Math 202
Hour Exam 2

Name: ____________________ Date: __________

9 Problems. 150 Points. Follow directions carefully. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 9 problems and 9 printed pages. Initial____.

Print the name of the people sitting either side of you :- ______________

Question	**Points Value**	**Points Awarded**	**Section Totals**
Short Answer 1	12		
Short Answer 2	12		
Short Answer 3	12		
Short Answer 4	12		
Short Answer 5	12		
Short Answer 6	12		
Long Answer 1	26		
Long Answer 2	26		
Long Answer 3	26		
Total	150		

Short Answer (12 points each) - keep explanations and calculations brief and where appropriate, answers should be exact.

1. Write down parametric equations for an ellipse with y-radius 3, x-radius 2 and centered at the point $(-2, 1)$.

2. Find the solution to the differential equation

$$y \ln(x) - \frac{1}{x}\frac{dy}{dx} = 0$$

with $y(1) = 1$.

3. Convert the following:

   (a) $(1, 2)$ from Cartesian coordinates to polar coordinates

   (b) $(2, \pi/3)$ from polar coordinates to Cartesian coordinates

4. Set up and then evaluate an integral to calculate the surface area of the volume of revolution obtained by rotating $f(x) = 2\sqrt{x}$ on the interval $[1, 3]$ about the x-axis (Note: You will only get half credit if you do not set up the integral).

5. Write down the nth term of the following sequence and determine if it converges, and if so, what it converges to:

$$\left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots\right\}.$$

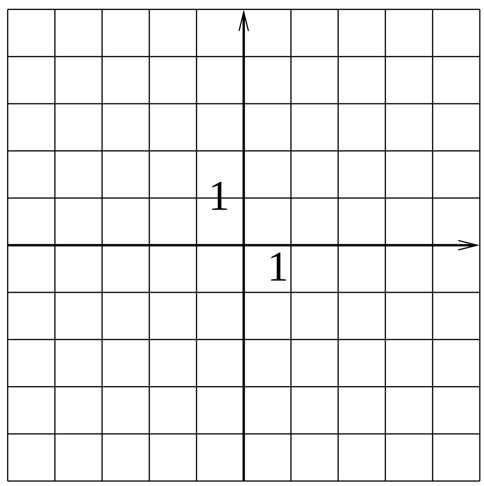
6. Suppose that $y = 3 \cdot 5^t$. Since y is an exponential function, it satisfies the differential equation $y' = ky$. Find k.

Long Answer (26 points each) - show work and provide explanations, an answer without supporting work is not worth much.

1. (This question continues over the page) Let C be the curve with parametric equations

$$x(t) = 2t - \pi \sin(t)$$
$$y(t) = 2 - \pi \cos(t)$$

(a) Sketch a graph of C for $-\pi \leq t \leq \pi$ on the axis below.



(b) The graph of C crosses itself at the point $(0, 2)$ forming a loop. Find the two different t values for when this happens.

(c) Explain why the area of the region bounded by the loop is equal to the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y(t)x'(t)dt$$

by considering the region in each of the four quadrants paying close attention to the direction of the parameterization and whether the region is above or below the x-axis.

(d) Calculate the area of the region bounded by the loop by evaluating the integral. You must show your work, but you can use tables.

2. (This problem continues on the next page) Let C be a half circle of radius R centered at the origin in the upper half plane and let L be a line segment of $y = 2x + 1$.

   (a) Write down a polar equation for the line segment L.

   (b) Write down a polar integral to calculate the arc length of L for $\alpha \leq t \leq \beta$.

   (c) Write down a Cartesian integral to calculate the length of the line segment L for $a \leq x \leq b$.

(d) Write down a Cartesian integral to calculate the length of C.

(e) Write down a polar integral to calculate the arclength of C.

(f) Use your results to explain for which situtations polar coordinates are useful and for which situations Cartesian coordinates are useful when calculating arclength.

3. Let T be the torus obtained by rotating the circle C of radius R and center $(0, a)$ about the x-axis where $r < a$.

   (a) Write down parametric equations for the circle C.

   (b) Calculate the surface area of the torus in terms of a and r (if you cannot find it in terms of a and r, you can choose values for both, but you will have points deducted).