ME 328 - H/W

Given: vehicle total mass 400kg

four springs with $k = 13\ kN/m$

Impact: $4.4\ m/s$ vertical

Find: b) ω_n (no damping) - nat. freq.

c) δ_{max} (no damping) - max deflection

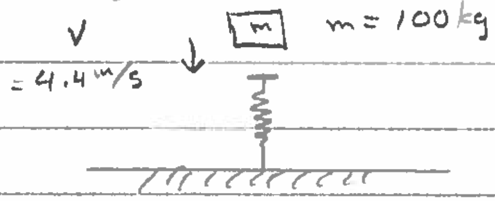
d) δ_{ST} - static deflection

e) show $c_c = 2280\ N\text{-sec}/m$

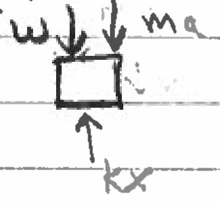
Assume: $\frac{1}{4}$ mass per wheel (100kg per spring)

linear elastic, massless spring

Solution a) FBD:



FBD (+Kinetic)



b) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{13{,}000\ N/m}{100\ kg}} = \underline{\underline{11.4\ rad/sec}}$

and: $f_n = \frac{\omega_n}{2\pi} = 1.8\ Hz$

c) $\delta_{max} = \delta_{ST}\left(1 + \sqrt{1 + \frac{v^2}{g\,\delta_{ST}}}\right)$

$$\delta_{ST} = \frac{W}{k} = \frac{mg}{k} = \frac{(100\ kg)(9.8\ m/s^2)}{13{,}000\ N/m} = 0.075\ m$$

$$\delta_{max} = 0.075m\left(1 + \sqrt{1 + \frac{(4.4\ m/s)^2}{(9.8\ \frac{m}{s^2})(0.075m)}}\right) = \underline{\underline{0.47\ m}}$$

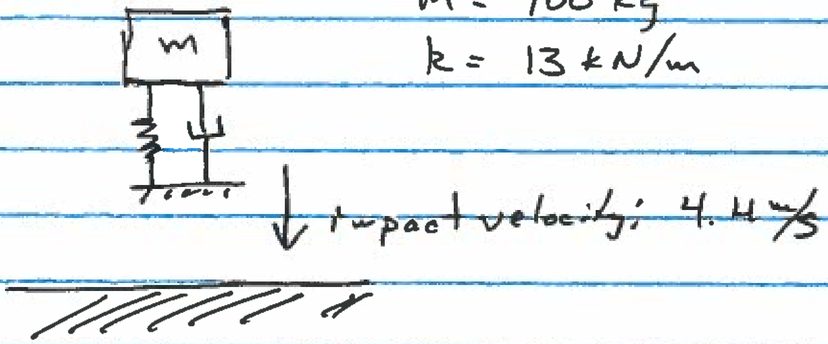
5 cont d) $\delta_{ST} = \frac{mg}{k} = \frac{(400\,kg)(9.8\,m/s^2)}{4(13{,}000\,N/m)}$ $\underline{\underline{0.075\,m}}$

e) $c_c = 2 m \omega_n$ (eq 2 in handout)

$= 2(100\,kg)(11.4\,rad/sec) = 2280\,kg/sec$

$= 2280\,\frac{kg}{sec} \cdot \frac{N}{kg\,m/s^2}$ $\underline{\underline{2280\,N\text{-}sec/m}}$

Given:



$m = 100\ kg$

$k = 13\ kN/m$

Plot displacement and acceleration vs time for $\xi = 0,\ 0.25,\ 1.0,\ 2.0$

use time marching:

$$X_{i+1} = X_i + 0.5(V_i + V_{i+1}) \cdot \Delta t$$

$$V_{i+1} = V_i + a_i * \Delta t$$

$$a_i = -(k/m) \cdot x_i - (c/m) \cdot v_i - a_g$$

for initial condition $i = 0$ $(t = 0)$, let $\xi = 1$

$X_1 = 0$

$V_1 = -4.4\ m/s$

$a_1 = (-k/m)(0) - (c/m)(-4.4\ m/s) - 9.8\ m/s^2$

$= 90.52\ m/s^2$

let $\Delta t = 0.01\ sec$

$V_2 = -4.4\ m/s + 90.52 \frac{m}{s^2} \cdot (0.01 sec) = -3.495 \frac{m}{s}$

$X_2 = 0 + 0.5(-4.4 - 3.495)(0.01 sec) = -0.039 m$

$a_2 = -(k/m)(-0.039 m) - (\frac{c}{m})(-3.495 \frac{m}{s}) - 9.8\ m/s^2$

$= 75.01\ m/s^2$

a) From graph, what is undamped natural freq?

$$T \simeq 0.55\text{ec}$$

$$\omega_n = \frac{2\pi}{T} = \underline{\underline{11.4}} \text{ rad/sec} \quad (1.82\,\text{Hz})$$

$$\left(\text{theory:} \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{13{,}000\,\text{N/m}}{100\,\text{kg}}} = \underline{\underline{11.4}}\,\frac{\text{rad}}{\text{sec}}\right)$$

b) What is displacement once oscillations have died out?

ANS from spreadsheet: $\underline{\underline{0.075\,\text{m}}}$

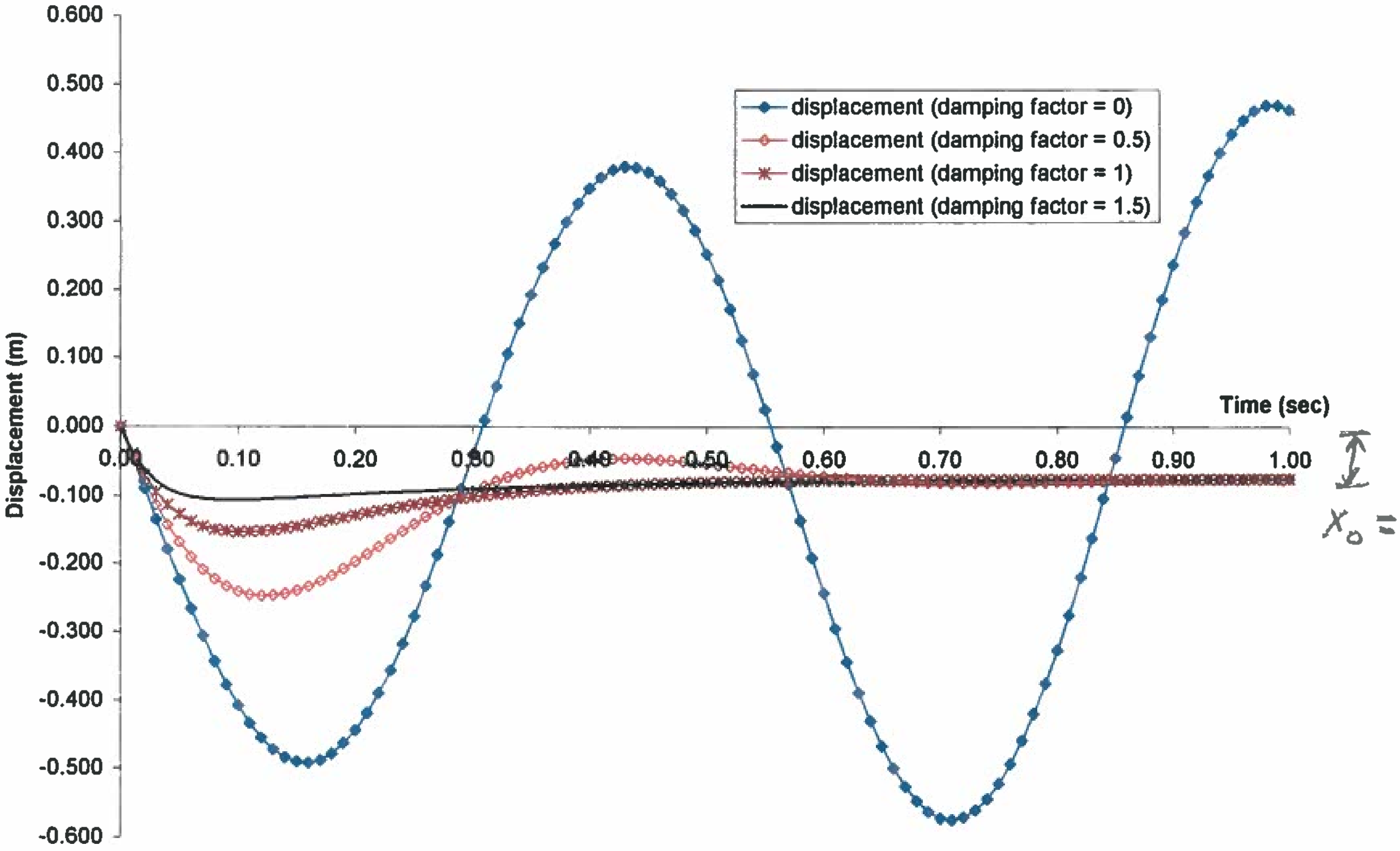
$$\left(\text{Theory} \quad \delta = \frac{W}{k} = \frac{100\,\text{kg}\,(9.8\,\text{m/s}^2)}{13{,}000\,\text{N/m}} = 0.075\,\text{m}\right)$$

c) Is the displacement graph "valid"?
The zero damping system displacement increases over time. This is an artifact of the assumption of constant acceleration through each time step. Therefore, without damping, the data becomes unreliable for large time. However, for shorter times, and with damping the results match theory well (see a & b above).

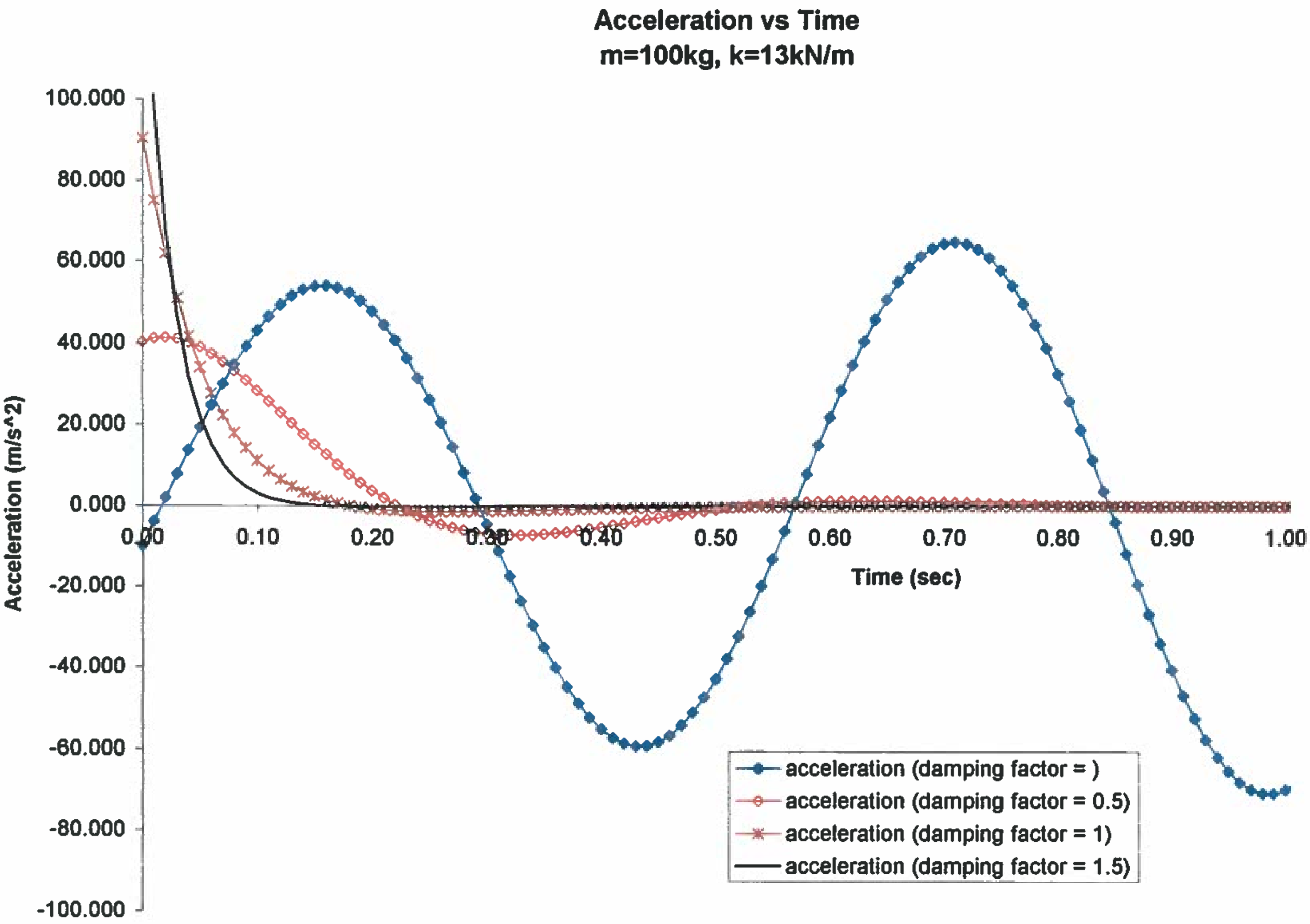
	A	B	C	D	E
1	mass, kg	time	displacement (damping factor = 0); equation 4	velocity, m/s; equation 5	acceleration (damping factor = 0); equation 6
2	100	0	0	-4.4	=-(A$4/A$2)*C2-(A$10/A$2)*D2-9.81
3	spring rate, N/m	0.01	=C2+0.5*(D2+D3)*(B3-B2)	=D2+E2*(B3-B2)	=-(A$4/A$2)*C3-(A$10/A$2)*D3-9.81
4	13000	=B3+B$3	=C3+0.5*(D3+D4)*(B4-B3)	=D3+E3*(B4-B3)	=-(A$4/A$2)*C4-(A$10/A$2)*D4-9.81
5	nat freq (eq 3)	=B4+B$3	=C4+0.5*(D4+D5)*(B5-B4)	=D4+E4*(B5-B4)	=-(A$4/A$2)*C5-(A$10/A$2)*D5-9.81
6	=SQRT(A4/A2)	=B5+B$3	=C5+0.5*(D5+D6)*(B6-B5)	=D5+E5*(B6-B5)	=-(A$4/A$2)*C6-(A$10/A$2)*D6-9.81
7	c-crit (eq 2)	=B6+B$3	=C6+0.5*(D6+D7)*(B7-B6)	=D6+E6*(B7-B6)	=-(A$4/A$2)*C7-(A$10/A$2)*D7-9.81
8	=2*A2*A6	=B7+B$3	=C7+0.5*(D7+D8)*(B8-B7)	=D7+E7*(B8-B7)	=-(A$4/A$2)*C8-(A$10/A$2)*D8-9.81
9	damping (eq 1)	=B8+B$3	=C8+0.5*(D8+D9)*(B9-B8)	=D8+E8*(B9-B8)	=-(A$4/A$2)*C9-(A$10/A$2)*D9-9.81
10	=0*A8	=B9+B$3	=C9+0.5*(D9+D10)*(B10-B9)	=D9+E9*(B10-B9)	=-(A$4/A$2)*C10-(A$10/A$2)*D10-9.81
11		=B10+B$3	=C10+0.5*(D10+D11)*(B11-B10)	=D10+E10*(B11-B10)	=-(A$4/A$2)*C11-(A$10/A$2)*D11-9.81
12		=B11+B$3	=C11+0.5*(D11+D12)*(B12-B11)	=D11+E11*(B12-B11)	=-(A$4/A$2)*C12-(A$10/A$2)*D12-9.81
13		=B12+B$3	=C12+0.5*(D12+D13)*(B13-B12)	=D12+E12*(B13-B12)	=-(A$4/A$2)*C13-(A$10/A$2)*D13-9.81
14		=B13+B$3	=C13+0.5*(D13+D14)*(B14-B13)	=D13+E13*(B14-B13)	=-(A$4/A$2)*C14-(A$10/A$2)*D14-9.81
15		=B14+B$3	=C14+0.5*(D14+D15)*(B15-B14)	=D14+E14*(B15-B14)	=-(A$4/A$2)*C15-(A$10/A$2)*D15-9.81
16		=B15+B$3	=C15+0.5*(D15+D16)*(B16-B15)	=D15+E15*(B16-B15)	=-(A$4/A$2)*C16-(A$10/A$2)*D16-9.81
17		=B16+B$3	=C16+0.5*(D16+D17)*(B17-B16)	=D16+E16*(B17-B16)	=-(A$4/A$2)*C17-(A$10/A$2)*D17-9.81
18		=B17+B$3	=C17+0.5*(D17+D18)*(B18-B17)	=D17+E17*(B18-B17)	=-(A$4/A$2)*C18-(A$10/A$2)*D18-9.81
19		=B18+B$3	=C18+0.5*(D18+D19)*(B19-B18)	=D18+E18*(B19-B18)	=-(A$4/A$2)*C19-(A$10/A$2)*D19-9.81
20		=B19+B$3	=C19+0.5*(D19+D20)*(B20-B19)	=D19+E19*(B20-B19)	=-(A$4/A$2)*C20-(A$10/A$2)*D20-9.81
21		=B20+B$3	=C20+0.5*(D20+D21)*(B21-B20)	=D20+E20*(B21-B20)	=-(A$4/A$2)*C21-(A$10/A$2)*D21-9.81
22		=B21+B$3	=C21+0.5*(D21+D22)*(B22-B21)	=D21+E21*(B22-B21)	=-(A$4/A$2)*C22-(A$10/A$2)*D22-9.81
23		=B22+B$3	=C22+0.5*(D22+D23)*(B23-B22)	=D22+E22*(B23-B22)	=-(A$4/A$2)*C23-(A$10/A$2)*D23-9.81
24		=B23+B$3	=C23+0.5*(D23+D24)*(B24-B23)	=D23+E23*(B24-B23)	=-(A$4/A$2)*C24-(A$10/A$2)*D24-9.81
25		=B24+B$3	=C24+0.5*(D24+D25)*(B25-B24)	=D24+E24*(B25-B24)	=-(A$4/A$2)*C25-(A$10/A$2)*D25-9.81
26		=B25+B$3	=C25+0.5*(D25+D26)*(B26-B25)	=D25+E25*(B26-B25)	=-(A$4/A$2)*C26-(A$10/A$2)*D26-9.81
27		=B26+B$3	=C26+0.5*(D26+D27)*(B27-B26)	=D26+E26*(B27-B26)	=-(A$4/A$2)*C27-(A$10/A$2)*D27-9.81
28		=B27+B$3	=C27+0.5*(D27+D28)*(B28-B27)	=D27+E27*(B28-B27)	=-(A$4/A$2)*C28-(A$10/A$2)*D28-9.81
29		=B28+B$3	=C28+0.5*(D28+D29)*(B29-B28)	=D28+E28*(B29-B28)	=-(A$4/A$2)*C29-(A$10/A$2)*D29-9.81
30		=B29+B$3	=C29+0.5*(D29+D30)*(B30-B29)	=D29+E29*(B30-B29)	=-(A$4/A$2)*C30-(A$10/A$2)*D30-9.81
31		=B30+B$3	=C30+0.5*(D30+D31)*(B31-B30)	=D30+E30*(B31-B30)	=-(A$4/A$2)*C31-(A$10/A$2)*D31-9.81
32		=B31+B$3	=C31+0.5*(D31+D32)*(B32-B31)	=D31+E31*(B32-B31)	=-(A$4/A$2)*C32-(A$10/A$2)*D32-9.81

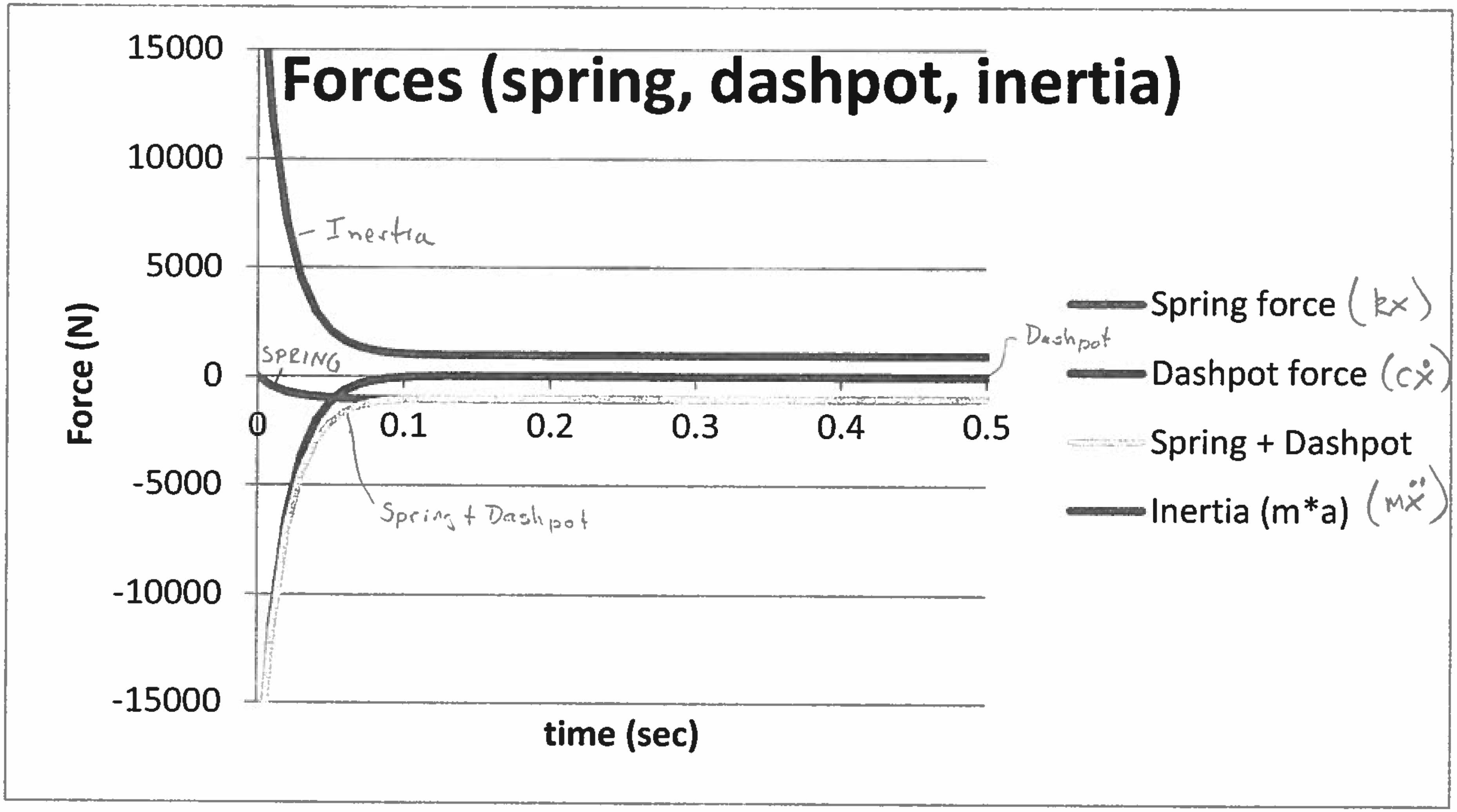
	A	B	C	D	E	F	G	H	I	J	K
1	mass, kg	time	displacement (damping factor = 0); equation 4	velocity, m/s; equation 5	acceleration (damping factor = 0); equation 6		mass, kg	time	displacement (damping factor = 0.5); equation 4	velocity, m/s; equation 5	acceleration (damping factor = 0.5); equation 6
2	100	0.00	0.000	-4.400	-9.810		100	0	0.000	-4.400	40.358
3	spring rate, N/m	0.01	-0.044	-4.498	-4.026		spring rate, N/m	0.01	-0.042	-3.996	41.214
4	13000	0.02	-0.090	-4.538	1.847		13000	0.02	-0.080	-3.584	41.442
5	nat freq (eq 3)	0.03	-0.135	-4.520	7.735		nat freq	0.03	-0.114	-3.170	41.107
6	11.40	0.04	-0.180	-4.443	13.561		11.40	0.04	-0.143	-2.759	40.274
7	c-crit (eq 2)	0.05	-0.224	-4.307	19.248		c-crit	0.05	-0.169	-2.356	39.007
8	2280.35	0.06	-0.266	-4.114	24.722		2280.35	0.06	-0.190	-1.966	37.369
9	damping (eq 1)	0.07	-0.306	-3.867	29.910		damping	0.07	-0.208	-1.592	35.421
10	0.00	0.08	-0.343	-3.568	34.743		1140.18	0.08	-0.222	-1.238	33.222
11		0.09	-0.377	-3.221	39.156			0.09	-0.233	-0.906	30.828
12		0.10	-0.407	-2.829	43.088			0.1	-0.241	-0.598	28.290
13		0.11	-0.433	-2.398	46.486			0.11	-0.245	-0.315	25.657
14		0.12	-0.455	-1.933	49.302			0.12	-0.247	-0.058	22.974
15		0.13	-0.472	-1.440	51.494			0.13	-0.247	0.172	20.281
16		0.14	-0.483	-0.925	53.032			0.14	-0.244	0.374	17.614
17		0.15	-0.490	-0.395	53.891			0.15	-0.239	0.551	15.004
18		0.16	-0.491	0.144	54.054			0.16	-0.233	0.701	12.480
19		0.17	-0.487	0.684	53.516			0.17	-0.225	0.825	10.065
20		0.18	-0.478	1.219	52.278			0.18	-0.217	0.926	7.779
21		0.19	-0.463	1.742	50.353			0.19	-0.207	1.004	5.638
22		0.20	-0.443	2.246	47.761			0.2	-0.197	1.060	3.653
23		0.21	-0.418	2.723	44.531			0.21	-0.186	1.097	1.835
24		0.22	-0.389	3.169	40.701			0.22	-0.175	1.115	0.188
25		0.23	-0.355	3.576	36.317			0.23	-0.164	1.117	-1.284
26		0.24	-0.317	3.939	31.433			0.24	-0.152	1.104	-2.582
27		0.25	-0.276	4.253	26.108			0.25	-0.142	1.078	-3.706
28		0.26	-0.232	4.514	20.409			0.26	-0.131	1.041	-4.661
29		0.27	-0.186	4.718	14.408			0.27	-0.121	0.995	-5.453
30		0.28	-0.138	4.862	8.180			0.28	-0.111	0.940	-6.089
31		0.29	-0.089	4.944	1.806			0.29	-0.102	0.879	-6.577
32		0.30	-0.040	4.962	-4.634			0.3	-0.094	0.813	-6.928

Displacement vs Time
m=100kg, k=13kN/m



$X_o = \frac{W}{k}$


Acceleration vs Time
m=100kg, k=13kN/m
Acceleration (m/s^2)
100.000
80.000
60.000
40.000
20.000
0.000
-20.000
-40.000
-60.000
-80.000
-100.000
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
Time (sec)
acceleration (damping factor =)
acceleration (damping factor = 0.5)
acceleration (damping factor = 1)
acceleration (damping factor = 1.5)



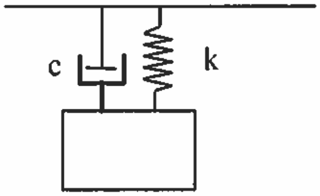
$m\dot{x} = -kx - c\dot{x}$ for all time

Design Decision Document
February 28, 2014
K. Lulay

Problem statement: determine spring constant and damping constant to satisfy the following criteria:

#	Criteria	Priority	Description
1	Total mass (given)	Essential	Total vehicle mass is 400kg (100kg per wheel)
2	Max total displacement limits	Essential	Max allowable displacement is 300mm from the "unloaded" height (spring is fully extended) when dropped from 2 meter height.
3	Ride feel	Important	No oscillating motion (the system should be critically or over damped)
4	Minimize impact force	Important	Deceleration forces should be minimized.

Assume:
1 degree of freedom system
Negligible mass of spring and dashpot, spring is linear, dashpot is viscous
The system can be modeled as:



General Solution method:
An Excel spreadsheet (created for an assignment) has been modified to help select spring constant (k) and damping constant (c). It is included with this document. The spread sheet using "time marching" to determine displacement, velocity, and accelerations of a damped system. The following kinematic equations are used:

$$x_{i+1} = x_i + 0.5*(v_i + v_{i+1}) * \Delta t \; ; \quad v_{i+1} = v_i + a_i * \Delta t \; ; \quad a_i = - (k/m)*x_i - (c/m)*v_i - a_g$$

The initial conditions of the vehicle at the moment of impact are as follows:

$x_0 = 0$ meters
$v_0 = - (2*a_g*h)^{1/2}$ where $a_g = 9.8 m/s^2$ and h is the height from which it was dropped (h=2m).
$v_0 = -6.26$ m/s
a_0 is calculated as: $a_0 = - (k/m)*x_0 - (c/m)*v_0 - a_g$

Criteria Evaluation

The first two criteria are considered “essential;” therefore, they must be satisfied. Criteria 3 and 4 are somewhat in opposition to each other. In order to have minimal oscillation, the system should be critically damped or over damped. To minimize deceleration forces, damping should be minimal (most of the initial impact force is due to high velocity and hence high force for the dashpot).

Design decision: since the deceleration forces exist for an extremely short duration at the moment of impact (and the driver’s seat, etc. will absorb some impact force), it was decided to give criterion 3 (ride feel) greater priority than criterion 4 (deceleration force). Therefore, the system will be designed to be critically damped ($\zeta = 1$).

Alternative solution: criteria 3 and 4 are both considered “important” and satisfying one will occur at the detriment of the other. The above recommendation (selecting $c=c_c$) was done to satisfy criterion 3 more than 4. To reduce impact force, $c<c_c$ could be selected, but there will be some oscillatory motion after impact (meeting criterion 3 less well). As an example, $\zeta = .75$ ($c=0.75c_c$) was investigated.

Calculations:

Determine critical damping:

$\zeta = c/c_c = 1$ Therefore, $c = c_c$; $c_c = 2m\,\omega_n$; $\omega_n = (k/m)^{1/2}$ $m = 100$ kg

In the attached spreadsheet, the value for spring constant, k (located in cell A4) is an input parameter. Once k has been entered in the spreadsheet, the natural frequency is calculated and from that, the critical damping constant is determined (c_c). The damping constant (c) is set equal to the critical damping constant. From those parameters, the displacement, velocity and acceleration of the mass (the vehicle) is calculated. The spring constant was varied until the maximum displacement was less than 300mm (criterion 2).

Results and Conclusion

Figure 1 shows that with k = 7100 N/m and critically damped (c= 1685kg/sec), that the maximum displacement is 0.3 meters. This satisfies all of the criteria except possibly criterion 4 (deceleration). It should be noted, that the Excel spreadsheet shows that if $\zeta = 0.6$, the deceleration is decreased from about 10g’s to about 8g’s, but with oscillation. Since the driver of the vehicle will be sitting in a cushioned seat, they will not personally experience this magnitude of deceleration.

Table 1 – comparison of deceleration and oscillation of two alternatives

Damping ratio, ζ	Damping factor, c	Spring rate, k	Maximum Impact Deceleration	Oscillation
1.0	1658kg/sec	7100 N/m	10g	None
0.6	1368kg/sec	13,000 N/m	7.8g	Some, dies out in <1 sec

Figure 1 - From Excel, displacement verses time for critically damped system ($\zeta = 1$).

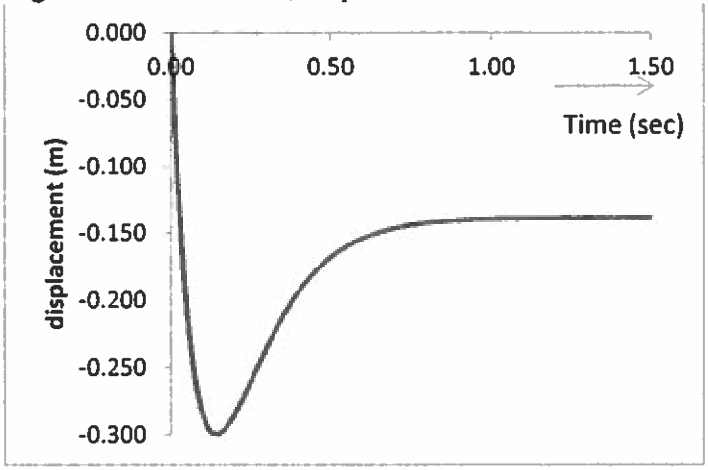


Figure 2 - From Excel, acceleration verses time for critically damped system ($\zeta = 1$).

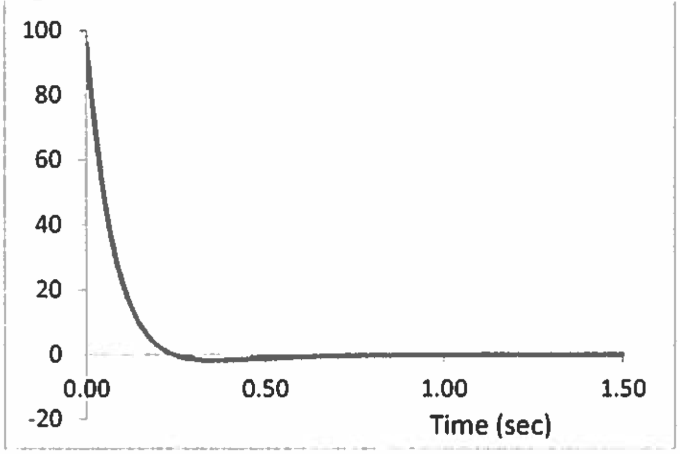
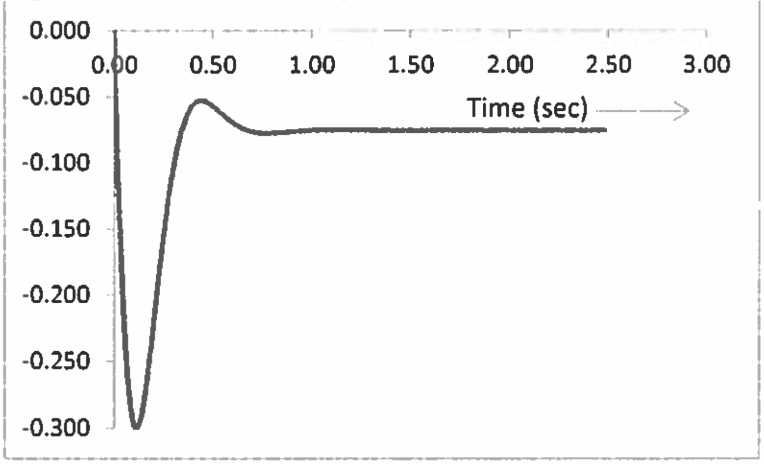


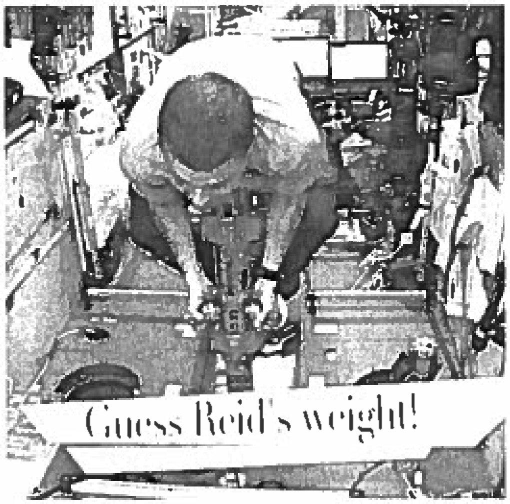
Figure 3 - From Excel, displacement verses time for underdamped system ($\zeta = 0.6$).



How do astronauts weight themselves in space? (actually, how do they determine their mass)

Answer – they become the "mass" of a mass spring system, the rate of oscillation (ω_n) is measured, spring constant (k) is known:

Mass = ω_n^2 k



https://www.airspacemag.com/daily-planet/how-do-astronauts-weigh-themselves-space-180953884/