Mth 201
Final Exam
Section A

Name: _____ Date: _____

12 Problems. 200 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 12 problems and 7 printed pages. Initial_____.

Print the names of the people sitting next to you._____.

Short Answer - minimum explanation and calculations necessary (12 points each).

1. Explain why a third degree polynomial must have a zero.

2. If $f(x)$ and $g(x)$ are increasing functions, explain why $f(g(x))$ is an increasing function. (Hint: In terms of derivatives, what would it mean for $f(g(x))$ to be increasing?)

3. Find the linear approximation of $y = x + \frac{1}{x}$ at $x = 2$.

4. Find the derivative

$$\frac{d}{dx}\int_1^3 \frac{\sin(t)}{t}dt.$$

5. Find the derivative

$$\frac{d}{dx}\int_1^x \frac{\sin(t)}{t}dt.$$

6. Evaluate the limit

$$\lim_{h \to 0} \frac{e^h - 1}{h}.$$

7. What is the **global maximum value** of $f(x) = e^x - x$ on the interval $[-3, 5]$?

8. If you drop a stone off the top of a building and it takes 6 seconds to hit the ground, how tall is the building?

Long Answer - show work and provide explanations, an answer without supporting work is not worth much (26 points each).

1. Find the derivatives of each of the following functions showing **all** your work (you will receive 0 credit if you give an answer without work).

   (a)
   $$f(x) = xe^x$$

   (b)
   $$g(z) = \frac{3z^2}{5z^2 - 7z}$$

   (c)
   $$h(x) = \ln(\ln(x))$$

   (d)
   $$k(x) = x^{x^2}$$

2. Answer the following showing **all** work (you will receive 0 credit for an answer with no work).

   (a) Evaluate the indefinite integral

   $$\int \frac{e^{\sqrt{y}}}{\sqrt{y}} dy.$$

   (b) Evaluate the definite integral

   $$\int_{-1}^{e-2} \frac{1}{t+2} dt.$$

   (c) Find the antiderivative $F(x)$ of

   $$f(x) = x^4 - x^2$$

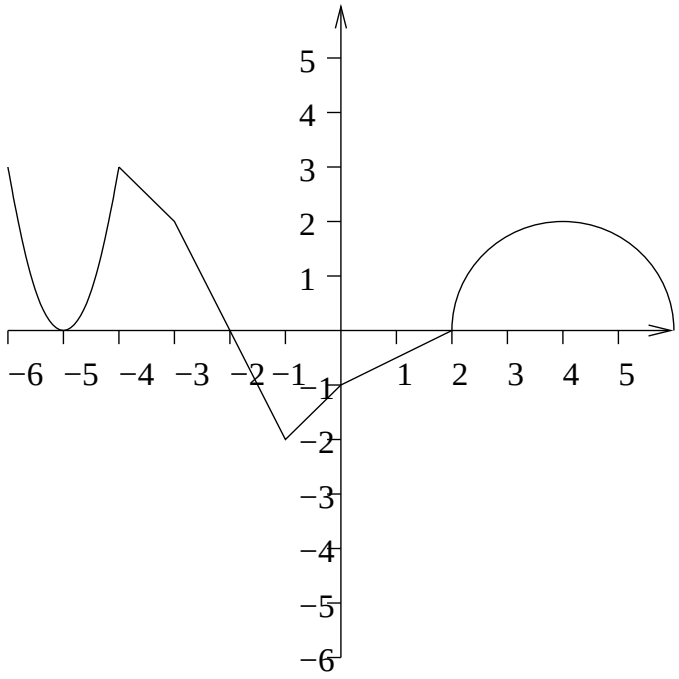
   with $F(1) = 0$.

   (d) Evaluate the indefinite integral

   $$\int \cos^2(x) + \sin^2(x) dx.$$

3. A cylindrical can is to hold $20\pi\ m^3$ of liquid. The material for the top and bottom costs \$10 per m^2 and material for the side costs \$8 per m^2. Find the radius r and height h of the most economical can.

4. Suppose that f illustrated below is the **derivative** of $F(x)$. Answer the following questions about $F(x)$.



(a) Find the intervals on which $F(x)$ is increasing and decreasing.

(b) Find all critical points of $F(x)$ and determine which are minimums, maximums or neither.

(c) If $G(x) = \int_1^x f(t)dt$ for the same f as illustrated above, fill in the following table for the values of $F(x)$.

x	-2	0	1	2	4	6
$G(x)$						