

## Section 7.6: Integration Using Tables and Computer Algebra Systems

### 1. TABLES OF INTEGRALS

Due to the technicality of the process of integration, we need additional methods to the ones we have already considered. One such method is to make use of a table of integrals. Specifically, a table of integrals is a list of standard integrals which can be compared to a given integral in an attempt to solve it using the solution from the table. It should be pointed out however that a table of integrals will not list every possible integral, but rather some standard types of integrals, so we may be required to perform some algebraic manipulation or technique from calculus to adjust to the same form as the table. We illustrate with some examples.

**Example 1.1.** Evaluate

$$\int e^{-3x} \cos(4x) dx.$$

For this integral, we use formula 99 from the table of integrals with  $a = -3$  and  $b = 4$  to obtain

$$\int e^{-3x} \cos(4x) dx = \frac{e^{-3x}}{25} (-3 \cos(4x) + 4 \sin(4x)).$$

**Example 1.2.** Evaluate

$$\int \sin^6(4x) dx.$$

For this integral, we use iteration formula 73 from the table of integrals with  $n = 6$ . Since the formulas do not directly match up however, we first need to make the substitution  $u = 4x$ , so  $dx = \frac{1}{4} du$ . Calculating, we get

$$\begin{aligned} \int \sin^6(4x) dx &= \frac{1}{4} \int \sin^6(u) du = \frac{1}{4} \left( -\frac{1}{6} \sin^5(u) \cos(u) + \frac{5}{6} \int \sin^4(u) du \right) \\ &= \frac{1}{4} \left( -\frac{1}{6} \sin^5(u) \cos(u) + \frac{5}{6} \left[ -\frac{1}{4} \sin^3(u) \cos(u) + \frac{3}{4} \int \sin^2(u) du \right] \right) \\ &= \frac{1}{4} \left( -\frac{1}{6} \sin^5(u) \cos(u) + \frac{5}{6} \left[ -\frac{1}{4} \sin^3(u) \cos(u) + \frac{3}{4} \left( \frac{u}{2} - \frac{1}{4} \sin(2u) \right) \right] \right). \end{aligned}$$

We then finish by substituting  $u = 4x$  back into the equation.

**Example 1.3.** Evaluate

$$\int \arcsin(\sqrt{x}) dx.$$

This does not match formulas 87, 90 or 93 in the tables, so we need to modify the integrand first. We try the substitution  $u = \sqrt{x}$ , so  $du = 2\sqrt{x}dx$ . Calculating, we get

$$\int \arcsin(\sqrt{x})dx = \int \arcsin(u)2\sqrt{x}du = 2 \int u \arcsin(u)du.$$

Now this is simply formula 90, so we get

$$\begin{aligned} \int \arcsin(\sqrt{x})dx &= 2 \left( \frac{2u^2 - 1}{4} \arcsin(u) + \frac{u\sqrt{1-u^2}}{4} \right) + C. \\ &= \frac{2x - 1}{2} \arcsin(\sqrt{x}) + \frac{\sqrt{x}\sqrt{1-x}}{2} + C. \end{aligned}$$

## 2. INTEGRALS USING COMPUTER ALGEBRA SYSTEMS

Due to the great advances in computer technology, in current times the emphasis of computational mathematics has been placed on computers. In particular, nowadays there are many math programs on computers and even on many calculators which will perform integrals of many different types of functions. Though these tools are extremely useful however, they should always be used with caution - sometimes a CAS may not know how to integrate a particular function, sometimes it may integrate incorrectly, and other times it may give the answer in a strange form which does not match the answer we would expect. We illustrate with an example.

**Example 2.1.** Use your calculator to evaluate

$$\int x \csc(x) \cot(x) dx.$$

The calculator gives back

$$\int x \csc(x) \cot(x) dx$$

which means it does not know how to integrate this function. Recall however that a fairly easy substitution allowed us to solve this problem before.

**Example 2.2.** Use your calculator to evaluate

$$\int \frac{1}{\sin(x)} dx.$$

The calculator gives back

$$\ln \left( \frac{|\sin(x)|}{|\cos(x) + 1|} \right)$$

but the table gives  $\ln(|\csc(x) - \cot(x)|)$ . Though these look like two different functions, after algebraic manipulation, we see that they are in fact identical.