EXACT constraint

Knowing how much is enough—and how much too much—is a key to successful design. By James G. Skakoon

We have all sat in a café at a wobbling pedestal table, one that teeters side-to-side on two feet, resting now on the third, now on the fourth, our glass ominously threatening to spill. We've probably all tried to arrest the wobble with a coaster or two under one foot. What is wrong is obvious enough to mechanical engineers. Three points determine a plane, and a fourth is one too many.

This example represents a fundamental of design engineering, exact constraint, which has a well-developed theory applicable for design engineers. Applying it improves designs by avoiding over-constraint. Over-constrained designs lead to high stresses, tight tolerances, looseness (as in the wobbling table), binding, difficult assembly—generally bad performance.

I first discovered over-constraint as a child carpenter. Using a screw, I could not fasten two boards together square and tight as I wanted. The boards were either square and loose, or tight and angled, but never square and tight. My design was over-constrained. I have since learned to drill a clearance hole in the first board, which removes the axial positioning constraint so it tightens to the second without turning with the screw.

Alan R. Parkinson, dean of the Ira A. Fulton College of Engineering and Technology at Brigham Young Uni-

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Above: Tables with rigid pedestal-style bases are often adjusted in-situ to prevent wobbling.

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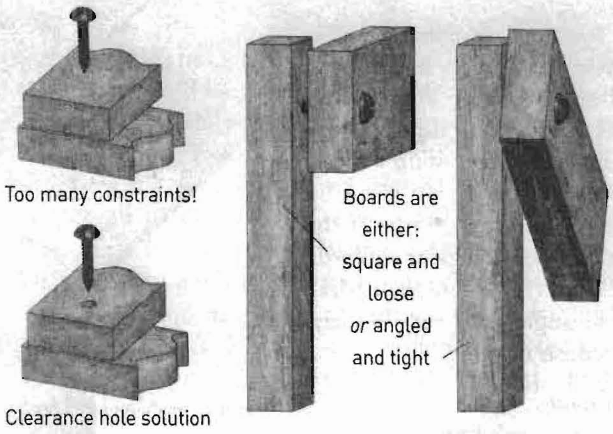
versity, has seen student teams struggle with over-constraint in their designs. "Sometimes exactly constrained things aren't necessarily intuitive, and it seems, as engineers, we're prone to want to add in additional constraints."

For example, having three bearings on one shaft is ill-advised because the third bearing will never line up perfectly. Inexperienced designers often overkill like this. Then they tighten dimensional parts tolerances or add assembly adjustments, covering up the over-constraint. Sure, you could loosen and retighten a pillow block to assemble the shaft, but you'd bow it, and likely wreck a bearing sooner or later. Experienced designers use two bearings with an adequately rigid shaft.

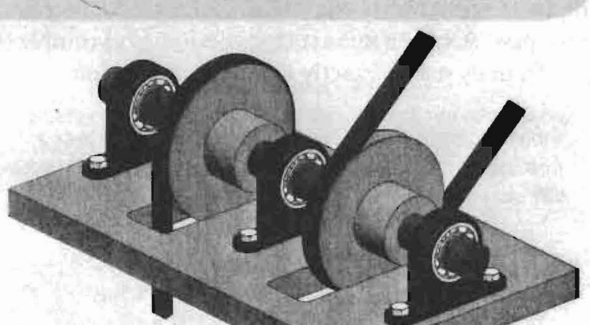


A clearance hole in the first board lets it be tight and square, simultaneously.

Neither New nor Used

Although not traditionally taught in mechanical engineering curricula, and not universally known among mechanical engineers, principles of exact constraint have been around for over a century. Designers of precision instruments have for decades used exact constraint, without which they simply would not achieve the precision required by many devices. Lawrence Kamm, a San Diego consulting engineer who has written about and promoted exact constraint, or minimum constraint design, as he calls it, agrees. "I was never taught it," he said. "I learned it from a book that was published in 1954, a book by T. N. Whitehead. It was intended for instrument designers, and it was a revelation."

Two bearings establish a shaft's axis in space. A third bearing will never align with the other two and the shaft.



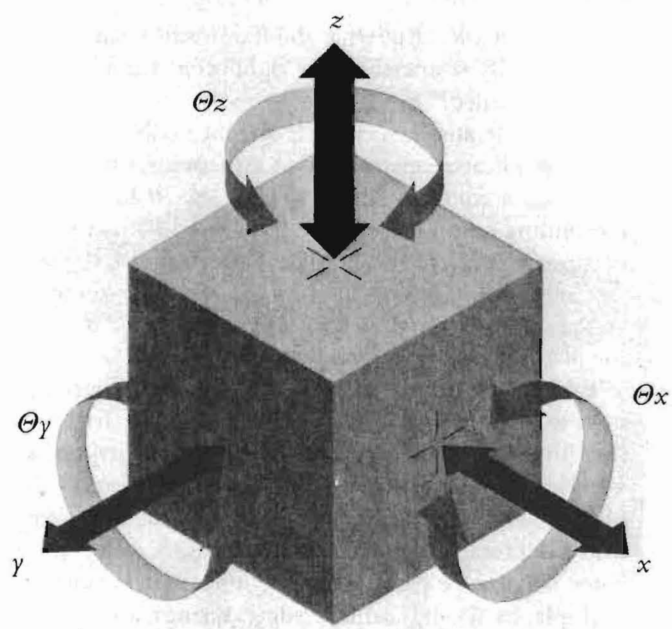
Parkinson has researched and published papers on smart assemblies, which incorporate features that absorb or cancel the effects of variation. While doing this work, he discovered that practicing design engineers like Kamm were applying the principles of exact constraint, even though it was a subject rarely taught in university courses. "As I learned about it [exact constraint design], I thought, 'This seems quite fundamental to the idea of machine design. And why don't our students know about this?' " Parkinson said.

Douglass Blanding, author of *Exact Constraint: Machine Design Using Kinematic Principles*, a helpful resource for design engineers applying exact constraint, sees the design community slowly becoming more aware of the subject. "It's been picked up by a couple of schools and ASPE [The American Society for Precision Engineering]," Blanding said, including the Massachusetts Institute of Technology among them. These are creditable organizations so, according to Blanding, people feel they need to get on board.

In Theory

An object in three-dimensional space has six degrees of freedom: three translations and three rotations. These are called x, y, z for the three translations and Θx, Θy, and Θz for the three rotations. Exact constraint means constraining these six degrees of freedom, no more and no less, to obtain the desired structure, or leaving one or more unconstrained to obtain the desired motion.

A common example for illustrating exact constraint is



The six degrees of freedom: Three translations and three rotations.

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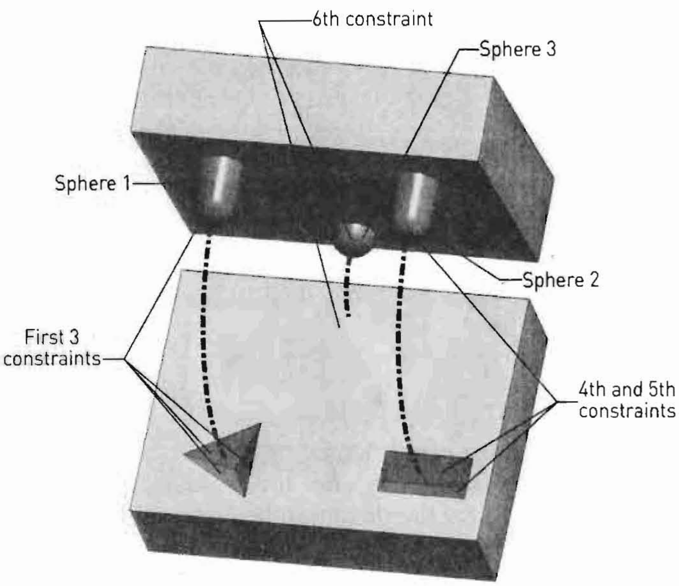
a kinematic connection. The first three constraints come from a sphere contacting three surfaces in a trihedral receptacle. These are the three translation constraints. The V-shaped groove with another sphere supplies two rotational constraints, and the plate surface with a third sphere, the final.

Remove the plate and it returns to the base in exactly the same position. No precision dimensions are required: ball diameter, ball position, socket positions, trihedral and V-groove dimensions and angles can vary widely without compromising exact constraint.



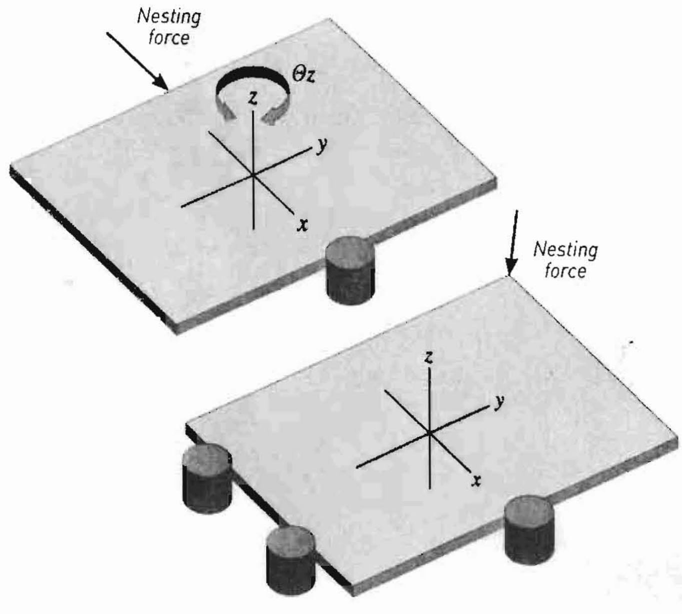
A typical kinematic connection exhibiting perfect three-dimensional constraint.

You might ask, "But what about upward? I can lift that kinematically connected plate right off its base! You call that constrained?"

The simple answer is that its weight holds it in place. The complicated answer is that a theoretical constraint is not just a contact point alone, but also includes a corresponding nesting force that maintains the contact. The nesting force is a force vector that goes through the contact point normal to the surfaces of contact, but these can be vectorially combined into a single force, in this case the plate's weight.

"Everybody underestimates that problem [necessary nesting force]," Blanding said. "As it turns out, friction is the thing that spoils everyone's result. [The part] doesn't go where it's supposed to go." The force's direction is often obvious by inspection alone, and its magnitude must counteract externally applied loads, as well as overcome friction. In practice, nesting forces are created, for example, by weight, cams, wedges, springs, and screws.

Exact constraint is easier to picture in two dimensions than in three. The principles are the same, but in two dimensions there are three degrees of freedom: two translations and one rotation (x, y, and Θz).

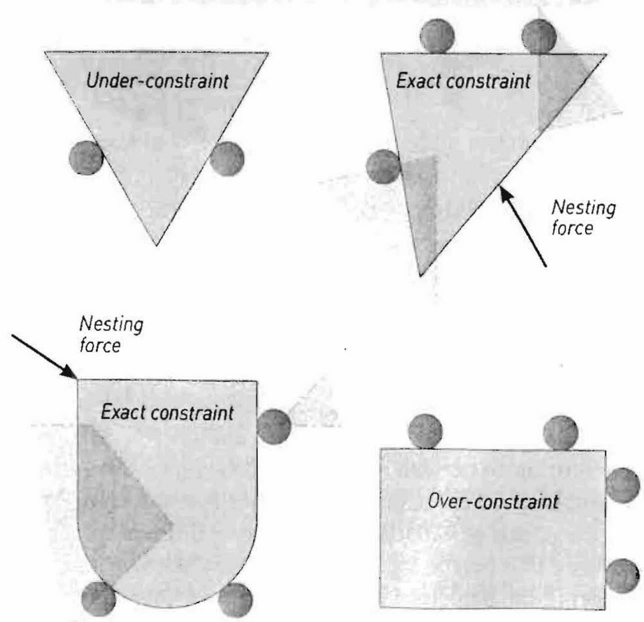


Top: A single constraint preventing translation in x.
Bottom: A plate fully constrained in two dimensions.

With one constraint, a contact point with its corresponding nesting force, the plate can still translate in y and rotate about z. Adding two more posts constrains the plate in a single, unambiguous position in 2-D space. That is, as long as the nesting force is enough to resist externally applied forces.

But not any three posts, or constraints, will do. It is helpful to test your eye at judging over-, under-, and exact constraint, and to visualize a suitable nesting force direction. Note that there is a window, called the nesting force window, through which suitable nesting forces must pass. A vector outside this window will tumble the part from its stable, exactly constrained position.

Some 2-dimensional constraint examples. Unless a nesting force passes through the nesting force window, the part will tumble.



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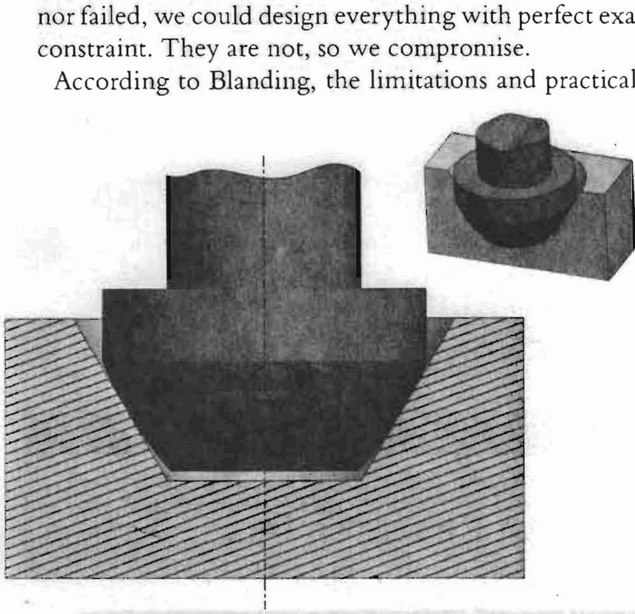
Theory to Practice

Are there practical applications of exact constraint design? Parkinson found one in his home inkjet printer. "Here's a device that is very inexpensive ... yet it's capable of laying down these very tiny drops of ink very precisely." Interested, he opened the cover to examine it with an engineer's eye, and discovered how. "It was an exactly constrained design," Parkinson said. "And I thought, 'That's the way you'd want to do that.' " Opening mine, I saw what he saw: a set of contacting points and nesting force springs constraining the ink cartridge into a repeatably exact location.

But applying exact constraint theory presents some practical challenges. If materials were inelastic and unyielding, and therefore components neither deformed nor failed, we could design everything with perfect exact constraint. They are not, so we compromise.

According to Blanding, the limitations and practicalities of the real world permit one to make assumptions. "If...the reason for using exact constraint is just to make something work without binding, you can make different assumptions," he said. "There are times when you say, 'If this thing rattles around a little bit in x, I don't care.' So you'll constrain it between two posts."



Surface matching compromises exact constraint, but distributes loads over larger areas.

Kamm's diverse precision designs range from semiconductor test equipment to self-aligning part grippers to space vehicle trainer components. Although he said, "The concept has always been a premise in every design I have made since I first understood the principle," he is also quick to acknowledge compromise measures such as load distribution. "There's not such a thing in physical reality as a point contact. Minimum constraint design doesn't mean that you can ignore things like stress and strain."

The trihedral receptacle shown in the kinematic connection example, although exactly constraining three translations, creates infinite stresses at the points of contact, as does any point contact. Furthermore, the trihedron's geometry is unpleasant for the shop. Instead, you can use a conical hole for circle contact to better distribute the load. Another step forward is to match or, better yet, slightly mismatch the contact surfaces' radii for best load distribution.

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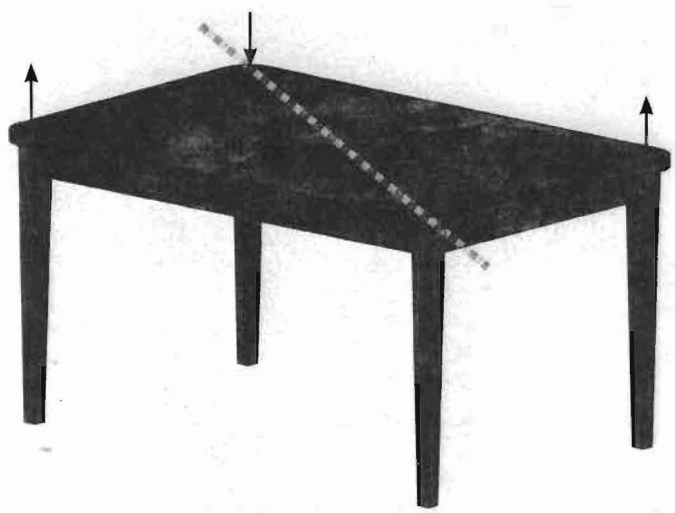
Some other useful compromises to exact constraint are pinned and bolted connections, ball bearings, and tapered roller bearings. Another is in-situ adjustment of over-constraint as in, for example, the thread-adjusted foot pads of a clothes dryer or washing machine.

Kamm points out another common deviation from strict adherence to the principle. "There is the question of stability," he said. "Not always should one limit oneself to minimum constraint design." He said that may sound like heresy, but he explained it with an example, the standard office chair, which instead of touching the floor at three points, has not four, but five casters. "Now why do that?" he asks, then continues, "And the answer is that if you sit on a chair with only three casters, you're liable to topple over."

So we add a fourth leg to a table to improve stability, and we accept the wobble, right? Not so fast. Pedestal-style tables with rigid bases always wobble, but square or rectangular tables with legs at the corners usually rest quite firmly on the floor. Why?

The table top twists just enough for the fourth leg to contact. This is called, variously, elastic constraint, redundant constraint, elastic averaging, and elastic design, and it increases load-carrying and improves stability. To use elastic constraint, combine a flexing feature with the proper nesting forces, and a redundant constraint then also contacts its mating surface.

Blanding, seeing deeper into exact constraint principles than most, offers a rigorous explanation for this 2-dimensional example. He observes that the top is not rigid, but instead has a degree of freedom, "namely, the flexibility," he said. This one additional degree of freedom requires one additional constraint to achieve exact con-



The table top bends on a diagonal so all legs contact the floor.

straint. "So three is actually incorrect, because it's like it has a hinge across the diagonal. The correct number of constraints is four, not three."

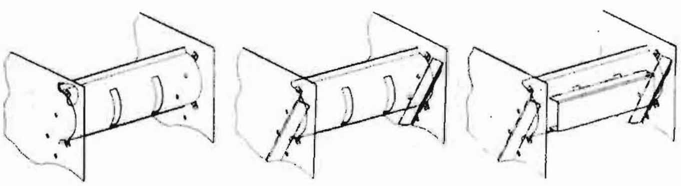
Practice Makes Perfect

Applying exact constraint principles to everyday design isn't complicated, even if the theory can be. Parkinson explained this by saying, "It seems to me that you could get a fair amount of benefit from just understanding what exact constraint means, and understanding a few basic examples. ... That might go pretty far."

Blanding echoed that sentiment: "Certainly an awareness-level understanding would benefit anyone."

A 1995 article in this magazine by Jon M. Kriegel, then an Eastman Kodak development engineer, described a troublesome sequence of design changes to a shaft support assembly in an office copier. At the start, the design was over-constrained, and parts variations precipitated binding of a shaft when tolerances combined unfavorably. Attempting to fix the binding, designers stiffened first one part, then another, until the assembly became so stiff that the screw fastening points fractured.

According to the article, considerable expense and time were lost, and the problem was never adequately solved until an expert educated those involved on exact constraint. If you recognize over-constraint when the problem first presents, or better yet, apply exact constraint during initial design, you will avoid altogether the all-too-common characteristic spiral: stiffening parts, tightening tolerances, and fighting assembly troubles. ■



A baffle in an office copier suffered from over-constraint. Adding stiffeners to the frame (middle) and to the baffle itself (right) failed to correct the problem.

For Further Reading

Douglass L. Blanding, *Exact Constraint: Machine Design Using Kinematic Principles*, New York: ASME Press, 1999.

Lawrence Kamm, *Designing Cost-Efficient Mechanisms*, Society of Automotive Engineers, 1993. Also published by McGraw-Hill, 1990.