

Mth 201  
Exam 3  
Section A

Name: \_\_\_\_\_

Date: \_\_\_\_\_

9 Problems. 100 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.
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I have verified that this exam contains 12 problems and 7 printed pages.  
Initial\_\_\_\_\_.

Print the names of the people sitting next to you.\_\_\_\_\_.

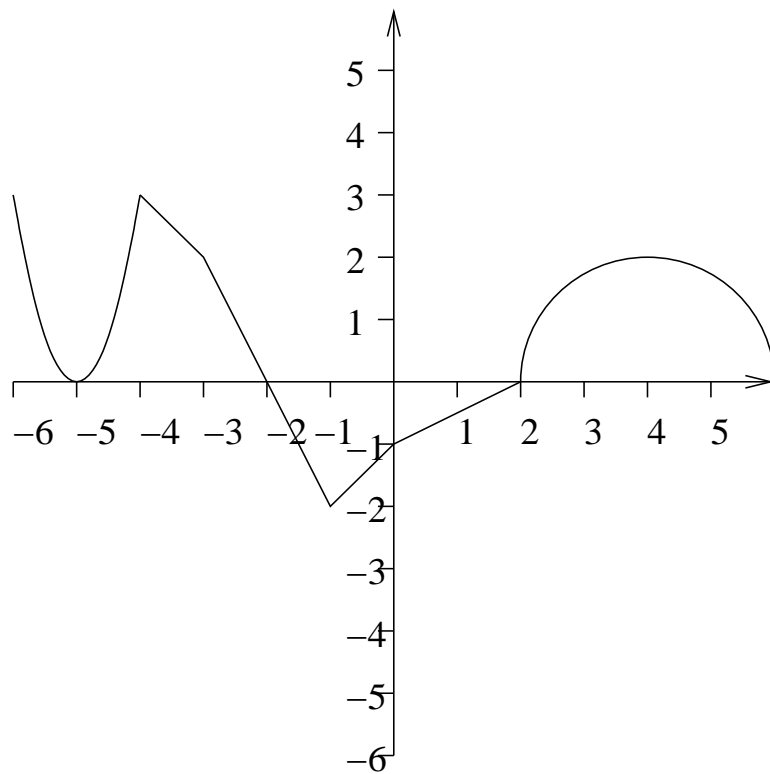
**Short Answer - minimum explanation and calculations necessary (6 points each).**

1. Since the function  $f(x) = x^2 - 4x + 1$  is continuous and differentiable on  $[0, 4]$  and  $f(0) = f(4)$ , the hypothesis of Rolle's theorem are satisfied. Find a number  $c$  in  $[0, 4]$  which satisfies the conclusion of Rolle's Theorem.
  
  
  
  
  
  
  
  
  
  
2. Find the global minimum and maximum of  $f(x) = x + \frac{1}{x}$  on the interval  $(0, 2)$ . (If there is no global maximum or minimum, **say so!**).

3. Answer either “a family of functions”, “a function”, or “a number” to the following:

- (a) An antiderivative is .....
- (b) A definite integral is .....
- (c) An indefinite integral is .....

4. If  $F(x) = \int_0^x f(t)dt$  where  $f(t)$  is illustrated below, find the **exact value** of  $F(4)$ .



5. A direction field is illustrated for a function below. Use it to sketch the antiderivative  $F$  which satisfies  $F(0) = -1$ .

6. Find the indefinite integral

$$\int \left(\frac{1}{x} + x\right) dx.$$

**Long Answer - show work and provide explanations, an answer without supporting work is not worth much (20 points each).**

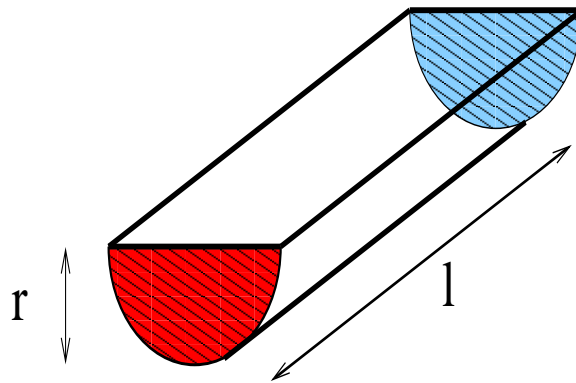
1. Suppose that

$$f(x) = \frac{x^2 - 1}{x}.$$

- (a) Write down the intervals on which  $f(x)$  is increasing and decreasing.
  
  
  
  
  
  
  
  
  
  
- (b) Find all critical points and determine which are local minimums, maximums or neither.
  
  
  
  
  
  
  
  
  
  
- (c) Write down the intervals on which  $f(x)$  is concave up and concave down.
  
  
  
  
  
  
  
  
  
  
- (d) Find all inflection points.
  
  
  
  
  
  
  
  
  
  
- (e) On the page overleaf, sketch a graph of  $f(x)$  labeling all interesting points.



2. You are building a canoe which is the shape of an open half cylinder with two **flat** half **circles** at the ends as illustrated below. You can only afford to buy  $10m^2$  of wood. Determine the dimensions (the values for  $l$  and  $r$ ) of the canoe which will maximize the volume inside the canoe. (Hint: The volume of a whole cylinder of height  $h$  and radius  $r$  is  $V = \pi r^2 h$ .)



3. A function  $f(x)$  is said to dominate a function  $g(x)$  as  $x \rightarrow \infty$  if the limit  $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$ . Determine with reasons which functions dominate which for each of the functions below (**Hint:** if you show that  $f(x)$  dominates  $g(x)$  and  $g(x)$  dominates  $h(x)$ , then  $f(x)$  must also dominate  $h(x)$ , so you shouldn't have to compare all 5 functions).

(a)  $f_1(x) = \ln(x)$ .

(b)  $f_2(x) = e^x$ .

(c)  $f_3(x) = x^2$ .

(d)  $f_4(x) = x^{\frac{1}{2}}$ .

(e)  $f_5(x) = \sin(x)$ .