NOTE TO STUDENTS: DID's do NOT need to be typed – handwritten (or combination typed and handwritten) is acceptable if neat and legible.

Design Increment Document, spring dashpot, DID #2018-2-8
ME328B project
February 8, 2018
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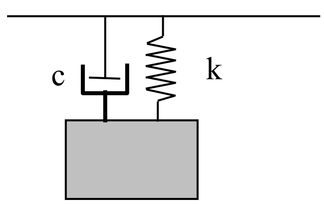
<u>Problem statement</u>: determine spring constant and damping constant to satisfy the following criteria:

#	Criteria	Priority	Description
1	Total mass (given)	Essential	Total vehicle mass is 400kg (100kg per wheel)
2	Max total displacement limits	Essential	Max allowable displacement is 300mm from the "unloaded" height (spring is fully extended) when dropped from 2 meter height.
3	Ride feel	Important	No oscillating motion (the system should be critically or over damped)
4	Minimize impact force	Important	Deceleration forces should be minimized.

<u>Assume</u>:
1 degree of freedom system
Negligible mass of spring and dashpot, spring is linear, dashpot is viscous
The system can be modeled as:



<u>General Solution method</u>:
An Excel spreadsheet (created for an ME401 assignment) has been modified to help select spring constant (k) and damping constant (c). It is included with this document. The spread sheet using "time marching" to determine displacement, velocity, and accelerations of a damped system. The following kinematic equations are used:

$$x_{i+1} = x_i + 0.5*(v_i + v_{i+1}) * \Delta t \; ; \quad v_{i+1} = v_i + a_i * \Delta t \; ; \quad a_i = -(k/m)*x_i - (c/m)*v_i - a_g$$

The initial conditions of the vehicle at the moment of impact are as follows:

$x_0 = 0$ meters

$v_0 = -(2*a_g*h)^{1/2}$ where $a_g = 9.8 m/s^2$ and h is the height from which it was dropped (h=2m).

$v_0 = -6.26$ m/s

a_0 is calculated as: $a_0 = -(k/m)*x_0 - (c/m)*v_0 - a_g$

Criteria Evaluation

The first two criteria are considered "essential;" therefore, they must be satisfied. Criteria 3 and 4 are somewhat in opposition to each other. In order to have minimal oscillation, the system should be critically damped or over damped. To minimize deceleration forces, damping should be minimal (most of the initial impact force is due to high velocity and hence high force for the dashpot).

Design decision: since the deceleration forces exist for an extremely short duration at the moment of impact (and the driver's seat, etc. will absorb some impact force), it was decided to give criterion 3 (ride feel) greater priority than criterion 4 (deceleration force). Therefore, the system will be designed to be critically damped ($\zeta = 1$).

Alternative solution: criteria 3 and 4 are both considered "important" and satisfying one will occur at the detriment of the other. The above recommendation (selecting $c=c_c$) was done to satisfy criterion 3 more than 4. To reduce impact force, $c<c_c$ could be selected, but there will be some oscillatory motion after impact (meeting criterion 3 less well). As an example, $\zeta = .75$ ($c=0.75c_c$) was investigated.

Calculations:

Determine critical damping:

$\zeta = c/c_c = 1$ Therefore, $c = c_c$; $c_c = 2m\,\omega_n$; $\omega_n = (k/m)^{1/2}$ $m = 100$ kg

In the attached spreadsheet, the value for spring constant, k (located in cell A4) is an input parameter. Once k has been entered in the spreadsheet, the natural frequency is calculated and from that, the critical damping constant is determined (c_c). The damping constant (c) is set equal to the critical damping constant. From those parameters, the displacement, velocity and acceleration of the mass (the vehicle) is calculated. The spring constant was varied until the maximum displacement was less than 300mm (criterion 2).

Results and Conclusion

Figure 1 shows that with k = 7100 N/m and critically damped (c= 1685kg/sec), that the maximum displacement is 0.3 meters. This satisfies all of the criteria except possibly criterion 4 (deceleration). It should be noted, that the Excel spreadsheet shows that if $\zeta = 0.6$, the deceleration is decreased from about 10g's to about 8g's, but with oscillation. Since the driver of the vehicle will be sitting in a cushioned seat, they will not personally experience this magnitude of deceleration.

Table 1 – comparison of deceleration and oscillation of two alternatives

Damping ratio, ζ	Damping factor, c	Spring rate, k	Maximum Impact Deceleration	Oscillation
1.0	1658kg/sec	7100 N/m	10g	None

0.6	1368kg/sec	13,000 N/m	7.8g	Some, dies out in <1 sec

Decision: k = 7100 N/m and critically damped (c= 1685kg/sec)

Formulae used in excel (note to students: hold CTRL and ~ down to show equations in cells)

mass, kg	time	displacement	velocity, m/s	acceleration	max disp
100	0	0	=-A18	=-(A$4/A$2)*C2-(A$12/A$2)*D2-A$14	=MIN(C2:C500)
spring rate, N/m	0.01	=C2+0.5*(D2+D3)*(B3-B2)	=D2+E2*(B3-B2)	=-(A$4/A$2)*C3-(A$12/A$2)*D3-A$14	
7100	=B3+B$3	=C3+0.5*(D3+D4)*(B4-B3)	=D3+E3*(B4-B3)	=-(A$4/A$2)*C4-(A$12/A$2)*D4-A$14	
nat freq (eq 3)	=B4+B$3	=C4+0.5*(D4+D5)*(B5-B4)	=D4+E4*(B5-B4)	=-(A$4/A$2)*C5-(A$12/A$2)*D5-A$14	
=SQRT(A4/A2)	=B5+B$3	=C5+0.5*(D5+D6)*(B6-B5)	=D5+E5*(B6-B5)	=-(A$4/A$2)*C6-(A$12/A$2)*D6-A$14	
c-crit (eq 2)	=B6+B$3	=C6+0.5*(D6+D7)*(B7-B6)	=D6+E6*(B7-B6)	=-(A$4/A$2)*C7-(A$12/A$2)*D7-A$14	
=2*A2*A6	=B7+B$3	=C7+0.5*(D7+D8)*(B8-B7)	=D7+E7*(B8-B7)	=-(A$4/A$2)*C8-(A$12/A$2)*D8-A$14	
damping factor	=B8+B$3	=C8+0.5*(D8+D9)*(B9-B8)	=D8+E8*(B9-B8)	=-(A$4/A$2)*C9-(A$12/A$2)*D9-A$14	
1	=B9+B$3	=C9+0.5*(D9+D10)*(B10-B9)	=D9+E9*(B10-B9)	=-(A$4/A$2)*C10-(A$12/A$2)*D10-A$14	
damping (eq 1)	=B10+B$3	=C10+0.5*(D10+D11)*(B11-B10)	=D10+E10*(B11-B10)	=-(A$4/A$2)*C11-(A$12/A$2)*D11-A$14	
=A10*A8	=B11+B$3	=C11+0.5*(D11+D12)*(B12-B11)	=D11+E11*(B12-B11)	=-(A$4/A$2)*C12-(A$12/A$2)*D12-A$14	
accel due to grav	=B12+B$3	=C12+0.5*(D12+D13)*(B13-B12)	=D12+E12*(B13-B12)	=-(A$4/A$2)*C13-(A$12/A$2)*D13-A$14	
9.81	=B13+B$3	=C13+0.5*(D13+D14)*(B14-B13)	=D13+E13*(B14-B13)	=-(A$4/A$2)*C14-(A$12/A$2)*D14-A$14	
Drop height, m	=B14+B$3	=C14+0.5*(D14+D15)*(B15-B14)	=D14+E14*(B15-B14)	=-(A$4/A$2)*C15-(A$12/A$2)*D15-A$14	
2	=B15+B$3	=C15+0.5*(D15+D16)*(B16-B15)	=D15+E15*(B16-B15)	=-(A$4/A$2)*C16-(A$12/A$2)*D16-A$14	
velocity at impact, m/s	=B16+B$3	=C16+0.5*(D16+D17)*(B17-B16)	=D16+E16*(B17-B16)	=-(A$4/A$2)*C17-(A$12/A$2)*D17-A$14	
=SQRT(2*A14*A16)	=B17+B$3	=C17+0.5*(D17+D18)*(B18-B17)	=D17+E17*(B18-B17)	=-(A$4/A$2)*C18-(A$12/A$2)*D18-A$14	
	=B18+B$3	=C18+0.5*(D18+D19)*(B19-B18)	=D18+E18*(B19-B18)	=-(A$4/A$2)*C19-(A$12/A$2)*D19-A$14	

Figures on next page

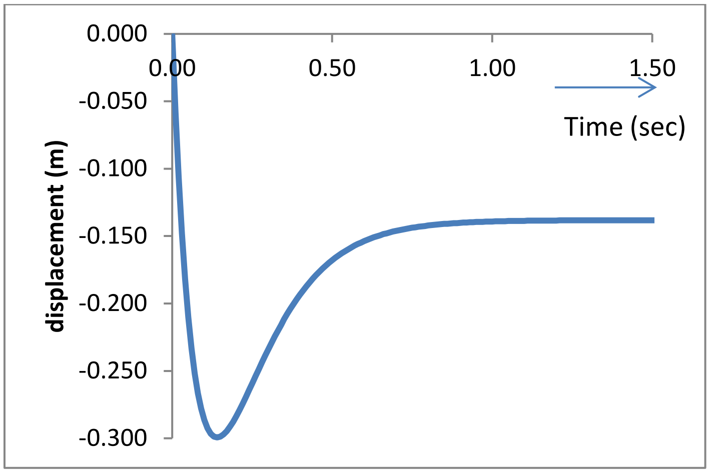


Figure 1 - From Excel, displacement verses time for critically damped system ($\zeta = 1$).

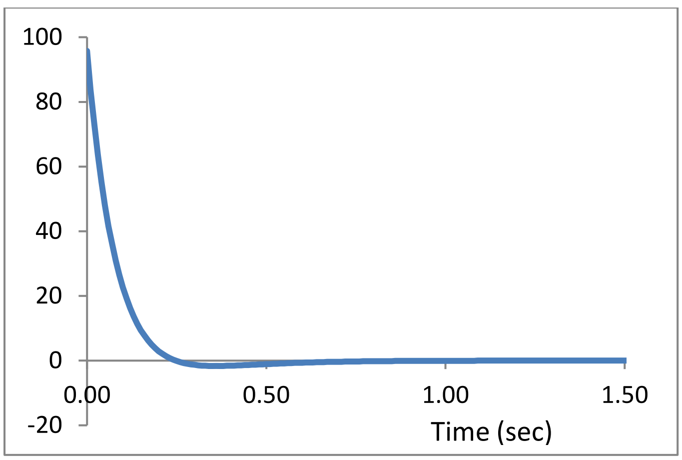


Figure 2 - From Excel, acceleration verses time for critically damped system ($\zeta = 1$).

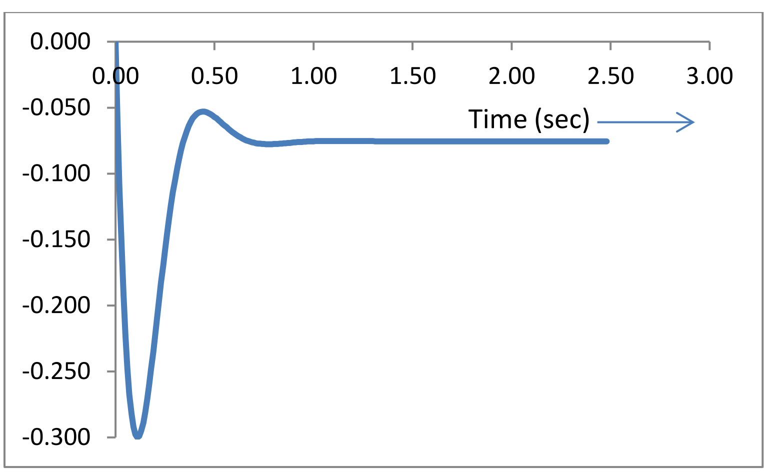


Figure 3 - From Excel, displacement verses time for underdamped system ($\zeta = 0.6$).