<u>STRESS ANALYSIS OUTLINE</u>

Stress analysis is typically conducted in order to prevent failure of a part or structure. It requires:

1) Determination of the stresses present in a loaded part
2) Application of a theory that relates the calculated stresses to the material's ability to carry a load.

<u>*Loosely*</u> speaking, "factor of safety" is defined as: "*material's ability to carry the load*" divided by "*the load being applied*".
Where:

- "*material's ability to carry the load*" is quantified by appropriate material properties
- *"the load being applied*" is quantified through appropriate stress calculations.

<u>Loading Conditions and Relevant Analyses (studied thus far)</u>

Quasi-static loads including:

- Fundamental loads: axial, torsional, bending (covered in EGR322 Strength of Materials)
- column loads (ME328)
- Hertzian contact (ME328) – stress produced by contact between spherical or cylindrical surfaces
- Stresses in bolts (later in ME328)
- FEA can be used to determine stresses in complex loaded parts
- Quasi-static failure is often referred to as "single cycle overload" – meaning the stress applied even a single time, causes failure. Other than buckling failure, quasi-static failure may be defined as either yielding or brittle fracture (see "Failure Mechanisms" below).

Non-static loads including:

- Impact – impact involves energy based analyses to determine peak force and deflections when two objects collide (one being a "massless spring" the other a "rigid mass"). In ME328 we required the assumption that **force-deflection relationship is the same during impact as the quasi-static condition**. The peak force can then be used to determine the stress during impact using quasi-static stress analysis tools (such as beam bending theory). Static failure theories (such as Distortion Energy or Maximum Normal Stress theory) can be applied to predict failure.
- Fatigue (aka cyclic) loading. To determine stresses, use quasi-static analysis tools (axial, bending, etc.) – but fatigue failure theory must be used to predict fatigue failure.

<u>Failure Mechanisms</u>

Yielding (aka plastic deformation) – may occur in ductile materials

- Cause: "single cycle overload" (too much stress applied even once). Example: paperclip bent (but not broken), dents in your car, etc.
- Mechanism in metals and alloys: dislocation slip
- Materials: ductile metals and alloys. Other ductile materials such as polymers "yield" due to other mechanisms other than dislocation slip; therefore, yield theories may or may not be applicable.
- Yielding is NOT fracture – it is permanent deformation (*generally* bad but not catastrophic)

- Relevant material property: yield strength (S_{ys}).
- Relevant theories to predict yielding:
  - Distortion Energy Theory (aka "von Mises") – determine the "von Mises" or "effective" stress. Failure: $\sigma_{effective} > S_{ys}$
  - Maximum shear stress theory (aka "Tresca") – determine the maximum shear stress in the part. Failure: $\tau_{max} > S_{ys}/2$

Brittle fracture – may occur in brittle materials

- Cause: "single cycle overload" (too much stress applied even once). Example – broken ceramic coffee mug.
- Mechanism: near instantaneous crack propagation with little or no plastic deformation.
- Materials: brittle materials (materials with ductility less than about 2%EL). "Ductile" materials under abnormal conditions such as operating at very cold temperatures, may experience brittle fracture rather than yielding.
- Brittle failure is sudden and catastrophic (does not give warning, and the part completely breaks). Unlike yielding, this **<u>is</u>** fracture (completely broken in-two).
- Relevant material property: tensile strength (S_{ut})
- Relevant failure theory: Maximum Normal Stress theory. Failure: $\sigma_1 > S_{ut}$
- Note: we did not analyze brittle materials with compression-only loads. But there are theories for compression loads in brittle materials – civil engineers deal with this regularly.

Buckling – may occur in relatively long slender columns with <u>compression</u> loads.

- Buckling is sudden and catastrophic (occurs without warning, and typically causes total collapse of a structure)
- Buckling is not a material failure (such as yielding or fracture) but rather is a failure of the structure itself.
- Relevant material properties: Modulus of Elasticity and for "intermediate length" columns, yield strength is relevant.

Fatigue failure

- Cause: fatigue failure is caused by loads that vary back and forth time-after-time-after-time (thousands of load cycles). Fatigue failure means that a crack has initiated, grown and the part has **completely broken in-two**. Example: bend a paperclip back and forth over and over until it breaks – that's low cycle fatigue failure.
- We have studied how many cycles a part may experience before fatigue failure (fracturing).
- Mechanism: very tiny cracks initiate and with each load application grow a very tiny amount (microns per cycle, initially). Eventually, the cracks may grow large enough to become detectable visually or with non-destructive examination techniques such as ultrasound. As the crack grows longer, it grows more with each load cycle. Fatigue cracks will eventually (after many load cycles) grow large enough that "one more cycle" will cause sudden and often catastrophic fracture (the part breaks).
- Materials: ALL materials may experience fatigue failure. In ME328, we only considered analysis for ferrous alloys (steel, cast iron). Regarding fatigue failure, there is no distinction in what we have studied between ductile and brittle materials.

- Relevant material properties: tensile strength (S_{ut}) and “fatigue strength” (S_{fail}) as determined from S-N diagram. In order to create an S-N diagram, the modified endurance limit and fS_{ut}, must first be determined.
- Failure theory: Goodman equation (or Soderberg, or Gerber equations).
- Comments: **fatigue failure is the most common causes of failure in mechanical design**. Why?
    - Most machines experience cyclic loading
    - Fatigue failure occurs at stresses well below stresses that cause static failure.
    - Fatigue failure is difficult to predict even in controlled laboratory settings.
    - Real-world loading conditions and material conditions are difficult to predict.
    - Due to the length of time typically experienced before fatigue failure occurs, the parts may become damaged (scratched, corroded, etc.) which may then lead to fatigue initiation, crack growth, and eventual final fatigue fracture.

Other failure mechanisms

- Corrosion (many types of corrosion)
    - May introduce stress concentrations which then lead to fatigue failure
- Creep – time dependent permanent deformation. Generally, occurs at elevated temperature.
- Hydrogen Embrittlement – occurs in high strength steels exposed to hydrogen…we’ll discuss later in ME328.
- …and the list goes on…

Stress concentrations:

All geometric discontinuities can cause stress to be amplified.

- Microscopic: grain boundaries, second phase microstructural features (such as precipitates in aluminum alloys and Fe_3C in steel), scratches, porosity, etc., all cause microscopic stress concentrations. We generally do NOT analyze these as their real effect is taken into account when material properties are measured.
- Macroscopic: any geometric change, such as holes, shoulders on shafts, etc. etc. etc. (almost all parts have some change in shape). It is important to be able to analyze these. Stress concentration charts are most commonly used to determine how much the stress is amplified due to the stress concentration.
- FEA is not always a reliable method for determining stress concentrations due to the finite size of mesh (very small elements are needed to accurately calculate stress in small radii, sharp corners, etc.).
- Stress concentration equation: $\sigma_{max} = K_t \, \sigma_{nom}$

This equation can be used to determine the stress at the point of concentration. The equation can be used to predict if local plastic deformation will occur at the stress concentration or not, but **linear elasticity must be maintained** in order to accurately determine the stress at the stress concentration. For example, consider nominal stress (σ_{nom}) of 75ksi and stress concentration factor (K_t) of 2. The predict stress at the concentration is = 2*75ksi = 150ksi. If the yield strength is less than 150ksi, yielding will occur and the material will not carry the 150ksi. It will just stretch, but the stress will be less than the predicted 150ksi. In actuality, it is the strain that is amplified by K_t and not necessarily the stress.