0.3600	1.7725	2.716		1.06	1.63	34.7
	0.37	0.3700	1.7725	2.813		1.08	1.71	37.0
	0.38	0.3800	1.7725	2.921		1.09	1.80	39.3
	0.39	0.3900	1.7725	3.045		1.11	1.90	41.8
	0.4	0.4000	1.7725	3.188		1.12	2.02	44.4
	0.41	0.4100	1.7725	3.356		1.13	2.15	47.2
	0.42	0.4200	1.7725	3.554		1.15	2.30	50.1
	0.43	0.4300	1.7725	3.795		1.16	2.49	53.3
	0.44	0.4400	1.7725	4.095		1.18	2.72	56.7
	0.45	0.4500	1.7725	4.481		1.19	3.01	60.4
	0.46	0.4600	1.7725	5.007		1.20	3.40	64.6



2010 - ME421 - FRACT MECH I

2) Given: Flat plate, w = unity
Thru-thickness edge-crack
Plot K_I vs a using
eqn for "short" crack & general.

Edge: $K_I = Y \sigma \sqrt{a}$

"short": $Y = 1.99$

general: $Y = 1.95 - 0.41\left(\frac{a}{w}\right) + 18.7\left(\frac{a}{w}\right)^2$
$- 38.48\left(\frac{a}{w}\right)^3 + 53.85\left(\frac{a}{w}\right)^4$

Let $\sigma = 1$

$K_I = Y\sqrt{a}$

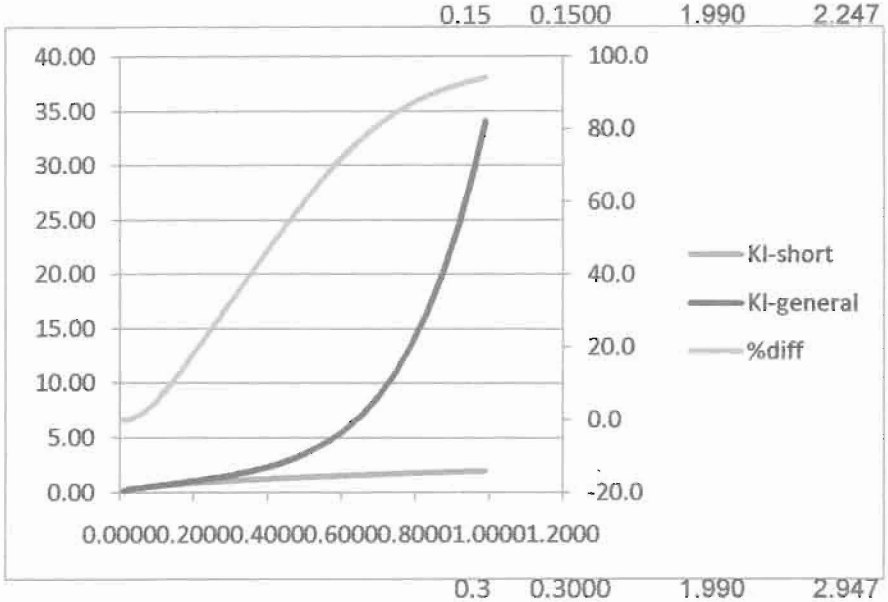
From Excel, for $\frac{a}{w} = 0.1$, the 2 eqns
differ by 5.4%

ME421-2010, FM 1, Problem 2

a (inch) varies
W (inch) 1
stress 1

PROBLEM:
Plot KI for edge crack using short crac

	a	a/W	Y-short	Y-general	KI-short	KI-general	%diff
	0.01	0.0100	1.990	1.988	0.20	0.20	-0.1
	0.02	0.0200	1.990	1.989	0.28	0.28	-0.1
	0.03	0.0300	1.990	1.994	0.34	0.35	0.2
	0.04	0.0400	1.990	2.001	0.40	0.40	0.6
	0.05	0.0500	1.990	2.012	0.44	0.45	1.1
	0.06	0.0600	1.990	2.025	0.49	0.50	1.7
	0.07	0.0700	1.990	2.041	0.53	0.54	2.5
	0.08	0.0800	1.990	2.059	0.56	0.58	3.4
	0.09	0.0900	1.990	2.080	0.60	0.62	4.3
>5%	**0.1**	**0.1000**	**1.990**	**2.103**	**0.63**	**0.66**	**5.4**
	0.11	0.1100	1.990	2.128	0.66	0.71	6.5
	0.12	0.1200	1.990	2.155	0.69	0.75	7.6
	0.13	0.1300	1.990	2.184	0.72	0.79	8.9
>10%	**0.14**	**0.1400**	**1.990**	**2.214**	**0.74**	**0.83**	**10.1**
	0.15	0.1500	1.990	2.247	0.77	0.87	11.4
					0.80	0.91	12.7
					0.82	0.96	14.1
					0.84	1.00	15.5
					0.87	1.04	16.9
					0.89	1.09	18.3
					0.91	1.14	19.7
					0.93	1.18	21.1
					0.95	1.23	22.5
					0.97	1.28	23.9
					1.00	1.33	25.3
					1.01	1.39	26.8
					1.03	1.44	28.2
					1.05	1.50	29.6
					1.07	1.55	31.1
	0.3	0.3000	1.990	2.947	1.09	1.61	32.5
	0.31	0.3100	1.990	3.011	1.11	1.68	33.9
	0.32	0.3200	1.990	3.077	1.13	1.74	35.3
	0.33	0.3300	1.990	3.147	1.14	1.81	36.8
	0.34	0.3400	1.990	3.220	1.16	1.88	38.2
	0.35	0.3500	1.990	3.296	1.18	1.95	39.6
	0.36	0.3600	1.990	3.375	1.19	2.03	41.0
	0.37	0.3700	1.990	3.458	1.21	2.10	42.5
	0.38	0.3800	1.990	3.546	1.23	2.19	43.9
	0.39	0.3900	1.990	3.638	1.24	2.27	45.3
	0.4	0.4000	1.990	3.734	1.26	2.36	46.7
	0.41	0.4100	1.990	3.835	1.27	2.46	48.1
	0.42	0.4200	1.990	3.941	1.29	2.55	49.5
	0.43	0.4300	1.990	4.053	1.30	2.66	50.9
	0.44	0.4400	1.990	4.170	1.32	2.77	52.3
	0.45	0.4500	1.990	4.294	1.33	2.88	53.7
	0.46	0.4600	1.990	4.424	1.35	3.00	55.0



3) Given: $\sigma = 20$ ksi

$w = 6$ in

$K_{Ic} = 50 \text{ ksi}\sqrt{\text{in}}$

$\sigma_{yp} = 60$ ksi

Find: a_{crit} edge crack

Assume: LEFM ($\frac{a}{w} < 0.8$), plane strain

$$K_I = Y\sigma\sqrt{a}$$

$$Y = 1.99 - 0.41\left(\frac{a}{w}\right) + 18.7\left(\frac{a}{w}\right)^2 - \ldots$$

(see prob 2)

Using Excel, plot K_I vs a

$$K_I = Y\sigma\sqrt{a}$$

Y is a fcn of $\frac{a}{w}$

$\sigma = 20$ ksi

at $a = 1.1$ inch $K_I \simeq K_{Ic}$ $(50 \text{ ksi}\sqrt{\text{in}})$

(Note: if "short crack" eqn is used
it results in $a = 1.6$ inch - not good)

ME421-2010, FM 1, Problem 3

a (inch)	tbd
W (inch)	6
stress	20
KIC	50

PROBLEM:

When does KI>KIC?

a	a/W	Y-short	Y-general	KI-short	KI-general	
0.05	0.0083	1.990	1.988	8.90	8.89	
0.1	0.0167	1.990	1.988	12.59	12.57	
0.15	0.0250	1.990	1.991	15.41	15.42	
0.2	0.0333	1.990	1.996	17.80	17.85	
0.25	0.0417	1.990	2.003	19.90	20.03	
0.3	0.0500	1.990	2.012	21.80	22.04	
0.35	0.0583	1.990	2.023	23.55	23.93	
0.4	0.0667	1.990	2.035	25.17	25.75	
0.45	0.0750	1.990	2.050	26.70	27.50	
0.5	0.0833	1.990	2.066	28.14	29.22	
0.55	0.0917	1.990	2.084	29.52	30.91	
0.6	0.1000	1.990	2.103	30.83	32.58	
0.65	0.1083	1.990	2.124	32.09	34.24	
0.7	0.1167	1.990	2.146	33.30	35.90	
0.75	0.1250	1.990	2.169	34.47	37.57	
0.8	0.1333	1.990	2.194	35.60	39.24	
0.85	0.1417	1.990	2.219	36.69	40.93	
0.9	0.1500	1.990	2.247	37.76	42.63	
0.95	0.1583	1.990	2.275	38.79	44.35	
1	0.1667	1.990	2.305	39.80	46.09	
1.05	0.1750	1.990	2.335	40.78	47.86	
1.1	**0.1833**	**1.990**	**2.367**	**41.74**	**49.65**	**KI>KIC**
1.15	0.1917	1.990	2.400	42.68	51.48	**KI>KIC**
1.2	0.2000	1.990	2.434	43.60	53.33	
1.25	0.2083	1.990	2.470	44.50	55.22	
1.3	0.2167	1.990	2.506	45.38	57.15	
1.35	0.2250	1.990	2.544	46.24	59.12	
1.4	0.2333	1.990	2.583	47.09	61.13	
1.45	0.2417	1.990	2.624	47.93	63.19	
1.5	0.2500	1.990	2.665	48.74	65.29	
1.55	0.2583	1.990	2.708	49.55	67.44	
1.6	0.2667	1.990	2.753	50.34	69.65	
1.65	0.2750	1.990	2.799	51.12	71.91	
1.7	0.2833	1.990	2.847	51.89	74.24	
1.75	0.2917	1.990	2.896	52.65	76.63	
1.8	0.3000	1.990	2.947	53.40	79.08	
1.85	0.3083	1.990	3.000	54.13	81.61	
1.9	0.3167	1.990	3.055	54.86	84.22	
1.95	0.3250	1.990	3.112	55.58	86.91	
2	0.3333	1.990	3.171	56.29	89.68	
2.05	0.3417	1.990	3.232	56.98	92.55	
2.1	0.3500	1.990	3.296	57.68	95.51	
2.15	0.3583	1.990	3.362	58.36	98.58	
2.2	0.3667	1.990	3.430	59.03	101.76	
2.25	0.3750	1.990	3.502	59.70	105.05	
2.3	0.3833	1.990	3.576	60.36	108.46	

Given:

4) center crack, $2a = 6"$

width of plate, $w = 18"$

$K_{Ic} = 30\ ksi\sqrt{in}$

$\sigma_{yp} = 70ksi$

Find: residual strength

Assume: LEFM (is $\frac{\sigma}{\sigma_{yp}} < 0.8$?), plane strain (thickness is unknown)

Soln

$$K_I = \sigma\sqrt{\pi a}\left(\frac{1}{\cos\frac{\pi a}{w}}\right)^{1/2}$$

Let $Y = \left(\frac{\pi}{\cos\frac{\pi a}{w}}\right)^{1/2} = 1.91$

$K_I = K_{Ic} = 30ksi\sqrt{in}$

$a = 3"$

$w = 18"$

$$\sigma = \frac{K_{Ic}}{Y\sqrt{a}} = \frac{30ksi\sqrt{in}}{1.91\sqrt{3}} = 9.1\ ksi$$

check LEFM: $\frac{9.1}{70} < 0.8$ ✓

5) Given: $W = 5''$

$B = 1''$

$a = 0.25''$ (edge crack)

$K_{Ic} = 20 \text{ ksi}\sqrt{\text{in}}$, $\sigma_{ys} = 80 \text{ ksi}$

Find: Is force required to fracture $> 80{,}000$ lb or not.

Assume: LEFM (valid if $\frac{\sigma}{\sigma_{ys}} < 0.8$)

Plane strain

Soln

$$K_I = \sigma Y \sqrt{a}$$

$$Y = 1.99 - 0.41\left(\frac{a}{W}\right) + \cdots \quad \text{(see previous prob)}$$

for $a = 0.25$ & $W = 5$

$Y = 2.012$

Let $K_I = K_{Ic} = 20 \text{ ksi}\sqrt{\text{in}}$

$$\sigma_{cr} = \frac{K_{Ic}}{Y\sqrt{a}} = \frac{20 \text{ ksi}\sqrt{\text{in}}}{2.012\sqrt{0.25}} = 19.9 \text{ ksi}$$

LEFM valid

$$\sigma = \frac{F}{A} \Rightarrow F = \sigma \cdot A = 19{,}900 \frac{\text{lb}}{\text{in}^2}(5\text{in} \times 1\text{in})$$

$$F_{crit} = 99{,}000 \text{ lb} > 80{,}000 \text{ lb}$$

$\therefore$ customer overload