

Math 202  
Hour Exam 1

Name: \_\_\_\_\_

Date: \_\_\_\_\_

9 Problems. 100 Points. Follow directions carefully. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 9 problems and 6 printed pages.  
Initial.....

Print the name of the people sitting either side of you :- \_\_\_\_\_

**Short Answer (8 points each) - no explanation or calculations necessary though where appropriate, answers should be exact. Print your answers to each question in the appropriate numbered box below.**

1. What trigonometric substitution is required to evaluate the integral

$$\int \frac{1}{x^2 \sqrt{x^2 + 4}} dx?$$

- (a)  $u = 2 \cos(x)$
- (b)  $u = 2 \sin(x)$
- (c)  $u = 2 \sec(x)$
- (d)  $u = 2 \tan(x)$

2. What method is required to evaluate the integral

$$\int \tan^6(x) \sec^4(x) dx?$$

- (a) Eliminate all but one  $\sec^2(x)$  using  $\sec^2(x) = \tan^2(x) + 1$  and then substitute  $u = \tan(x)$ .
- (b) Eliminate all but one  $\tan^2(x)$  using  $\sec^2(x) - 1 = \tan^2(x)$  and then substitute  $u = \sec(x)$ .
- (c) Eliminate all but one  $\sec^2(x)$  using  $\sec^2(x) = \tan^2(x) + 1$  and then substitute  $u = \sec(x)$ .



5. In order to apply integration by parts to the integral

$$\int x \arcsin(x) dx,$$

what choice of  $u$  and  $dv/dx$  should be made?

6. Describe **all** the concavity and increasing/decreasing properties of a function  $f(x)$  which guarantees that the trapezoid rule will give a lower estimate to the integral  $\int_a^b f(x) dx$  on the interval  $[a, b]$ .

**Long Answer (18 points each) - show work and provide explanations, an answer without supporting work is not worth much.**

1. Show that Gabriel's Horn (the solid obtained by rotating the region bounded between  $x = 1$  and  $y = 1/x$  about the  $x$ -axis) has finite volume.

2. Using the method of partial fractions (**showing all work**), evaluate the integral

$$\int \frac{2x^4 + 6x^2 + x + 2}{x^3 + x} dx.$$

3. A friend in engineering tells you that the volume of a circular cone of height  $H$  and base radius  $R$  is

$$\frac{\pi R^2 H}{6}.$$

They refuse to admit they are wrong unless you can present them with a proof of the actual correct answer. By realizing the cone as a volume of revolution, find its volume and prove your friend wrong!