ME 328 – Machine Design
Assignment 2, 2020

SKETCHING: For all problems in engineering, be sure to make sketches as appropriate. Free body diagrams are almost always appropriate for stress analysis problems. For bending problems, shear and moment diagrams are appropriate. 1 picture=1000 words (approximately).

GENERAL HINT: when doing stress analysis using metric units, most engineering materials have strength on the order of MPa (10^6 N/m^2). If you use "newtons" and "millimeters" as your units for force and length, this works out well since 1 N/mm^2 is 10^6 N/m^2 (1 N/mm^2 = 1MPa). Life is less stressful if you use N/mm^2.

SHARED WISDOM: always **include units each and every step** of your calculations, and always **show the equations you use in variable form** before using numbers. This practice requires a small investment of time, but pays HUGE DIVIDENDS later! It will help you troubleshoot problems (find where you make mistakes) and help you 3 months from now remember what you were doing (critically important for practicing engineers**). Now is the time to develop good habits**. And this is **required** in the standard problem-solving format – which is **required**.

Also, do not combine multiple equations into a large "super equation" – keep them separate! For example, determine the axial stress in a 0.5 inch diameter bar with 5000 pound load:

DO SOMETHING LIKE THIS:
$\sigma = F/A$,
 $F = 5000\text{lb}$
 $A = (\pi/4)\,(d)^2 = (\pi/4)\,(0.5\text{in})^2 = 0.196\text{in}^2$
$\sigma = 5000\text{lb}/0.196\text{in}^2 = 25{,}500\text{lb/in}^2 = 25.5\text{ksi}$

NOT THIS:
$\sigma = F/A = F/\{\,(\pi/4)\,(d)^2\} = 5000\text{lb}/\{\,(\pi/4)\,(0.5\text{in})^2\} = 25{,}500\text{lb/in}^2 = 25.5\text{ksi}$

NEVER EVER NEVER NEVER THIS:
$\sigma = 5000/\{\,\pi/4\,(0.5)^2\} = 25{,}500\text{lb/in}^2 = 25.5\text{ksi}$

It is much easier to double check your answers and to troubleshoot (find mistakes) in small equations than mega-equations!

1. Educational Purpose: *This problem is meant to help you understand how to apply static failure theories which requires applying engineering judgment to make a decision.*
   How much torque can a 0.500 inch diameter bar carry before failure for the following materials? Failure is defined as either yielding or fracture – whichever comes first. How will these two materials fail (yield or fracture)? What failure theory(s) would be appropriate to apply to each of these? For material properties see textbook appendices or MatWeb (I'm

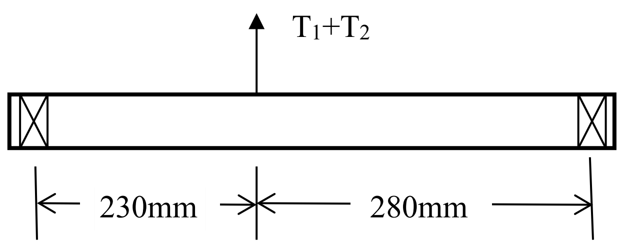
okay with MatWeb material properties for SCHOOL WORK – NEVER for REAL ENGINEERING WORK!). Hint: if you find tensile strength but not yield strength, that should tell you something (there is probably a reason for lacking yield strength data). Also, ductility can help you determine appropriate failure theory (less than about 3%EL could be considered to be brittle).

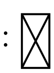
a) ASTM class 20 gray cast iron
b) 2024-T3

The following two problems are in PDF format on the course web page. Educational Purpose*: these problems are meant to stretch you beyond classroom lectures. You will need to look up appropriate equations to solve the problems. These problems also introduce you to a design consideration (angular deflection) for bearings. We will discuss bearings only very briefly later this semester.*

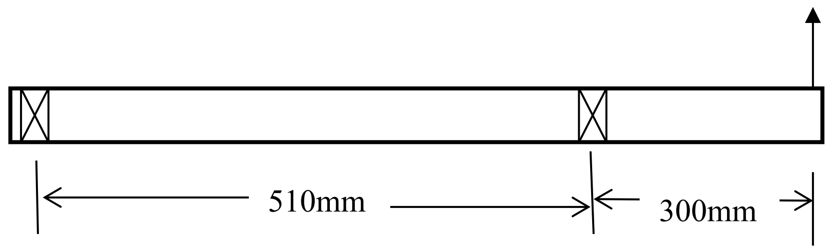
2. Problem 3-69 in the posted PDF
3. **DESIGN HW:** 4-36 (4-36 is based on 3-69) in the posted PDF. This problem may appear to be quite challenging; however, by "breaking" it into small pieces, each small piece should be manageable. You will need to use appropriate beam deflection charts that show angular deflection for various loads (pdf of such a chart is on the course web page in the homework table). NOTE: angles shown in such charts are typically in radians, not degrees. Here is what is required for this problem:

   a) Use Excel to create a graph which shows the relationship between angular deflection and shaft diameter at each of the two bearing locations for the following loading condition (from problem 4-36):



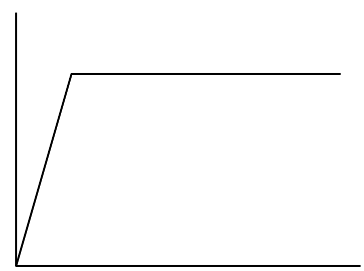
A symbol used for bearings: 

   b) Use Excel to create a graph which shows the relationship between angular deflection and shaft diameter at each of the two bearing locations for the loading condition shown below (also from 4-36). Hint, it is unlikely you will find this loading condition in any beam deflection chart. You will have to think back to your days in statics and FEA. Remember "equivalent systems"? Hint again: Replace the cantilever load (1800N+270N) with an equivalent force and couple-moment…but where? Think about this…if you have the equivalent force and moment at a location such that the force will not cause any shaft deflection (hmmm, where?) then you won't need to include that force in your deflection analysis; only the moment. Are there any loading conditions shown in the beam deflection chart that has only an applied moment?



c) Hopefully, you have assumed that the shaft will remain linear elastic (not plastically deformed). If it is linear, then you can add parts (a) and (b) together to solve problem 4-36 (adding the results from two different loads like this is referred to as "superposition" – just in case you've forgotten that). Do so to answer 4-36, and show a single graph with three lines (one each from parts a, b, and c to this problem).

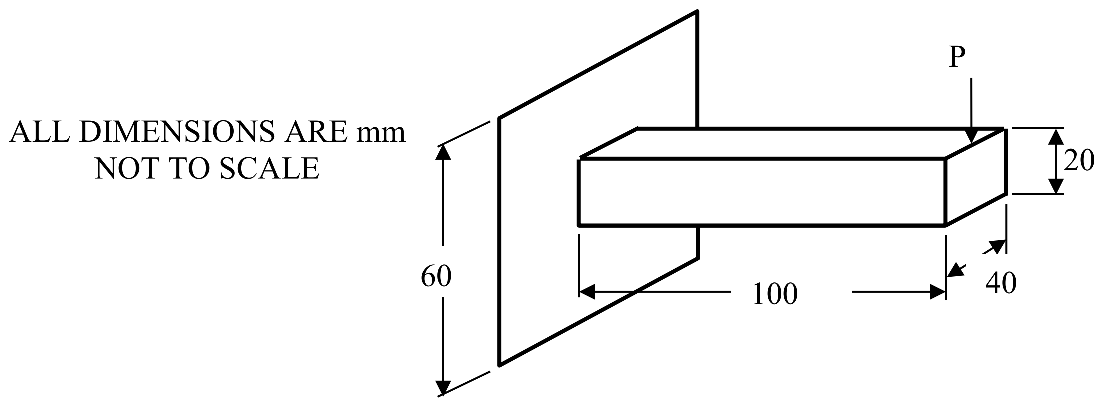
4. Educational purpose: *help understand what happens to bending stress if the material is not linear-elastic. It is also a "precursor" to stress concentrations – helping you to identify they exist and are difficult to properly model with FEA.*
For this problem, assume the aluminum alloy is elastic/perfectly plastic, and has yield strength of 300MPa (as shown in the σ–ε diagram sketch below). Perfectly plastic means that once the yield stress is reached the stress remains constant beyond that.



Using finite element analysis, determine the bending stress in the cantilever beam shown below. The load, P, is 5kN. You may choose any appropriate element type.

a. Provide a hardcopy of the FEA results which should show how the bending stress varies in the beam from end to end, from top to bottom. Do the results agree with the equation s = My/I --- remember, both bending moment (M) and distance from the neutral axis (y) varies in the beam.
b. Where specifically do you expect yielding to occur first as the load is increased sufficiently? Briefly explain why.
c. For a section of the beam near the attachment (90mm from the loaded end), calculate the bending stress at the top of the beam and compare it with the FEA results (very briefly discuss).
d. Sketch the stress distribution from top to bottom through the thickness at a cross section 90mm for the loaded end.
e. **For extra credit**: Repeat part "d", but for a load of 10kN. Hint, the 5kN load will not cause yielding, but the 10kN load will cause yielding. An assumption that is typically made for beam bending: "the material is linear elastic". Once yielding occurs, the

assumption is not valid and therefore the equation: $\sigma = My/I$ is no longer valid. Briefly explain the difference in the stress distribution between parts d and e. **You do NOT need to model this with FEA.**



Educational purpose of following questions: *learn the basics of power transmission through chain-drives, belt-drives, and gear trains. All three of those forms of power transmission share commonality in determining speed and torque relations. We will discuss gears to a limited extent later in the semester, but you will need to understand speed and torque relations sooner than that for the project.*

5. A sprocket (or sprocket-wheel) is the gear-like machine element used with chain-drives. Consider the two different chain-driven power trains (parts a and b below). For both of these, assume the load transmitting force in the belt is 100 pounds (tension – you can't push a rope…or a chain) and assume the "slack" side has no tension. Draw a free body diagram of both the driving and driven sprockets. Then determine the torque in the driving sprocket (the sprocket receiving power from the motor) and the torque in the driven sprocket (the power is transmitted to the driven sprocket from the driving sprocket through the chain).
   a) The driving sprocket is 10" in diameter and the driven sprocket is 5" in diameter.
   b) The driving sprocket is 5" in diameter and the driven sprocket is 10" in diameter.

6. Using a bicycle (yours or a friends), select the lowest gear-speed (what you would use for going up a steep hill). Answer the following (feel free to use your textbook or online sources, especially to look up terminology):
   a) How many "teeth" are on the driving sprocket, N_{in}? The "driving sprocket" is the sprocket attached to the pedals.
   b) What is the diameter of the driving sprocket, d_{in}? To determine the diameter of the sprocket, measure the distance from the center of the chain on one side of the sprocket to the center of the chain on the opposite side.
   c) What is the diametral pitch of the driving sprocket, P_{in}? The diametral pitch has units of number of teeth per inch, $P=N/d$; where N is the number teeth and d is the diameter of the sprocket ("pitch diameter").

d) How many "teeth" are on the driven sprocket, N_{out}? The "driven sprocket" is the sprocket attached to the wheel.
e) What is the diameter of the driven sprocket, d_{out}?
f) What is the diametral pitch of the driven sprocket, P_{out}?
g) If the pedals were being operated at 100RPM (revolutions per minute), what would the speed of the wheel be (in RPM)? Hint, if the tire rotates one-half revolution for each full peddle revolution, then the answer would be 50RPM.
h) Based on your bike measurements above, show that the following is true:
$P_{out} = P_{in} = P = N_{in}/d_{in} = N_{out}/d_{out}$
i) Show that the following is also true:
$d_{in}/d_{out} = \omega_{out}/\omega_{in}$ where ω_{in} and ω_{out} are the angular velocities (RPM or radians per second) for the driving and driven sprockets, respectively.
j) Assuming power loss in the chain-drive system is negligible (typically, a reasonable assumption), and power is torque times angular velocity (power = $T\omega$; ω should be in radian per second); therefore, $T_{in}\omega_{in} = T_{out}\omega_{out}$. If the bicyclist applies 50ft-lb torque to the pedal, what is the torque in the wheel?

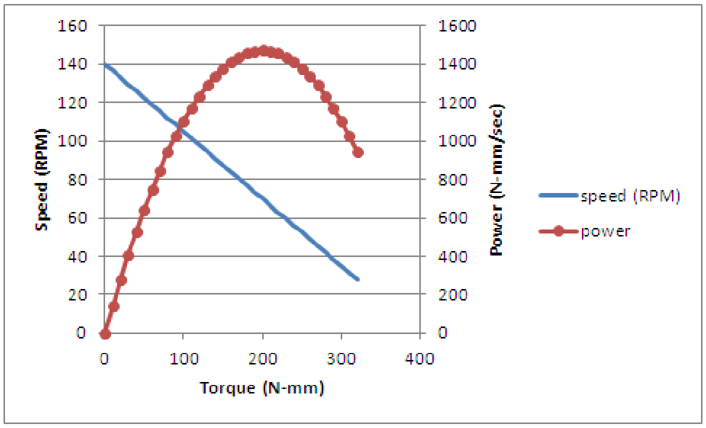
7. Educational purpose: *this problem builds on the previous problem. This problem now introduces power transmission. This too, will be important for the project.* Gear ratios are expressed as fractions or ratios: $\omega_{driving}$ / ω_{driven} or typically $\omega_{driving}$:ω_{driven} (input speed to output speed). A ratio of 2:1 means the output of the chain (or gear) is spinning at half the speed of the input sprocket (or gear). A ratio of 0.9:1 means the output (tire) is spinning faster than the input (bike peddle or motor) – this is sometimes referred to as "overdrive". Consider an electric motor with the performance curve shown below. This is a typical curve for electric DC motors – i.e. there is a near-linear relation between speed (rpm) and torque (T). At a certain high torque level, the motor stalls (stops turning) and spins at a maximum speed when no load is applied (free spinning). Power is the product, $T\omega$.

You are part of the design team for winch system. The project criteria are shown in the table:

Criterial Table

#	**Criteria**	**Priority**	**Description**
1	Motor	Essential	Use motor performance curve data provided
2	Drum	Essential	Diameter of the lifting drum is 20mm
3	Load	Essential	Capable of lifting a 1kg load
4	Speed	Important	Lifting the 1 kg load as fast as possible within the given constraints.

Your task is to determine the following based on the criteria in the Criterial Table:
a) What gear ratio best meets the needs?
b) How long will it take to raise the load 1 meter assuming constant velocity? Assume a_g=10m/s^2.



Electric motor performance curve (at 12VDC)

8. Educational purpose: *mechanical engineering has its own jargon. It is important for students to be conversant in this jargon.* Define or describe the following terms, use sketches if appropriate, and cite your sources (URL's are acceptable if cited): struts, gusset, plenum, strain gage (*single element* and *strain gage rosette*), OEM (Original Equipment Manufacturer).