ASME International

Name SOLUTION
Course ME401
Sheet ______ of ______
Date ______

Given: 0.50 in dia bar

Find: - Torque req'd for failure
- what failure theory is appropriate
- how will it fail

Mat'l a) ASTM class 20 gray cast iron
b) AISI 1040 HR

Assume: uniform torque throughout

Solution:

a) & b) $\tau_{max} = \dfrac{Tr}{J}$

$\longrightarrow T = \dfrac{J\tau_{max}}{r}$

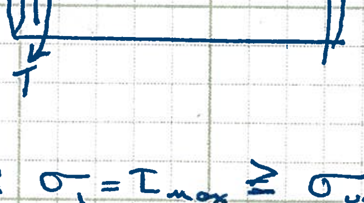
$J = \dfrac{\pi D^4}{32}$ eq 3-38

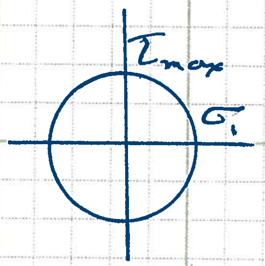
$= \dfrac{\pi (0.5 in)^4}{32} = 6.14 \times 10^{-3} in^4$

$r = 0.25 in$

a) ASTM class 20: $\sigma_{UT} = 22 ksi$ (Table A-24)

No yield strength is listed in Table A-24 because gray cast iron has low ductility (i.e. it is 'brittle'). It will fail due to brittle fracture, max normal stress thry is appropriate:





$\sigma_1 = \tau_{max}$ (pure shear loading)

failure: $\sigma_1 = \tau_{max} \geq \sigma_{UT}$

$\longrightarrow T = \dfrac{(6.14 \times 10^{-3} in^4)(22{,}000 psi)}{0.25 in} = \underline{\underline{540 \text{ in-lb}}}$

(ALTERNATIVE - NEXT PG) *

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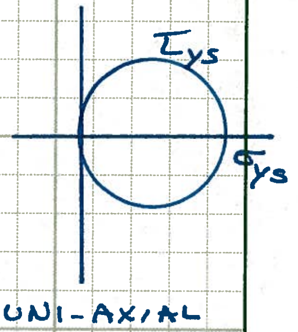
b) AISI 1040 HR

-ductile mat'l, will fail due to yielding

- use max shear or von Mises failure theory.



Max shear theory $\tau_{ys} = \frac{1}{2}\sigma_{ys}$

1040 HR:

$\sigma_{ys} = 42$ ksi (Table A-20)

$\tau_{ys} = \frac{1}{2}(42\,\text{ksi}) = 21$ ksi

$$\longrightarrow \quad T = \frac{(6.14\times10^{-3}\,\text{in}^4)(21\,\text{ksi})}{0.25\,\text{in}} = \underline{\underline{516\ \text{in-lb}}}$$

* CAST IRON:

a) alternative. Shear modulus of rupture $S_{su} = 26$ ksi

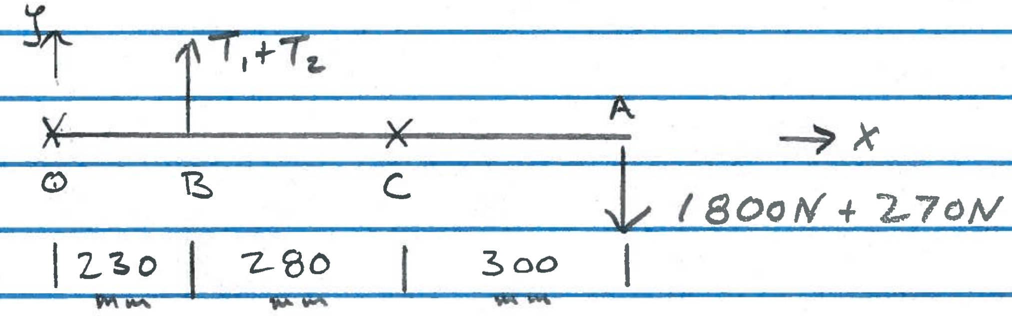
S_{su} is the "ultimate shear strength" that is applicable for brittle failure due to "pure shear" load.

$$T = \frac{J\tau_{max}}{r} = \frac{(6.14\times10^{-3}\,\text{in}^4)(26\,\text{ksi})}{0.25}$$

$$T = 638\ \text{ksi} = \underline{\underline{640\ \text{ksi}}}$$ (somewhat better answer)

3-69

Given: 30 mm dia shaft as shown:



Pulleys at B & A, $T_2 = 0.15\,T_1$. $D_B = 250$ mm

Find: $D_A = 400$ mm

a) T_1 & T_2. Soln: $\Sigma T = 0$

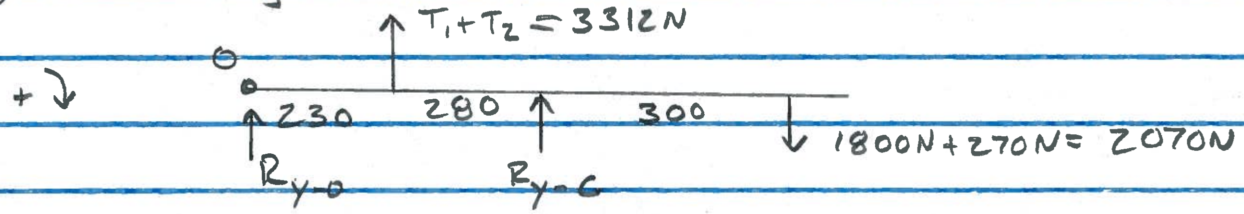
$\Sigma T = (1800N - 270N)200\text{mm} + (0.15T_1 - T_1)125\text{mm}$

$= 0$

$= 306$ kN-mm $-(0.85T_1)125$mm $= 0$

$T_1 = 2880$ N-m $T_2 = 432$ N-mm

b) Find brg reactions (@ O & C)



$\Sigma M_B = 0 = (-3312N)230\text{mm} - R_{yc}(230+280\text{mm}) + (230+280+300\text{mm})2070N$

$R_{yc} = 1794N\ (\uparrow)$

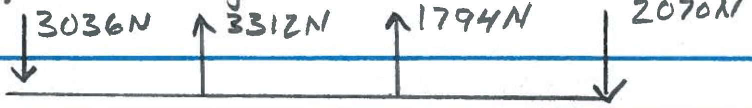
$\Sigma F_y = 0 = R_{yO} + 3312N + 1794N - 2070N$

$R_{yO} = 3036N\ (\downarrow)$

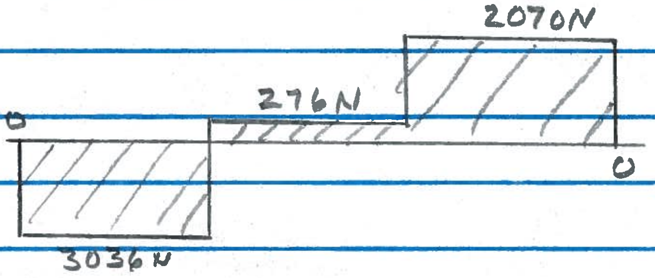
3-69

c) Draw V, M dig's

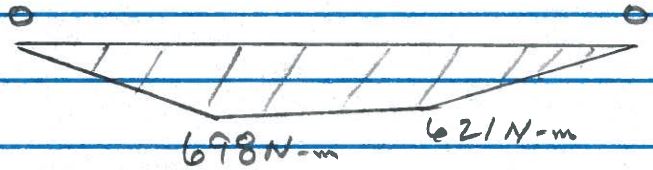
loading



Shear



Moment



d) Find bending stress & torsional shear at pt B (M_{max})

$$\sigma = \frac{Mc}{I}$$

$M = 698{,}000\ \text{N-mm}$

$c = \frac{1}{2} d = \frac{1}{2}(30\text{mm}) = 15\ \text{mm}$

$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(30\text{mm})^4$

$I = 39{,}700\ \text{mm}^4$

$$\sigma = \frac{(698{,}000\ \text{Nmm})(15\text{mm})}{39{,}700\text{mm}^4} = 263\ \text{N/mm}^2$$

$\underline{\underline{\sigma = 260\ \text{MPa}}}$

3-69

d)

$$\tau = \frac{Tr}{J}$$

$T = (1800\,N - 270\,N)\,200\,mm$

$T = 306{,}000\ N\text{-}mm$

$r = 15\ mm$

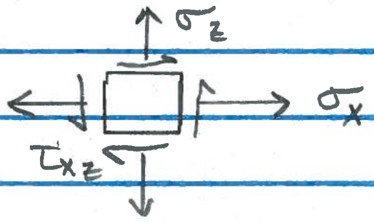
$J = \frac{\pi}{32} d^4 \qquad d = 30\,mm$

$J = 79{,}500\ mm^4$

$$\underline{\underline{\tau = 58\ MPa}}$$

e) at B (M_{max}) find principal stresses & max shear

- consider a point at to of shaft at B:



σ_x = bending stress

$\sigma_x = 260\ MPa$

$\sigma_z = 0$

τ_{xz} = torsional shear

$= 58.\ MPa$

$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \frac{260\,MPa + 0}{2} + \left[\left(\frac{260\,MPa}{2}\right)^2 + (58\,MPa)^2\right]^{1/2}$$

$$= 130\,MPa + 142\,MPa = \underline{\underline{270\,MPa}}$$

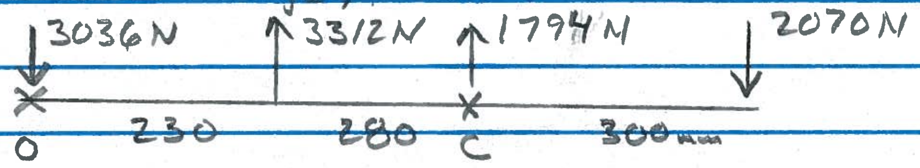
$$\sigma_3 = 130 - 142 = \underline{\underline{-12\,MPa}}$$

$$\tau_{max} = 142\,MPa = \underline{\underline{140\,MPa}}$$

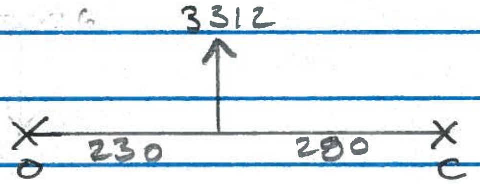
4-36

From 3-69, steel shaft

Determine minimum shaft diameter such that brg deflection is no greater than 0.06° (for both brgs)



(a) consider deflections at O & C for:



From deflection chart (#5)

AT PT O:

$$\Theta_o = \frac{Pab(L+b)}{6LEI}$$

where
$P = 3312 \text{ N}$
$a = 230 \text{ mm}$
$b = 280 \text{ mm}$
$L = 230 + 280 = 510 \text{ mm}$
$E = 210{,}000 \text{ N/mm}^2$ (steel)
$I = \frac{\pi}{64} d^4$

If $d = 20 \text{ mm}$, $I = 7853 \text{ mm}^4$

then $\Theta_o = 0.0334$ radians

4-36

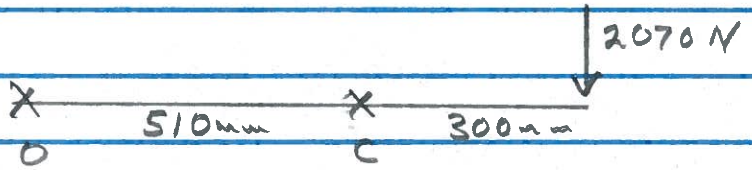
at C:

$$\theta_c = \frac{Pab(L+a)}{6LEI}$$

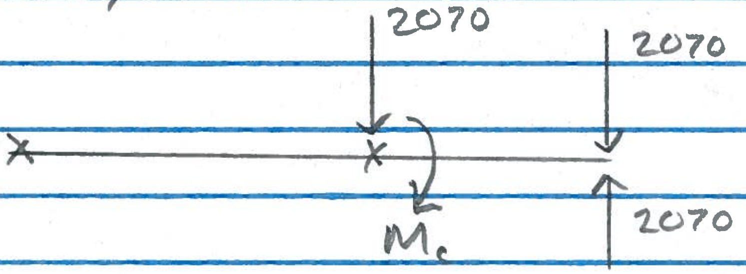
If $d = 20\text{mm}$ $\theta_c = 0.0313$ rad.

SEE ATTACHED EXCEL

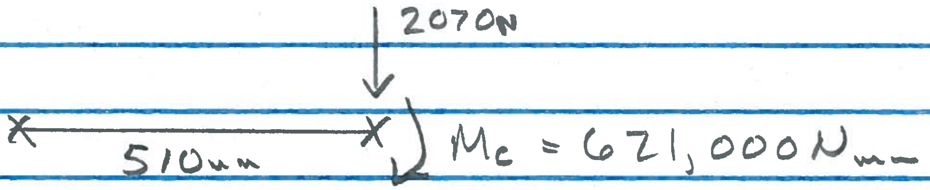
b) Consider deflections at O & C for:



Equivalent system:



$$M_c = (2070\text{ N})\,300\text{mm}$$
$$= 621{,}000\text{ N mm}$$



The 2070N load at C causes <u>no deflection</u> in shaft

4-36

Chart condition (#7) w/ M at end:

$$\theta_o = \frac{M_o L}{6EI} \qquad \theta_c = \frac{M_o L}{3EI}$$

(#7 show M at left side, so "flip it")

+ where $M_o = 621{,}000$ Nmm
$L = 510$ mm
$E = 210{,}000$ MPa
$I = \frac{\pi}{64} d^4$
If $d = 20$mm $I = 7853$ mm^4

$\theta_o = 0.0320$ rad $\qquad \theta_c = 0.0640$ rad

See Excel attached.

c) 'Add' parts a & b.
NOTE: Loads on both pulleys cause deflections in the same direction at both brgs (so add, not subtract).
FOR BRG O, ≥56mm dia shaft is req'd
FOR BRG C, ≥62mm dia shaft is req'd

Therefore, <u>62 mm</u> dia is minimum acceptable

Problem 4-36 in Shigley:

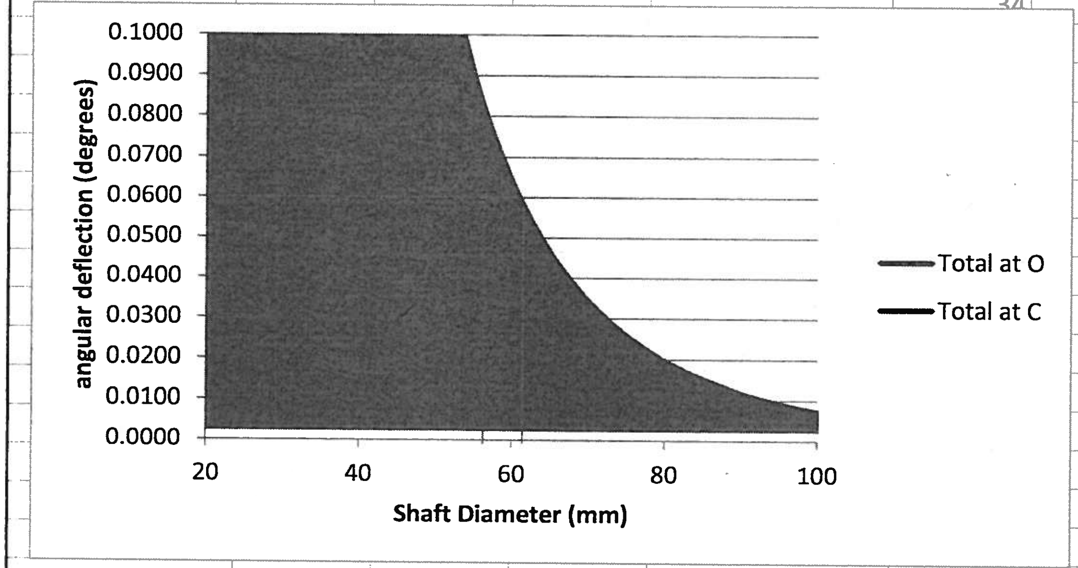
determine shaft deflection at brgs as a fcn of shaft diameter

GIVEN		units
length a	230	mm
length b	280	mm
total length	510	mm
applied F	3312	N
applied M	621000	N-mm
mod of Elast	210000	Mpa

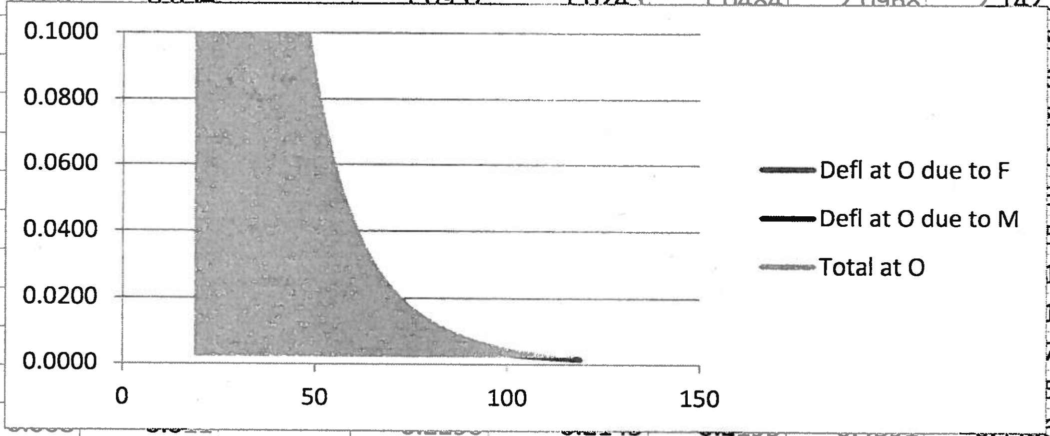
CHECKED, 20mm

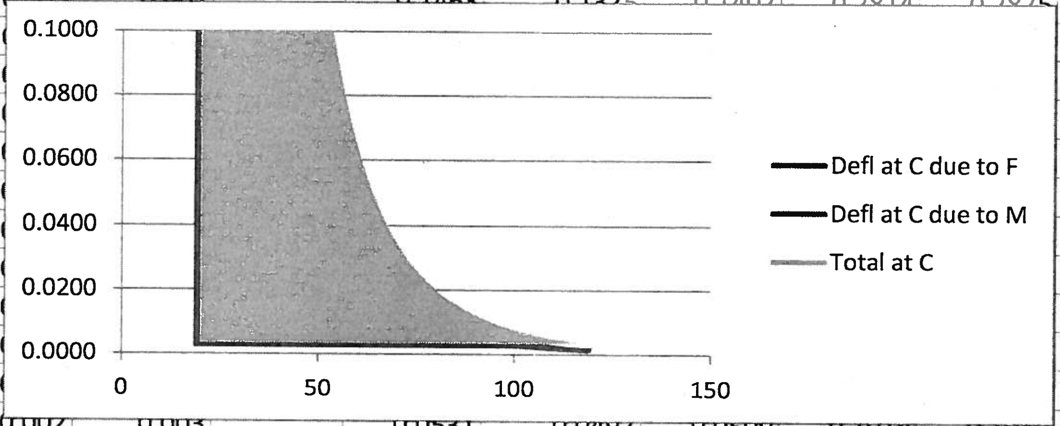
degrees-to-radians 57.29565

Shaft dia, mm	I, pi*d^4/64	In radians:	Defl at O due to F	Defl at C due to F	Defl at O due to M
19	6397		0.041	0.038	0.039292
20	***7854***		***0.033***	***0.031***	***0.032004***
21	9547		0.027	0.026	0.02633
22	11499		0.023	0.021	0.021859
23	13737		0.019	0.018	0.018298
24	16286		0.016	0.015	0.015434
25	19175		0.014	0.013	0.013109
26	22432		0.012	0.011	0.011205
27	26087		0.010	0.009	0.009635
28	30172		0.009	0.008	0.008331
29	34719		0.008	0.007	0.00724
30	39761		0.007	0.006	0.006322
31	45333		0.006	0.005	0.005545
32	51472		0.005	0.005	0.004883
33	58214		0.005	0.004	0.004318
	65597		0.004	0.004	0.003832
	73662		0.004	0.003	0.003412
	82448		0.003	0.003	0.003049
	91998		0.003	0.003	0.002732
	102354		0.003	0.002	0.002456
	113561		0.002	0.002	0.002213
	125664		0.002	0.002	0.002
	138710		0.002	0.002	0.001812
	152745		0.002	0.002	0.001646
	167820		0.002	0.001	0.001498
	183985		0.001	0.001	0.001366
	201289		0.001	0.001	0.001249
	219787		0.001	0.001	0.001144
	239531		0.001	0.001	0.001049
	260577		0.001	0.001	0.000965
49	282980		0.001	0.001	0.000888
50	306797		0.001	0.001	0.000819



Defl at C due to M	Total at O	Total at C	In degrees:	Defl at O due to F	Defl at C due to F	Defl at O due to M	Defl at C due to M	Total at O	Total at C	Shaft dia, mm
0.078584	0.080	0.117		2.3485	2.1999	2.2513	4.5025	4.5998	6.7024	19
0.064007	***0.065***	***0.095***		***1.9129***	***1.7918***	***1.8337***	***3.6673***	***3.7466***	***5.4592***	20
0.052659	0.054	0.078		1.5738	1.4741	1.5086	3.0171	3.0823	4.4913	21
0.043718	0.045	0.065		1.3065	1.2238	1.2524	2.5048	2.5590	3.7287	22
0.036596	0.037	0.054		1.0937	1.0245	1.0484	2.0968	2.1421	3.1213	23
0.030868									2.6327	24
0.026217									2.2361	25
0.022411									1.9114	26
0.019271									1.6436	27
0.016662									1.4211	28
0.01448									1.2350	29
0.012643									1.0784	30
0.011089									0.9458	31
0.009767									0.8330	32
0.008636									0.7365	33
0.007664									0.6536	34
0.006825	0.007	0.010		0.2040	0.1910	0.1955	0.3910	0.3995	0.5821	35
0.006097	0.006	0.009		0.1822	0.1707	0.1747	0.3494	0.3569	0.5200	36
0.005464	0.006	0.008		0.1633	0.1530	0.1565	0.3131	0.3199	0.4661	37
0.004912	0.005	0.007		0.1468	0.1375	0.1407	0.2814	0.2875	0.4189	38
0.004427									0.3776	39
0.004									0.3412	40
0.003624									0.3091	41
0.003291									0.2807	42
0.002996									0.2555	43
0.002732									0.2330	44
0.002497									0.2130	45
0.002287									0.1951	46
0.002099									0.1790	47
0.001929									0.1645	48
0.001777	0.002	0.003		0.0531	0.0497	0.0509	0.1018	0.1040	0.1515	49
0.001639	0.002	0.002		0.0490	0.0459	0.0469	0.0939	0.0959	0.1398	50





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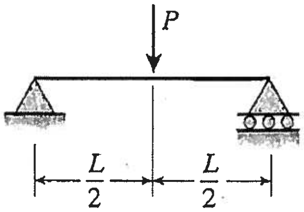
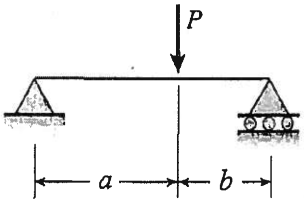


$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \qquad v' = -\frac{P}{16EI}(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{\max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

5



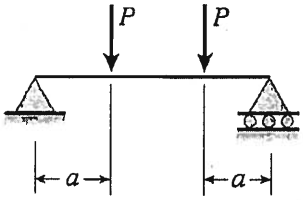
$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L + b)}{6LEI} \qquad \theta_B = \frac{Pab(L + a)}{6LEI}$$

$$\text{If } a \ge b, \quad \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI} \qquad \text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

$$\text{If } a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad \text{and} \quad \delta_{\max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\,LEI}$$

6



$$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2) \qquad v' = -\frac{P}{2EI}(aL - a^2 - x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2) \qquad v' = -\frac{Pa}{2EI}(L - 2x) \qquad (a \le x \le L - a)$$

$$\delta_C = \delta_{\max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \qquad \theta_A = \theta_B = \frac{Pa(L - a)}{2EI}$$

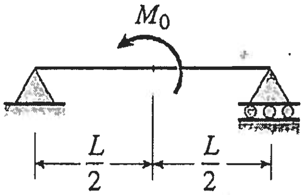
7

$$v = -\frac{M_0x}{6LEI}(2L^2 - 3Lx + x^2) \qquad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\delta_C = \frac{M_0L^2}{16EI} \qquad \theta_A = \frac{M_0L}{3EI} \qquad \theta_B = \frac{M_0L}{6EI}$$

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \text{and} \quad \delta_{\max} = \frac{M_0L^2}{9\sqrt{3}\,EI}$$

8



$$v = -\frac{M_0x}{24LEI}(L^2 - 4x^2) \qquad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = 0 \qquad \theta_A = \frac{M_0L}{24EI} \qquad \theta_B = -\frac{M_0L}{24EI}$$

(Continued)

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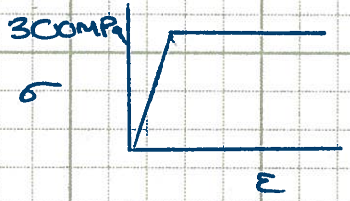
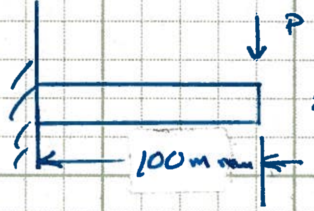
Given: Elastic-perfectly plastic mat'l
$\sigma_{YS} = 300\ MPa$



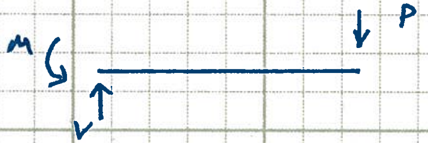
Cantilever beam:

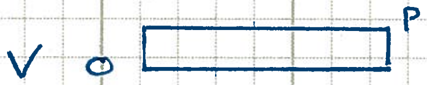


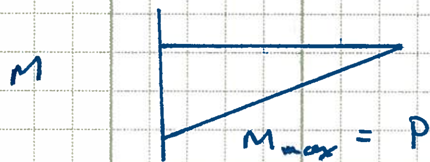
20mm high
40mm wide

a) For P = 5 kN - how does the stress vary from end to end, top to bottom?

ANS:







$M_{max} = P \cdot L = (5kN)(90\ mm) = 450\ kN\text{-}mm$

Bending stress:

$\sigma = \frac{My}{I}$ Assuming linear mat'l

∴ stress is proportional to M - varies linear from zero at free end to max at "wall".

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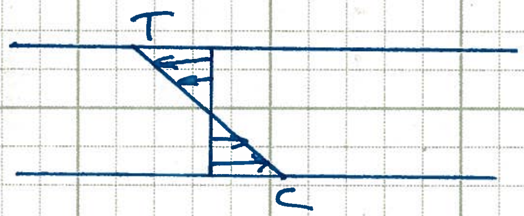
stress varies linearly through the thickness (y is distance from neutral axis).

∴ stress is max where y is max & M is max

b. As P increases, where will yielding begin at?

ANS: At top or bottom (assuming $\sigma_{ys}^{T} = \sigma_{ys}^{C}$) at the wall.

c. sketch stress distribution at 90mm from wall:



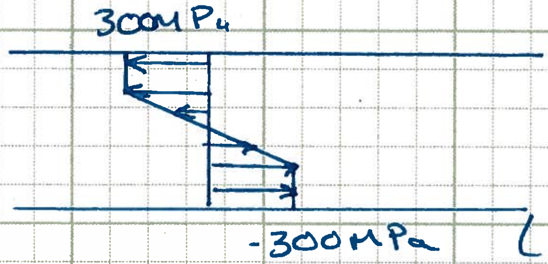
Assuming linear elastic

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{(450{,}000\,\text{N-mm})(10\,\text{mm})}{\frac{1}{2}(40\,\text{mm})(20\,\text{mm})^3}$$

$$= \underline{\underline{170}}\ \underline{\text{MPa}}$$

d. repeat c, except for a sufficient load to cause yielding:



(assuming $\sigma_{ys}^{C} = \sigma_{ys}^{T}$)

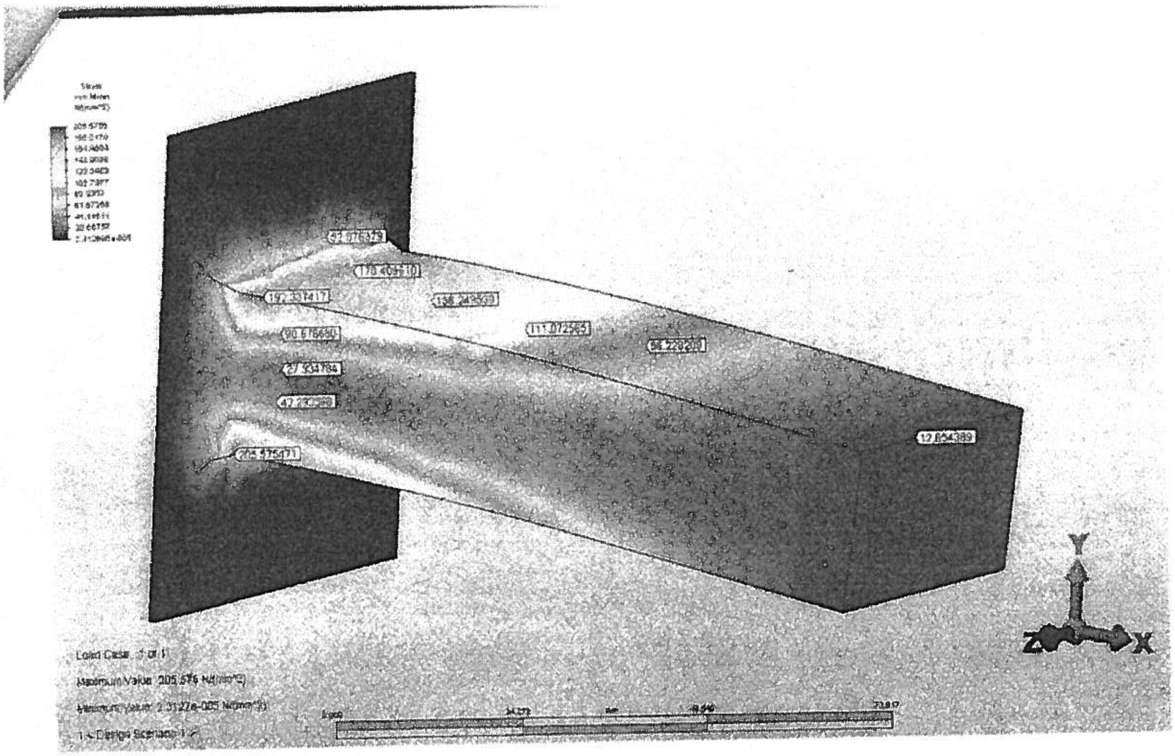


Figure 1: 5kN load applied to beam; von Mises stress results.

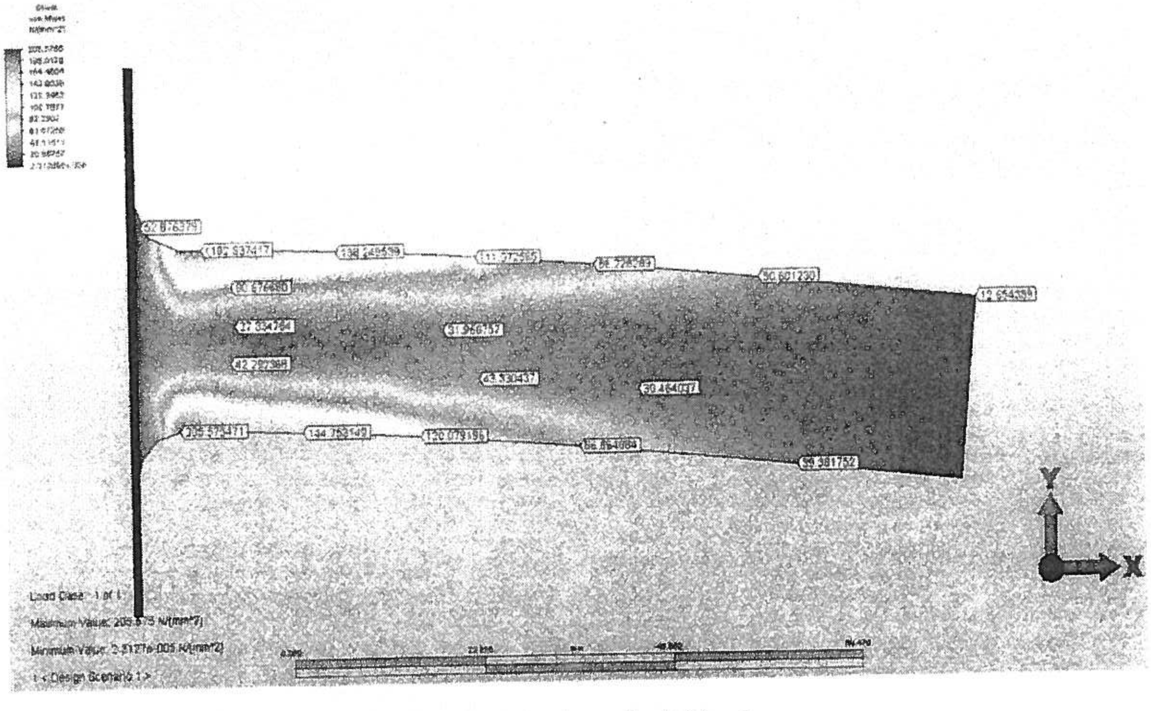
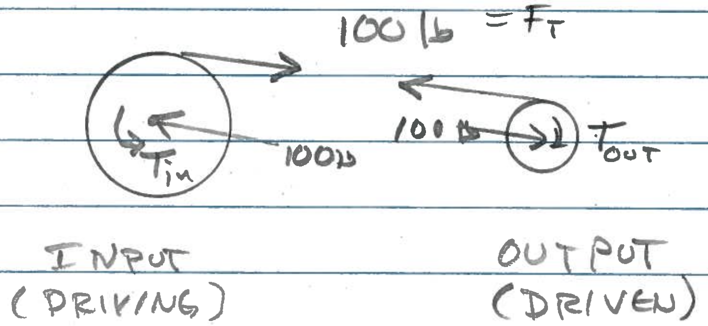


Figure 2: Side view of loaded beam.

5 Draw FBD of 2 chain drives discussed below.

Tension = F_T = 100 pound (tight side)
= ϕ (slack side)

a) $d_{in} = 10''$ $d_{out} = 5''$



$$T_{in} = \tfrac{1}{2} d_{in} E_T = \tfrac{1}{2}(10'')(100\text{ lb}) = \underline{500\text{ in-lb}}$$
$$T_{OUT} = \tfrac{1}{2} d_{OUT} F_T = \tfrac{1}{2}(5'')(100\text{ lb}) = \underline{\underline{250\text{ in-lb}}}$$

b) $d_{in} = 5''$ $d_{out} = 10''$

$$T_{in} = \tfrac{1}{2} d_{in} F_T = \tfrac{1}{2}(5\text{ in})(100\text{ lb}) = \underline{250\text{ in-lb}}$$
$$T_{OUT} = \tfrac{1}{2} d_{OUT} F_T = \tfrac{1}{2}(10\text{ in})(100\text{ lb}) = \underline{\underline{500\text{ in-lb}}}$$

6) Varies depending on bike

7) Given the performance curve for a DC motor (speed vs. T & power vs. T)

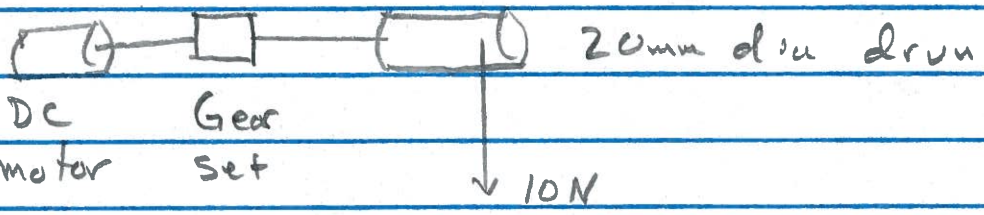
Drum dia = 20mm

"Weight" = 1 kg (10N) $a_g = 10 m/s^2$

Determine gear ratio ($\omega_{drive} : \omega_{driven}$)

a) to lift the 1kg load the fastest.

b) how long will it take to lift 1m?

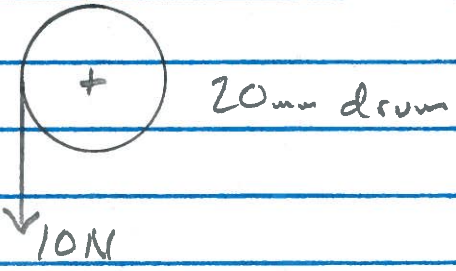


Solution

- assume no losses of power

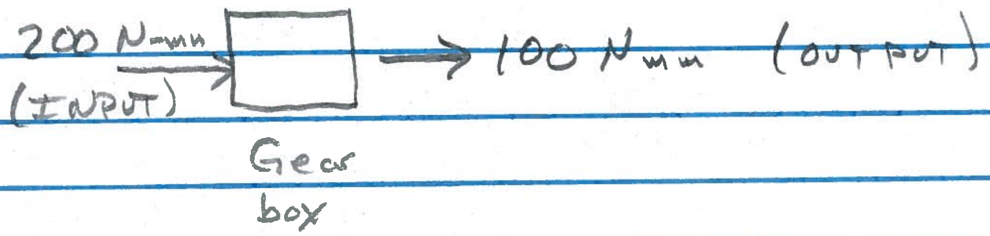
This motor has maximum power output when a 200 Nmm torque is applied.

Determine gear ratio that will load the motor at 200 N-mm.



when lifting 1 kg at constant speed

$$T = W\left(\tfrac{1}{2}d\right) = 10N\,(10\,mm)$$
$$= 100\ N_{mm}$$



$$\frac{\omega_{IN}}{\omega_{OUT}} \Rightarrow \frac{\omega_{driving}}{\omega_{driven}} = \frac{T_{OUT}}{T_{IN}} = \frac{100\ N_{mm}}{200\ N_{mm}}$$

RATIO: 1:2 OR $\underline{\underline{0.5:1}}$

b) How fast? At 200 Nmm load, the motor spins at 65 RPM. WITH 0.5:1, the drum will spin at 130 RPM.

$$\text{Circum} = \pi D = \pi(20mm) = 62.8mm$$

- Each rev moves the wt 62.8mm

$$1m = \frac{0.0628m}{rev} \cdot \frac{130\,rev}{min} = 8.16\ m/min$$

$$\Rightarrow 0.122\ min/meter \quad \left(\underline{\underline{7.3\ sec/m}}\right)$$