Math 202
Final Exam

Name: _____ Date: _____

11 Problems. 200 Points. Follow directions carefully. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 11 problems and 8 printed pages. Initial_____.

Print the name of the people sitting either side of you :- _____

Short Answer - no explanation or calculations necessary though where appropriate, answers should be exact. The points value for each question is specified below.

1. Does the series

$$\sum \frac{1}{\sqrt{n}}$$

converge or diverge?

10 points

2. Solve the differential equation

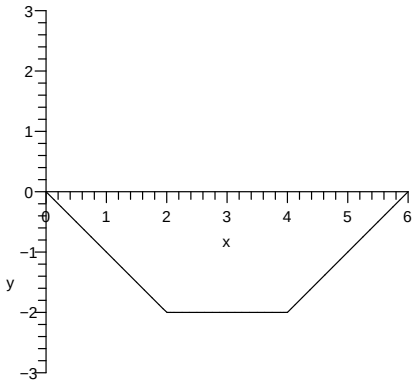
$$\frac{dy}{dx} = e^y \cos(x)$$

with the initial value $y(0) = 0$.

10 points

3. Find the value of the midpoint sum over 3 subintervals for the function illustrated below.

10 points



Extra Credit: Is it an overestimate, underestimate or exact? Briefly explain why.

10 points

4. For the following integrals, only one of the integration methods will work. For each integral, you must identify which method must be used and give the additional information requested below:

   - Substitution - specify u
   - Integration by Parts - specify u and dv/dx
   - Trigonometric Substitution - specify whether to use $\tan(\theta)$, $\sin(\theta)$ or $\sec(\theta)$
   - Partial Fractions - break up as partial fractions

   DO NOT evaluate any of the integrals!

   (a)
   $$\int x^2 \cos(x) dx$$
   10 points

   (b)
   $$\int x e^{x^2} dx$$
   10 points

   (c)
   $$\int \sqrt{(1-x^2)} dx$$
   10 points

   (d)
   $$\int \frac{1}{x^2-1} dx$$
   10 points

5. Give an example of a series which converges but does not converge absolutely (so it converges conditionally).

10 points

6. Does the **sequence**

$$a_n = \sin\left(\frac{1}{n}\right)$$

converge or diverge? If it converges, say what it converges to and if it diverges, briefly explain why.

10 points

7. Write down parametric equations for the equation of a circle centered at $(1, 1)$ of radius 2.

10 points

Long Answer (25 points each) - show work and provide explanations, an answer without supporting work is not worth much.

1. (a) Find the Maclaurin expansion for the function

$$\frac{\sin(x)}{x}.$$

You may assume the Maclaurin series for $\sin(x)$ if you already know it, but you must give a general nth term.

(b) Use your answer to part (a) to explain why

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 0.$$

(c) Use your answer to part (a) to find a power series representation for the indefinite integral

$$\int \frac{\sin(x)}{x} dx.$$

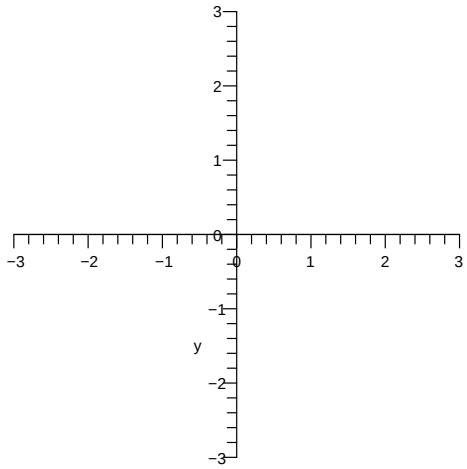
2. Find the interval of convergence of the power series

$$f(x) = x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + 36x^6 + \dots$$

3. Define the polar function r as

$$r = \sin(\theta).$$

(a) On the axis below, sketch a graph of r with $0 \leq \theta \leq \pi$.



(b) The graph of r should be very familiar geometrically. What is it?

(c) Use the arc length formula to calculate the arc length of r with $0 \leq \theta \leq \pi$.

(d) Extra Credit: You could have used your answer to part (b) to answer part (c) instead of calculating an integral. Briefly explain why.

10 points

4. Find the **exact** volume of the solid obtained by revolving the region **above** the graph of $y = x^2$ and below the line $y = 9$ about the line $y = 9$ (see illustration below).