ASME International

Name SOLUTIONS
Course ME328
Sheet ____ of ____
Date STRESS CONC.

1. Given: $\sigma_x = 1$ psi on right side (applied)

Thickness = 0.5 in.

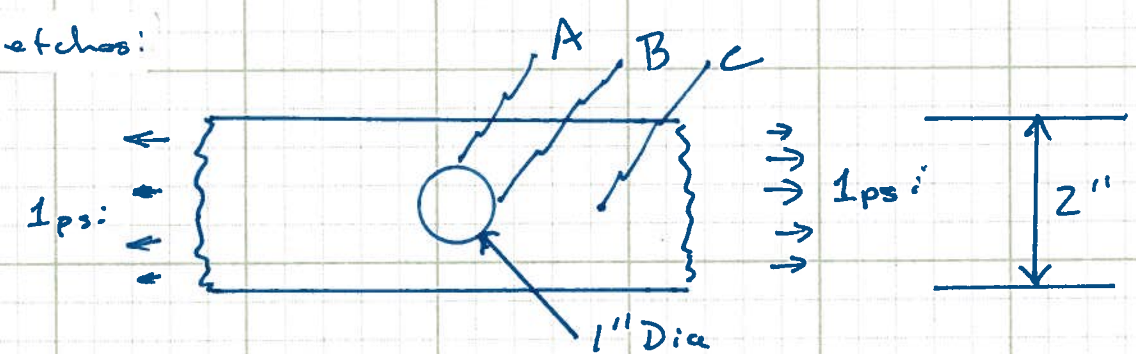
$E = 30$ Mpsi, $\nu = 0.27$, $\sigma_{ys} \equiv S_{ys} = 100$ ksi

Find: - see below

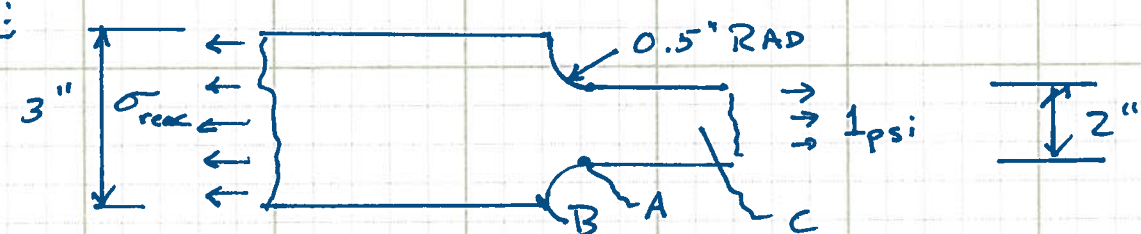
Assume: Linear elastic
Uniform load if stress concentration not present

sketches:

hole



fillet:



$$\sigma_{reaction} = \frac{2}{3}\sigma_{app} = 0.67 \text{ psi}$$

a) Determine σ_{max} (due to stress concentration)

$$\sigma_{max} = K_t\, \sigma_0 \qquad \text{Hole: } \sigma_0 = \frac{F}{(w-d)t} = \frac{1 \text{ lb}}{(2''-1'')0.5''}$$

$$\sigma_0 = 2 \text{ psi}$$

$$K_t \rightarrow \text{Fig A-15-1} \quad \frac{d}{w} = \frac{1.0}{2.0} = 0.5 \quad K_t = 2.19 = 2.2$$

$$\sigma_{max} = 2.2(2\text{ psi}) = \underline{\underline{4.4 \text{ psi}}} \quad (\text{Hole}).$$

ASME International

Name
Course
Sheet of
Date

Fillet: Fig A-15-5

$\frac{r}{d} = \frac{0.5}{2.0} = 0.25$

$\frac{D}{d} = \frac{3.0}{2.0} = 1.5$

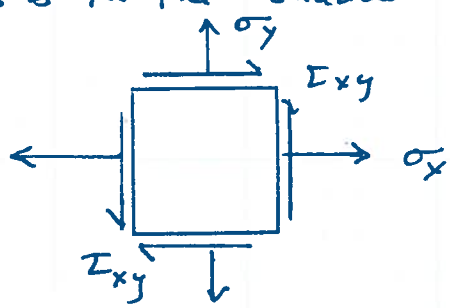
$\sigma_o = 1\text{psi} \; \left(\frac{F}{A}\right)$

$K_t = 1.6$

$\sigma_{max} = 1.6\,(1\text{psi}) = \underline{\underline{1.6\text{psi}}}$

b) Draw stress element at A & C & corresponding Mohr's circle. Determine σ_x at B. A is at stress conc, B is in the "shadow" and C is far away.

HOLE



AT A: $\sigma_x = \sigma_{max} = 4.4$ psi

$\sigma_y = \phi \quad \tau_{xy} = 0$

AT B: $\sigma_x = 0$

AT C: $\sigma_x = 1$ psi

$\sigma_y = 0 \quad \tau_{xy} = 0$

FILLET AT A: $\sigma_x = \sigma_{max} = 1.6$ psi

$\sigma_y = 0 \quad \tau_{xy} = 0$

AT B: $\sigma_x = 0$

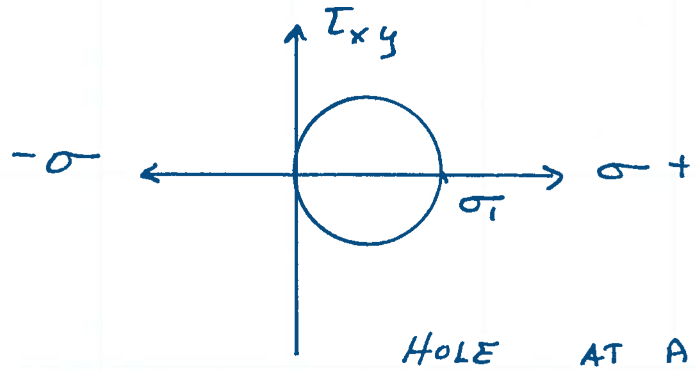
AT C: $\sigma_x = 1$ psi

$\sigma_y = 0 \quad \tau_{xy} = 0$

ASME International

Name
Course
Sheet of
Date

HOLE
& FILLET



HOLE AT A: $\sigma_1 = 4.4$ psi

$\sigma_2 = \sigma_3 = 0$

AT C: $\sigma_1 = 1$ psi

$\sigma_2 = \sigma_3 = 0$

FILLET AT A: $\sigma_1 = 1.6$ psi

$\sigma_2 = \sigma_3 = 0$

AT C: $\sigma_1 = 1$ psi

$\sigma_2 = \sigma_3 = 0$

c) From FEA, σ_x at B is zero for both parts (hole & fillet) and it is 1 psi at C. AT the stress concentration (point A) for the hole $\sigma_x = 4.4$ (same as hand calc) and for the fillet $\sigma_x = 1.67 \approx 1.7$. For the hand calculation $\sigma_x = 1.6$. The difference is within the precision level of reading the chart.

ASME International

Name
Course
Sheet of
Date

d) What would the applied stress have to become to cause "local" yielding?

Hole.

$$(\sigma_x)_{hole} = 4.4\ psi \quad w/\ \sigma_{applied} = 1\ psi$$

When $(\sigma_x)_{hole} = S_{ys}$ yielding will occur (this is uniaxial stress condition).

$$\frac{4.4 psi}{S_{ys}} = \frac{1 psi}{?} \qquad S_{ys} = 100 ksi \text{ (given)}$$

$$? = \frac{(S_{ys})(1 psi)}{4.4 psi} = 22.7\ ksi$$

When the applied load reaches 22.7 ksi (23 ksi) local yielding will occur at the hole

Fillet:

$$? = \frac{(S_{ys})(1 psi)}{1.6 psi} = 62.5\ ksi \quad (63 ksi)$$

62.5 ksi applied load will cause onset of local yielding at the fillet.

ASME International

Name
Course
Sheet of
Date

e) What is σ_{app} to cause "bulk" yielding.

Bulk yielding occurs when the nominal stress (σ_o) reaches S_{ys} (again, this is uniaxial loading). $S_{ys} = 100$ ksi

Hole: $\sigma_o = 2$ ksi (the stress in the cross-section with the hole - the nominal area is $0.5 in^2$)

$$\frac{\sigma_o}{S_{ys}} = \frac{\sigma_{app}}{?} \Rightarrow \{\sigma_{app} = 1 \text{ ksi}\}$$

$$\frac{2 \text{ ksi}}{100 \text{ ksi}} = \frac{1 \text{ ksi}}{?} \qquad ? = \underline{\underline{50 \text{ ksi}}} \quad (\text{Hole})$$

When the applied load reaches 50ksi, the bar will permanently elongate.

Fillet: $\sigma_o = 1$ psi

$$\frac{1 \text{ psi}}{100 \text{ ksi}} = \frac{1 \text{ psi}}{?}$$

$$\underline{\underline{? = 100 \text{ ksi}}} \quad (\text{Fillet})$$

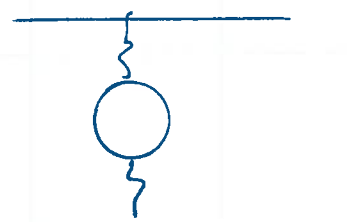
f) If the bars were brittle w/ $\sigma_{UT} = S_{UT} = 100\text{ksi}$:
What applied load would cause fracture?

ANS: one stress at the concentration reaches S_{UT}, the bar will fracture.

Hole:

$$\frac{4.4\text{ psi}}{S_{UT}} = \frac{1\text{ psi}}{?}$$

$$? = \underline{\underline{22.7\text{ ksi}}}$$ will cause fracture at the hole



Fillet:

$$\frac{1.6\text{psi}}{S_{UT}} = \frac{1\text{psi}}{?} \qquad ? = \underline{\underline{62.5\text{ ksi}}}$$

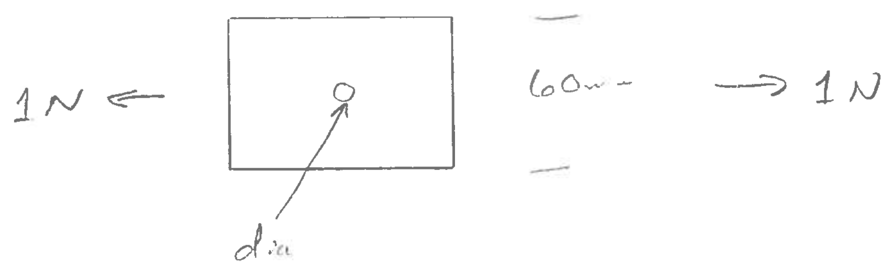
fracture

Soln

For the stress concentration problems, there is difference between charts & FEA. FEA does a poor job with stress concentrations! It takes a very fine mesh to properly model most stress concentrations, and even then the engineer should be cautious when using FEA data.

Soln

2 Given: Flat plate, axial load of 1 N
60 mm wide, 5 mm thick
Hole in middle
Find: σ_{max} vs dia (dia 1 → 20 mm)
Assume: - load is uniform except near hole
- elastic



side Note: 1 N (or 1 lb) is a convenient load (unit load) if applied load is not defined. Once the applied load is known, the resulting stress is scaled. For example, if the actual load is 2500 N, then multiply the results of this 1 N analysis by 2500.

Solution:

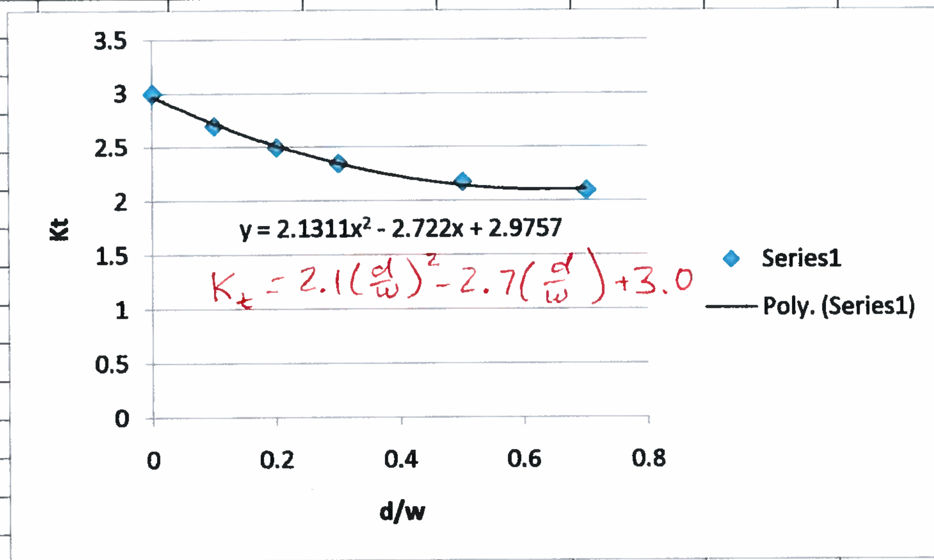
$$\sigma_{max} = K_t \, \sigma_o \qquad \sigma_o = \frac{F}{(w-d)t} = \frac{1\,N}{(60_{mm} - dia)\,5mm}$$

K_t is determined in attached Excel

Soln 4

a) Determine an equation relating stress concentration factor to d/w based on an axially loaded flat plate with a hole in the middle. Use Chart A-15-1

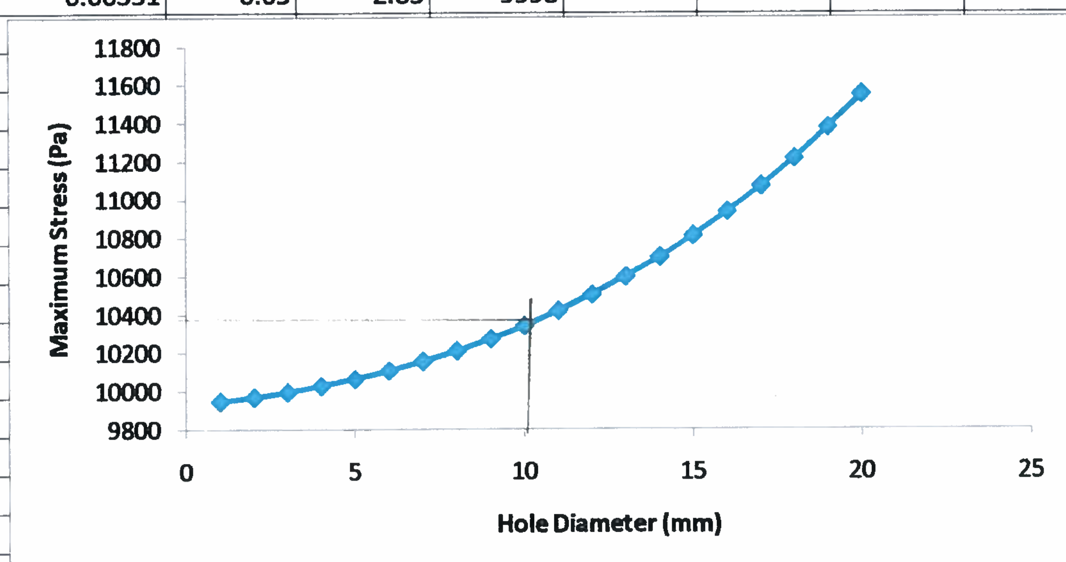
d/w	Kt
0	3
0.1	2.7
0.2	2.5
0.3	2.35
0.5	2.18
0.7	2.1



b) Use the generated equation to determine the maximum stress near the hole as a function of hole diameter (ranging from 1mm to 20mm dia)

$K_t = 2.1(\frac{d}{w})^2 - 2.7(\frac{d}{w}) + 3.0$

Hole diameter, mm	nominal stress, Mpa	d/w	Kt	max stress, Pa
1	0.00339	0.016667	2.94	9950
2	0.00345	0.033333	2.89	9971
3	0.00351	0.05	2.85	9998
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18	[illegible]	0.3	[illegible]	[illegible]
19	0.00488	0.316667	2.33	11377
20				



Note, stress concentration factor is greater for smaller holes, however, due to less material with larger holes, the stress is greater for larger holes (for a given width)

c) for a 10 mm dia hole in this plate, $\frac{d}{w} = \frac{10\,mm}{60\,mm} = 0.17$

$K_t = 2.5 \rightarrow 2.6$

$K_t = 2.6$

$\sigma_{max} = 10{,}400$ Pa for 1 N load

A brittle mat'l, will fail when

$\sigma_{max} = S_{UT} = 150$ MPa

$$\frac{1\,N}{10{,}400\,Pa} = \frac{x}{150\,MPa}$$

$x = \underline{\underline{14.4\,kN}}$

ASME International

Name

Course

Sheet of

Date

d) $d = 10\,mm$, 2024-T3, what force will cause local yielding?

From A-24, $S_{ys} = 345\ MPa$

$\sigma_{max} = 10{,}400\ Pa$ w/ 1 N load:

$$\frac{1\,N}{10{,}400\,Pa} = \frac{x}{345\,MPa} \qquad x = 33.2\,kN$$

33 kN applied force will cause onset of local yielding.

e) $d = 10\,mm$, 2024-T3, what force will cause bulk yielding?

ANS: when $\sigma_0 \geq S_{ys}$ bulk yielding will occur (yielding through the cross-section with the hole).

$$\sigma_0 = \frac{F}{(w-d)t} = \frac{1\,N}{(60\,mm - 10\,mm)(5\,mm)} = 0.004\,MPa$$

$$= 4000\ Pa$$

$$\frac{1\,N}{4000\,Pa} = \frac{x}{345\,MPa} \qquad \underline{\underline{x = 86\,kN}}$$