Classicality, the Ensemble Interpretation, and Decoherence: Resolving the Hyperion Dispute

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Abstract We analyze seemingly contradictory claims in the literature about the role played by decoherence in ensuring classical behavior for the chaotically tumbling satellite Hyperion. We show that the controversy is resolved once the very different assumptions underlying these claims are recognized. In doing so, we emphasize the distinct notions of the problem of classicality in the ensemble interpretation of quantum mechanics and in decoherence-based approaches that are aimed at addressing the measurement problem.

1 Introduction

The question of how the classical world of our experience can be explained from within quantum mechanics continues to fuel a lively debate. At the heart of this problem of the quantum-to-classical transition is the superposition principle, which is formally grounded in the linearity of the Hilbert space. Since quantum states are represented by vectors in a Hilbert space, we may form linear combinations of these vectors. The superposition principle then states that such linear combinations correspond again to a new quantum state. Superpositions cannot be interpreted as classical ensembles of their components states. Instead, the phenomenon of interference shows that all components in the superposition must be understood as, in some sense, simultaneously present.

A particularly counterintuitive instance are systems described by a superposition of macroscopically distinguishable positions. One way such superpositions may dynamically arise is via a von Neumann measurement [12] of a microscopic system

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prepared in a certain superposition state. Unitary evolution applied to the composite system–apparatus combination may then lead to an entangled superposition state whose components refer to the pointer of the apparatus being located at distinct positions on the dial. Another, and rather different, example is the coherent spreading of initially localized wave packets. Suppose a free particle is at time $t = 0$ described by a wave packet of the form

$$\psi(x, t = 0) = \left(\frac{1}{\sqrt{\pi}\sigma}\right)^{1/2} \exp\left[-\frac{x^2}{2\sigma^2}\right], \tag{1}$$

then the position probability density $|\psi(x, t)|^2$ at $t > 0$ is given by

$$|\psi(x, t)|^2 = \frac{1}{\sqrt{\pi}\sigma(t)} \exp\left[-\frac{x^2}{\sigma^2(t)}\right], \tag{2}$$

where the width $\sigma(t)$ of the wave packet grows as

$$\sigma(t) = \sigma\sqrt{1 + \hbar^2 t^2/(m^2\sigma^4)}, \tag{3}$$

with m denoting the mass of the particle. For microscopic particles the spreading of the wave packet occurs on extremely short timescales. For example, for an electron ($m \approx 10^{-30}$ kg) and $\sigma(t = 0) = 1$ Å, we obtain $\sigma(t = 1 \text{ s}) = 10^{16}\sigma = 10^6$ m. This coherent spreading was the core obstacle encountered by Schrödinger when he initially tried—inspired by ideas laid out in de Broglie's Ph.D. thesis of 1924 [3]—to directly identify narrow wave packets with particles [11]. The concept of particles is virtually synonymous with localizability, a feature evidently not described by a wave packet that may rapidly and coherently disperse over macroscopic distances.

On the other hand, it seemed possible to uphold a peaceful correspondence between wave packets and well-localized objects in at least two situations. The first example, studied by Schrödinger in 1926 [11], is the special case of coherent states for the quantum harmonic oscillator, where the wave packet remains narrow at all times $t > 0$ and where its peak oscillates back and forth similar to a classical point mass. The second example, which will be of most interest for the purpose of this paper, is represented by macroscopic systems for which the spreading described by (3) is *typically* very slow.

However, the rate of spreading may be drastically enhanced in chaotic systems irrespective of their size, and thus the problem of coherent spreading of the wave packet over large regions in space may reappear even at the level of macroscopic systems. This situation was studied by Zurek [17] using the example of Hyperion, a chaotically tumbling moon of Saturn. Chaotic systems exhibit an exponential sensitivity to the initial phase-space parameters. Wave packets may diverge or become squeezed in the position or momentum direction at an exponential rate given by the Lyapunov exponent λ. Suppose the initial spread in momentum is Δp_0 and exponential squeezing in this direction occurs, $\Delta p(t) = \Delta p_0 e^{-\lambda t}$. In the quantum setting, it follows from the uncertainty principle that the initial spread in position must be at least on the order of $\hbar/\Delta p_0$, and the required conservation of the phase-space volume and unitary dynamics then imply that the wave packet undergoes coherent spreading

according to $\Delta x(t) \sim (\hbar/\Delta p_0)e^{\lambda t}$. Zurek estimated that within ~ 20 years the quantum state of Hyperion would evolve into a highly nonlocal coherent superposition of macroscopically distinguishable orientations of the satellite's major axes, thus setting up a measurement-free version of Schrödinger's cat. In the absence of decoherence, it would always be possible to choose an appropriate observable that would confirm the existence of this superposition, either through a direct projective measurement onto the superposition state itself or by means of an interference measurement in the component basis.

Using the standard model for quantum Brownian motion, Zurek then showed that decoherence rapidly suppresses such coherent spreading, locally degrading the superposition into an apparent (improper) ensemble of narrow position-space wave packets. The superposition initially confined to the satellite is rapidly dynamically dislocalized into the composite satellite–environment system via environmental entanglement. This implies that there exists no local measurement that could be performed on the satellite that would in practice reveal the presence of the superposition. The different orientations of Hyperion are thus dynamically environment-superselected [14–16] as the robust quasiclassical states between which coherence becomes locally suppressed, thereby ensuring effective classicality for the satellite. Of course, this conclusion is rather insensitive to the particular decoherence model: Any environmental monitoring of the position and orientation of the satellite, such as that mediated by the ubiquitous scattering of environmental particles [5], will bring about such decoherence.

Although Zurek's conclusions are intuitively reasonable and in agreement with general insights gained from studies of environmental entanglement and decoherence [6, 10, 18], they were subsequently criticized by Wiebe and Ballentine [13]. These authors revisited the problem of the quantum–classical correspondence for Hyperion by presenting detailed numerical studies for an explicit model of the satellite. Based on their results, they concluded that even in the absence of environmental interactions there is no problem with the quantum–classical correspondence for Hyperion. *A fortiriori*, this conclusion led the authors to claim that "decoherence is not essential to explain the classical behavior of macroscopic bodies" [13, p. 022109-1], in contrast with Zurek's original argument [17] and the commonly accepted wisdom about the role of decoherence in the problem of the quantum-to-classical transition [5, 6, 10, 18].

In the following we will not only show that the studies of Zurek, and of Wiebe and Ballentine, address different problems, but also demonstrate that the conclusions drawn from the results of these studies are based on distinct sets of assumptions. By presuming a strictly epistemic ensemble (statistical) interpretation of quantum mechanics, Wiebe and Ballentine take the view from the outset that there is no measurement problem, while this is precisely the problem addressed by Zurek. In this way, the calculations of Wiebe and Ballentine do not challenge the conclusions of Zurek's analysis regarding the role of decoherence, contrary to the claims put forward in [13]. In bringing out the fundamental differences between the two studies, we thus show that the controversy is rooted in an instance of "comparing apples and oranges." Finally, while this article is motivated by the specific example of Hyperion and the corresponding investigations of Zurek, and of Wiebe and Ballentine, it has a much

broader scope: It sheds light on tacit assumptions in the ensemble interpretation that effectively amount to presuming characteristic features of classicality usually derived from decoherence.

2 What Classicality? An Analysis of Different Paradigms

In their study [13], Wiebe and Ballentine define proper quantum–classical correspondence for Hyperion via the condition

$$\Delta_{\text{qm–cl}} \equiv \sum_m \left| P_{\text{cl}}(m) - P_{\text{qm}}(m) \right| \ll 1, \tag{4}$$

where $P_{\text{cl}}(m)$ and $P_{\text{qm}}(m)$ are the classical and quantum probability distributions, respectively, for Hyperion's angular momentum along the z axis, which is the space-fixed axis perpendicular to the orbital plane of the satellite. In the classical case, the probability distribution is given by the (classically) incompletely defined initial state evolved under the appropriate equations of motion. In the quantum setting, Hyperion is described by a pure-state superposition of eigenstates $|m\rangle$ of the angular-momentum operator $\hat{J}_z$,

$$|\psi\rangle = \sum_m c_m |m\rangle, \tag{5}$$

and $P_{\text{qm}}(m)$ is the corresponding probability for finding the value m in a measurement of the operator-observable $\hat{J}_z$,

$$P_{\text{qm}}(m) = |\langle m|\psi\rangle|^2 = |c_m|^2. \tag{6}$$

If the inequality (4) is fulfilled, i.e., if the classical and quantum probability distributions $P_{\text{cl}}(m)$ and $P_{\text{qm}}(m)$ agree reasonably well, then Wiebe and Ballentine take this result as saying that there is no quantum–classical problem for Hyperion, i.e., that Hyperion behaves classically. Through numerical studies of an explicit model, the authors find that the inequality (4) indeed holds to a sufficient degree.

However, agreement of classical and quantum probability distributions for a single observable is not a sufficient criterion for proper quantum–classical correspondence. In the classical setting, both the value of angular momentum along the z axis and the position (orientation) of Hyperion are simultaneously well-defined; any probabilistic aspect is simply due to our (practically motivated, but not fundamentally required) ignorance about the initial state. In the quantum setting, on the other hand, the position operator and $\hat{J}_z$ do not commute. This allows for two possible scenarios.

In the first scenario, despite the noncommutativity of these two operators, we may suppose that the eigenstates of $\hat{J}_z$ are also *approximate* position eigenstates for Hyperion. In this case, a measurement of $\hat{J}_z$ would be unable to distinguish the coherent superposition of macroscopically distinct positions of Hyperion from the corresponding classical mixture of positions. Therefore $\hat{J}_z$ would simply be the wrong choice of observable for detecting this nonclassical superposition. However, there always exists a projective observable that would optimally verify the existence of the superposition,

while *practical* difficulties in measuring such an observable can in turn be *explained* by environmental interactions, namely, by environment-induced superselection [14–16].

In the second scenario, we shall conversely assume that a measurement of $\hat{J}_z$ is indeed sensitive (in the above sense) to the superposition of macroscopically distinct positions of Hyperion. If we measure $\hat{J}_z$ and obtain a certain outcome, we may thus conclude that we have measured such a superposition—i.e., that we have verified the presence of coherence between different positions. But Wiebe and Ballentine *a priori* do not regard this thus-confirmed existence of the superposition as a nonclassical state of affairs. Instead, Wiebe and Ballentine explicitly state that the only role of quantum states is to describe "the probabilities of the various possible phenomena" [13, p. 022109-1]. Thus they presume the ensemble, or statistical, interpretation of quantum mechanics [2].

The key assumption of this type of interpretation is to consider the quantum state as only representing the statistical properties of an ensemble of similarly prepared systems. The ensemble interpretation thus implies that the entire formal body of quantum mechanics (for example, a probability amplitude) has no direct physical meaning, in the sense of having no direct correspondence to the entities of the physical world (see also the comment by Leggett [7]). This interpretation effectively points toward the need for some hidden-variables theory to fully specify the state of individual systems, but it does not actually specify what this "complete theory" would be.

A signature of the ensemble view is its interpretation of superpositions. In his review paper [2], Ballentine used the example of the momentum eigenstate of a single electron, which yields a plane wave in configuration space (i.e., a superposition of all spatial positions). In the ensemble interpretation, this quantum state is viewed as representing an (conceptual, infinitely large) ensemble of single electrons with the same momentum, but evenly spread out over all positions. In other words, superpositions are interpreted as representing the results of an ensemble of yet-to-be-performed measurements, while the occurrence of the individual measurement outcomes is not dynamically explained (as in explicit hidden-variables or physical-collapse theories) or represented (as in the collapse postulate) within the quantum-mechanical formalism.

Thus, in this interpretation, the superposition considered by Zurek is simply viewed as describing the probability distribution for finding the satellite at a particular position and orientation upon measurement. The measurement problem and the problem of the quantum-to-classical transition, however, are precisely concerned with explaining the workings of this instrumentalist algorithm in terms of a physical theory. Furthermore, and this is the important point, if Hyperion is considered as a closed system (i.e., if decoherence is absent) there always exists some observable for Hyperion that would confirm the presence of the coherent superposition of macroscopically distinguishable orientations which Zurek has shown to result from the combination of classical chaotic dynamics and quantum unitary evolution.

The key question thus is: Why is it so prohibitively difficult to confirm in practice the presence of coherence in such cat-like superpositions? This question is not at all addressed by Wiebe and Ballentine. In the case of Schrödinger's cat, the projec-

tive observable directly verifying the presence of the superposition would be prover-bially nonclassical, namely, of the form $\hat{O}_{\text{cat}} \propto (|\text{alive}\rangle\langle\text{dead}| \pm |\text{dead}\rangle\langle\text{alive}|)$. This observable would therefore have no counterpart in the classical setting, which would exclude any possibility of comparing the corresponding classical and quantum prob-ability distributions. Wiebe and Ballentine's particular choice of $\hat{J}_z$ as the observable of interest allows them to avoid such an obstacle. Although $\hat{J}_z$ corresponds (assum-ing the second scenario above) to the measurement of a nonclassical superposition of positions, it still also represents the measurement of a classical quantity (namely, angular momentum) taking well-defined values in the classical model of Hyperion. It is only this peculiar feature of the chosen observable, together with the adoption of an ensemble interpretation of quantum mechanics, that permits Wiebe and Ballentine's analysis to proceed.

It is easy to anticipate that once the problem of the quantum-to-classical transition is reduced to the comparison of quantum and classical probability distributions for a single observable (which, as discussed above, is deliberately chosen such as to make sense also in the classical setting), environmental interactions and the resulting deco-herence processes will play but a minor role when compared to a framework in which measurability and the existence of quasiclassical observables is to be *explained* from within quantum mechanics. Indeed, when Wiebe and Ballentine investigate the influ-ence of classical noise on the probability distributions for $\hat{J}_z$, aiming to simulate en-vironmental interactions and decoherence-like processes (see, however, [1, 6, 8, 10, 18] on important limitations on simulating decoherence by noise), they (unsurpris-ingly) find only a comparably small degree of smoothing of these distributions. From this finding they conclude that environmental interactions—and thus what the authors label "decoherence"—are insignificant to the problem of classicality and present this argument as a challenge to Zurek's claims.

But this is a fallacious conclusion. Decoherence allows one to treat the super-position locally as an apparent ensemble of quasiclassical configurations, here pre-sumably coherent states well localized in both position and angular momentum [9]. This leads to the superselection of the preferred quasiclassical observables that were simply picked out by Wiebe and Ballentine. (Other practical obstacles, such as the suitable preparation of measurement apparatuses and the design of appropriate cou-plings between system and apparatus, will naturally also play a role.) The similarity between classical and quantum probability distributions for a particular observable, as shown by Wiebe and Ballentine, has simply no bearing on Zurek's argument.

3 Conclusions

Our discussion demonstrates the importance of a careful distinction between mathe-matical calculations and their proper (physical) interpretation. The focus of Zurek's argument is to show how coherent spreading of the wave packet over macroscopic distances may become relevant also for macroscopic objects, including celestial bod-ies long regarded as prime examples of classical systems, and how decoherence leads to a local (improper) ensemble of narrow wave packets describing quasiclassical trajectories in the usual sense of the emergent-classicality program of decoherence.

Wiebe and Ballentine's analysis, on the other hand, is solely concerned with a comparison of distributions of measurement outcomes whose probabilistic aspect has two fundamentally different sources (classical "ignorance" vs. quantum "randomness"). What these authors have demonstrated is that, *if* we measure the operator-observable $\hat{J}_z$ on Hyperion and *if* we assume the usual measurement axioms of quantum mechanics, then the resulting distribution of measurement outcomes will be reasonably close to the classical distribution. In other words, they have successfully shown that decoherence does not play a crucial role in restoring proper quantum–classical correspondence in the Ehrenfest [4] or quantum-Liouville sense for a particular observable that they deem to be most natural.

But as we have demonstrated in this paper, contrary to the authors' claims these results do not challenge Zurek's conclusion regarding the importance of decoherence for the problem of the quantum-to-classical transition. Zurek's point is that there always exist *some* observable that could confirm the presence of the nonclassical superposition state of Hyperion. Decoherence, then, explains the practical difficulty in measuring such observables and allows us, for all practical purposes, to describe measurement locally in effectively classical terms and thus to ignore the Schrödinger-cat problem *in practice*. The empirical adequacy of Wiebe and Ballentine's *a priori* belief in the absence of any such problem may thus be regarded as a *consequence* of the ubiquitous action of decoherence. We therefore suggest that these authors' conclusion that "it is not correct to assert that environmental decoherence is the root cause of the appearance of the classical world" [13, p. 022109-13] is a *non sequitur*. In this way, we hope that our analysis has quite peacefully resolved the controversy.

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