Mth 201
Hour Exam 1

Name: _____ Date: _____

10 Problems. 100 Points. Follow directions carefully. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

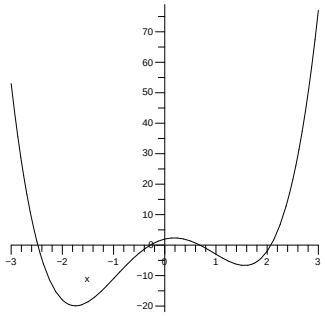
I have verified that this exam contains 10 problems and 9 printed pages. Initial____.

Print the name of the people sitting either side of you :- _____

Short Answer (7 points each) - Minimal explanation and calculations necessary though where appropriate, answers should be exact.

1. Determine the equation for the exponential function $f(x) = Ca^x$ which passes through the points $(2, 6)$ and $(3, 3)$.

2. Could the following be a complete graph of a polynomial of degree 5? You must give a reason for your answer.



3. Determine the inverse function of

$$f(x) = \frac{3x + 2}{2x - 1}.$$

4. Circle the invalid mathematical step in the following argument which claims to prove that $\ln(2) = 0$.

   (a) We know that $2 = 1 + 1$.

   (b) Therefore, $\ln(2) = \ln(1 + 1)$.

   (c) Using the logarithm properties, we have $\ln(1 + 1) = \ln(1) + \ln(1)$.

   (d) The logarithm properties also tell us that $\ln(1) = 0$.

   Concluding, we have $\ln(2) = \ln(1 + 1) = \ln(1) + \ln(1) = 0 + 0 = 0$.

5. Find the domain of the function

$$f(x) = \frac{2x}{x^2 - 1}.$$

6. Evaluate the limit

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}.$$

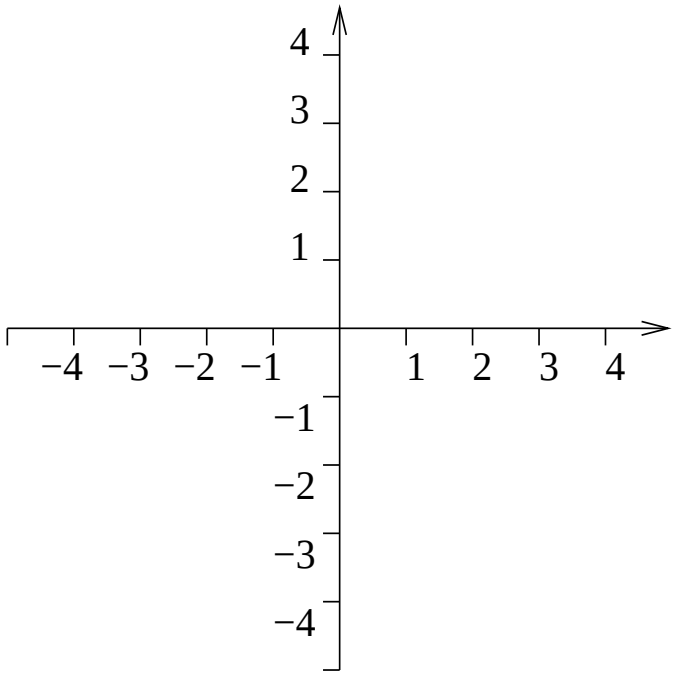
7. On the following axis, sketch the graph of a function $y = f(x)$ which satisfies **all** of the following:

(a)
$$\lim_{x\to\infty} f(x) = -1$$

(b)
$$\lim_{x\to-\infty} f(x) = 1$$

(c)
$$f(2) = 0$$

(d)
$$f'(2) = -1$$



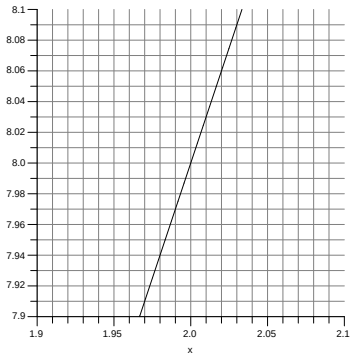
Long Answer (17 points each) - show work and provide explanations, an answer without supporting work will not receive credit.

1. Let $f(x) = 3x + 2$. Using direct substitution, we know that

$$\lim_{x \to 2} f(x) = 8.$$

To find the value of δ so that $|x-2| < \delta$ guarantees that $|f(x)-8| < 0.1$ we can use our calculator or algebra.

(a) If you use your calculator and try $\delta = 0.1$ you get the following screen. Explain using this window how you know this value of δ is not an appropriate choice to guarantee that $|f(x) - 8| < 0.1$.



(WARNING: This question continues over the page)

(b) Use algebra to find the correct value for δ.

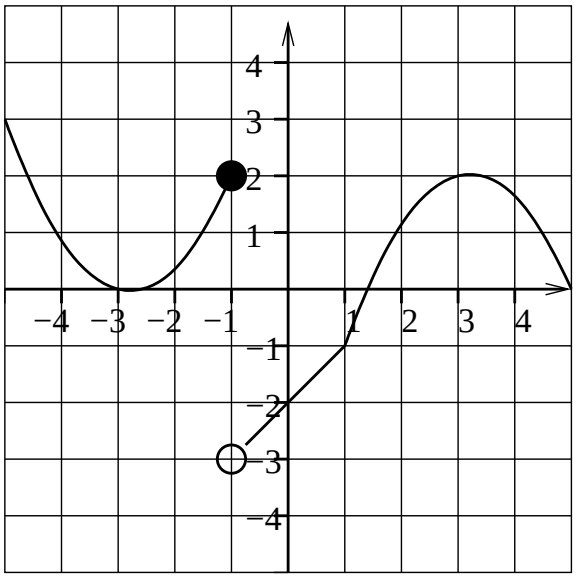
2. (a) Find the derivative function of

$$f(x) = \frac{1}{x}$$

using the limit definition of the derivative. **Warning**- you need to show **all** work - you cannot apply any rules of differentiation since they have not yet been developed in this class!

(b) Use your answer to part (a) to find the equation for the tangent line at $x = 3$.

3. The following is a graph of the function $f(x)$. Answer the following questions about $f(x)$.



(a) Evaluate

$$\lim_{x \to -1^+} f(x)$$

(b) Evaluate

$$\lim_{x \to -1^-} f(x)$$

(c) Use your answer to (a) and (b) to explain why the limit does not exist at $x = -1$.

WARNING There are more parts to this question over the page.

(d) Is $f(x)$ continuous at $x = -1$? Explain why or why not.

(e) Determine an equation for the tangent line to $f(x)$ at $x = 2$ (your answer will be an estimate based on what you can derive from the graph but you should explain how you get the equation).