Mth 201
Final Exam
Section A

Name: ______________________ Date: ____________

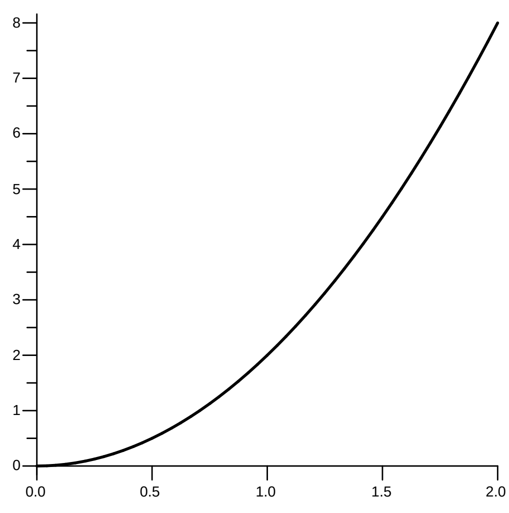
12 Problems. 200 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 12 problems and 10 printed pages. Initial____.

Print the names of the people sitting next to you.____.

Short Answer - minimum explanation and calculations necessary (12.5 points each).

1. Estimate the area under the graph illustrated below in the interval $[0, 1.5]$ using a right hand Riemann sum with 3 subdivisions. You



2. If $f(x)$ and $g(x)$ are differentiable functions which are increasing for all x, and $f(x) > 0$ and $g(x) > 0$ for all x, explain why the product $f(x) \cdot g(x)$ is an increasing function for all x.

3. Find the the equation of the tangent line to $y = x \ln(x)$ at $(1, 0)$.

4. Evaluate the limit

$$\lim_{x \to 0} x \ln(x).$$

5. Find the x-coordinate(s) of the **global minimum** of

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6xe^x$$

on the interval $[-1, 4]$

6. If $g(1) = 2$, $f(1) = -1$, $g'(1) = 0$ and $f'(1) = 7$, what is $(f/g)'(1)$?

7. Give an example of a function which is continuous everywhere but not differentiable everywhere and state at what x-value(s) is it not differentiable.

8. If $F'(x) = 2x^2$ and $F(2) = 1$, find $F(3)$.

Long Answer - show work and provide explanations, an answer without supporting work is not worth much (25 points each).

1. Find the **derivatives** of each of the following functions showing **all** your work or providing explanations (you will receive 0 credit if you give an answer without work or explanations).

   (a)
   $$f(x) = 1 + x^2 e^x$$

   (b)
   $$s(t) = \frac{(\tan(3t^2)}{t}$$

   (c)
   $$h(x) = 4^x$$

(d)

$$k(x) = \int_x^1 \sin^{-1}(t^2)dt$$

(e)

$$l(x) = x^{x^2}$$

2. Answer the following showing **all** work (you will receive 0 credit for an answer with no work).

   (a) Evaluate the indefinite integral

$$\int x\sqrt{x^2 - 1}dx.$$

   (b) Evaluate the definite integral

$$\int_1^1 \frac{e^x \ln(x+1)e^{x^{x^2}}}{2x^2 + 2x + 1}dx.$$

   (c) Find the antiderivative $F(x)$ of

$$f(x) = \frac{1}{x} + 2x$$

   with $F(1) = 0$.

(d) Evaluate the definite integral

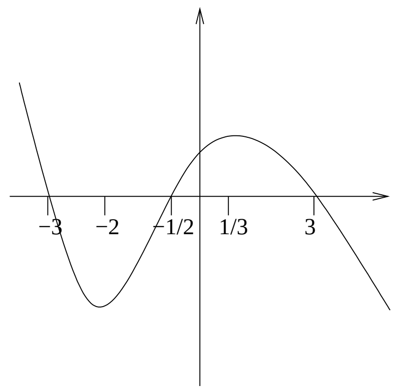
$$\int_{-4}^{-2} \frac{x^2+1}{2x} dx.$$

(e) Find the most general antiderivative $F(x)$ of

$$f(x) = \frac{e^x}{1+e^x}.$$

3. An 8 foot long ladder is leaning against a wall. The top of the ladder is sliding down the wall at the rate of 2 feet per second. How fast is the bottom of the ladder moving along the ground at the point in time when the bottom of the ladder is 4 feet from the wall.

4. Suppose that f illustrated below is the **derivative** of $F(x)$. Answer the following questions about $F(x)$.



(a) Give the intervals on which $F(x)$ is increasing.

(b) Give the intervals on which $F(x)$ is decreasing.

(c) Determine the x-values of any minimum or maximum values.

(d) Give the intervals on which $F(x)$ is concave up.

(e) Give the intervals on which $F(x)$ is concave down.

(f) Determine the x-values of any inflection points.

(g) Using your results, sketch a graph of $F(x)$ if you know that $F(0) = 0$.