Mth 202
Final Exam
Section B

Name: ______________________________ (2 Points)

12 Problems. 250 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 12 problems and 12 printed pages. Initial____.

Print the names of the people sitting next to you.____.

Question	**Points Value**	**Points Awarded**	**Section Totals**
Name	2		
Short Answer 1	15		
Short Answer 2	15		
Short Answer 3	15		
Short Answer 4	15		
Short Answer 5	15		
Short Answer 6	15		
Short Answer 7	15		
Short Answer 8	15		
Long Answer 1	32		
Long Answer 2	32		
Long Answer 3	32		
Long Answer 4	32		
Total	250		

Short Answer - minimum explanation and calculations necessary (15 points each).

1. Find the solution of the differential equation

$$\frac{dy}{dx} = y^2 + 1$$

with $y(1) = 0$.

2. Write down parametric equations for an ellipse centered at $(-1, 1)$ with y-radius 5 and x radius 3.

3. Evaluate the improper integral

$$\int_0^\infty \frac{1}{(2x+1)^3}dx.$$

4. Let

$$r = \frac{1}{\sin(\theta) - \cos(\theta)}.$$

Convert this polar function into Cartesian coordinates and rewrite it so that y is a function of x.

5. Determine the area of the region bounded between the graphs of

$$y = x \text{ and } y = x^2.$$

6. Prove that the arclength of a line

$$y = mx + c$$

for $a \leq x \leq b$ is equal to

$$\sqrt{(1 + m^2)}(b - a).$$

7. Determine whether the following series converges, and if it does, evaluate its sum:

$$\sum_{n=1}^{\infty} \frac{12}{(-5)^n}.$$

8. Suppose that the Maclaurin series of a function $f(x)$ is

$$\sum_{n=0}^{\infty} (-1)^n \frac{2x^n}{3^n(n!)}.$$

Use it to determine $f^{(22)}(0)$ i.e. the 22nd derivative of $f(x)$ at $x = 0$.

Long Answer - show work and provide explanations, an answer without supporting work is not worth much (32 points each).

1. Answer the following questions. You must show all your work and cannot use tables (except the basic tables 1-18).

   (a) Evaluate

$$\int (x+1)\ln(x)dx$$

(b) Evaluate **one of** the following two integrals:

$$\text{either} \qquad \int \frac{x^2+1}{x^2-1}dx \qquad \text{or} \qquad \int \frac{\sqrt{x^2-9}}{x^3}dx.$$

Only calculate **one** of these integrals - you will receive credit for only one (whichever you do first).

2. (a) Sketch the region R bounded by $x = 1 - y^2$, the lines $y = 1$ and $y = 2$ and the y-axis.

(b) Determine the volume of the solid obtained by rotating the region R about the x-axis. You may use either the method of shells or discs.

3. (This question continues over the page)

   (a) On the same axes, sketch the two polar curves $r = 6\cos(\theta)$ and $r = 4 - 2\cos(\theta)$ for $0 \le \theta \le 2\pi$.

   (b) Determine all points of intersection of the polar curves $r = 6\cos(\theta)$ and $r = 4 - 2\cos(\theta)$ for $0 \le \theta \le 2\pi$.

(c) Determine the area bounded inside the polar curve $r = 6\cos(\theta)$ but outside the curve $r = 4 - 2\cos(\theta)$.

4. (This question continues over the page) The goal of this problem is to use a Taylor polynomial to approximate $\ln(0.9)$ and then attempt to find a Taylor series for $\ln(x)$ around $x = 1$.

   (a) Determine the fifth degree Taylor polynomial for $f(x) = \ln(x)$ **around** $x = 1$.

   (b) Use Taylor polynomial of degree 5 to approximate the value of $\ln(0.9)$.

(c) Determine a formula for the general nth derivative of $\ln(x)$ at $x = 1$ i.e $f^{(n)}(1)$ where $f(x) = \ln(x)$ (Hint: See if you can see a pattern above when you were determining the fifth degree Taylor polynomial).

(d) Use your answer to write down the Taylor series for $f(x) = \ln(x)$ around $x = 1$.