Mth 201A
Hour Exam 3

Name: _____ Date: _____

10 Problems. 100 Points. Follow directions carefully. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 10 problems and 9 printed pages. Initial____.

Print the name of the people sitting either side of you :- _____

Short Answer (7 points each) - Minimal explanation and calculations necessary though where appropriate, answers should be exact.

1. The function $f(x) = x^2 - 1$ is continuous and differentiable on the interval $[0, 2]$. Find a number c with $0 \leq c \leq 2$ which satisfies the conclusion of the mean value theorem.

2. Evaluate the limit
$$\lim_{x \to \infty} \frac{5e^{-x} + 5x}{x}.$$

3. Calculate the exact value of the integral

$$\int_{-3}^{0} \sqrt{9 - x^2}.$$

4. Find the most general antiderivative of the function

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}.$$

5. By way of example, show that a function can have a critical point which is neither a minimum or maximum value.

6. Find the global maximum and minimum values of

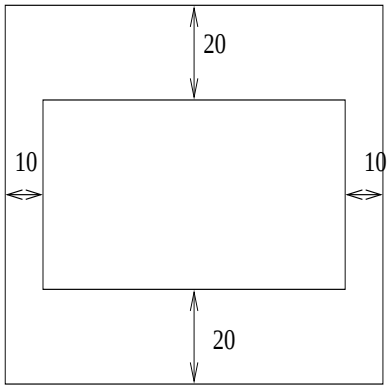
$$x^3 + 3x^2 + 3x$$

on the interval $[-3, 3]$.

7. Choose the values for b and c so that the function $f(x) = x^2 + bx + c$ has a minimum value of 4 at $x = 1$.

Long Answer (17 points each) - show work and provide explanations, an answer without supporting work will not receive credit.

1. A rectangular building is to cover $20,000$ feet. Zoning regulations require 20 feet of frontage roads at the front and rear of the building and 10 feet alleyways either side. Find the dimensions of the smallest piece of property the building can be constructed on.



2. Suppose that

$$f(x) = x \ln(x).$$

(a) Write down the intervals on which $f(x)$ is increasing and decreasing.

(b) Find all critical points and determine which are local minimums, maximums or neither.

(c) Write down the intervals on which $f(x)$ is concave up and concave down.

(d) Find all inflection points.

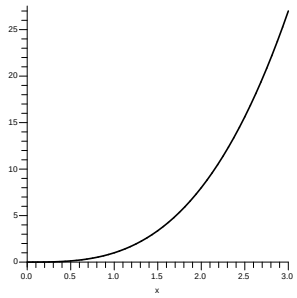
(e) Use the information you have derived to sketch a graph of $f(x)$ below labeling all interesting points.

3. Suppose that $f(x) = x^3$.

(a) Use the left hand sum with 3 subdivisions to estimate

$$\int_0^3 x^3 dx.$$

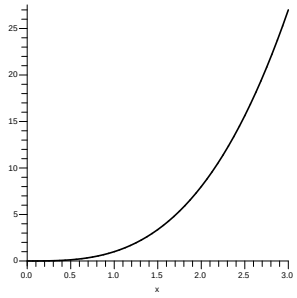
and illustrate the rectangles of the left hand sum on the graph of $f(x)$ below.



(b) Use the right hand sum with 3 subdivisions to estimate

$$\int_0^3 x^3 dx.$$

and illustrate the rectangles of the right hand sum on the graph of $f(x)$ below.



(c) Use your illustrations to explain why the left hand sum will always be an undersetimate and the right hand sum will always be an overestimate to the actual value of

$$\int_0^3 x^3 dx.$$

(d) Find a value of n which guarantees that the left and right hand sums are within 0.01 of eachother.

(e) Use your results to guess the value of

$$\int_0^3 x^3 dx.$$

(f) (EXTRA CREDIT) Explain why the value of n found in part (d) guarantees that the actual value of

$$\int_0^3 x^3 dx$$

is within 0.01 of the left hand sum.