MATH 129
FINAL EXAM REVIEW PACKET
(Revised Spring 2008)

The following questions can be used as a review for Math 129. These questions are not actual samples of questions that will appear on the final exam, but they will provide additional practice for the material that will be covered on the final exam. When solving these problems keep the following in mind: Full credit for correct answers will only be awarded if all work is shown. Exact values must be given unless an approximation is required. Credit will not be given for an approximation when an exact value can be found by techniques covered in the course. The answers, along with comments, are posted as a separate file on http://math.arizona.edu/~calc.

1. Suppose the rate at which people get a particular disease (measured in people per month) can be modeled by $r(t) = 10\pi \sin\left(\frac{\pi}{3}t\right) + 30$. Find the total number of people who will get the disease during the first three months ($0 \le t \le 3$).

2. If $\int_1^3 f(u)du = 7$, find the value of $\int_1^2 f(5-2x)dx$.

3. Evaluate $\int \frac{t}{\sqrt{t+1}} dt$

4. Use the method of integration by parts:

a) Evaluate $\int \frac{\ln\left(z^2+1\right)}{z^2} dz$

b) Evaluate $\int x \arcsin(x^2) dx$.

c) Let g be twice differentiable with $g(0) = 6$, $g(1) = 5$, and $g'(1) = 2$. Find $\int_0^1 x \cdot g''(x) dx$.

5. Evaluate the following integrals (you can use the table of integrals):

a) $\int \cos^2(3\theta + 2) d\theta$

b) $\int \frac{2}{4t^2 - 9} dt$

c) $\int \frac{dy}{\sqrt{y^2 + 8y + 15}}$

6. Use the method of partial fractions to evaluate $\int \frac{3y^3 + 5y - 1}{y^3 + y} dy$.

7. Use the method of trigonometric substitution to evaluate $\int \frac{dx}{(5-x^2)^{3/2}}$.

8. The velocity v of the flow of blood at a distance r from the central axis of an artery with radius R is proportional to the difference between the square of the radius of the artery and the square of the distance from the central axis. Find an equation for v using k as the proportionality constant. Find the average rate of flow of blood. Recall that the average value of a function over $[a, b]$ is given by $\frac{1}{b-a}\int_a^b f(x)dx$.

9. In the study of probability, a quantity called the expected value of X is defined as $E(X) = \int_{-\infty}^{\infty} xf(x)dx$. Find $E(X)$ if $f(x) = \begin{cases} \frac{1}{7}e^{-x/7} & x \ge 0 \\ 0 & x < 0 \end{cases}$.

10. a) Find an approximation of $\int_0^1 e^{-t^2}dt$ using the midpoint rule with $n = 2$. (Show your work).
b) Does the midpoint rule give an overestimate or underestimate for the integral in part a?

11. Determine if the improper integral converges or diverges. Show your work/ reasoning. If the integral converges, evaluate the integral.

a) $\int_0^{\infty} \frac{1}{x^2+4}dx$ b) $\int_1^{\infty} \frac{1}{2^x}dx$ c) $\int_0^1 \frac{e^x}{\left(e^x-1\right)^2}dx$ d) $\int_{\pi/6}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}}dx$

12. According to a book of mathematical tables, $\int_0^{\infty} e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$. Use this formula and substitution to find $\int_m^{\infty} e^{-\left(\frac{x-m}{s}\right)^2}dx$.

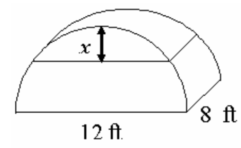
13. Suppose f is continuous for all real numbers and that $\int_0^{\infty} f(x)dx$ converges. Determine which of the following converge. Explain or show your work clearly. Assume $a > 0$.

a) $\int_0^{\infty} a \cdot f(x)dx$ b) $\int_0^{\infty} f(ax)dx$ c) $\int_0^{\infty} \left(a + f(x)\right)dx$ d) $\int_0^{\infty} f(a+x)dx$

14. Use an appropriate comparison to determine if the improper integral converges or diverges. If the integral converges, find an upper bound for the integral (see p356).

a) $\int_2^{\infty} \frac{d\theta}{\sqrt{\theta^3+2}}$ b) $\int_1^{\infty} \frac{1+\sin^2 x}{(x+3)^3}dx$ c) $\int_1^{\infty} \frac{(1+\sin^2 x)x^2}{x^3+3}dx$

15. Use the concept of slicing and the variable shown at the right to set up the definite integral needed to find the volume of the solid.



16. Consider the region bounded by $y = -3x + 6$, $y = 3\sqrt{x}$, and the *x*-axis. Sketch and shade in this region. Set up the integral(s) needed to find the area if we use the following:
a) slices that are perpendicular to the *x*-axis. b) slices that are perpendicular to the *y*-axis.

17. Consider the region bounded by $y = 5e^{-x}$, $y = 5$, and $x = 3$. Find the volume of the solid obtained by rotating the region around the following: a) the *x*-axis b) the line $y = 5$

18. Consider the region bounded by $y = x^3$, $y = 8$, and the *y*-axis. Find the volume of the solid obtained by rotating the region around the *y*-axis.

19. Consider the region bounded by the first arch of $y = \sin x$ and the *x*-axis. Find the volume of the solid whose base is this region and whose cross-sections perpendicular to the x-axis are the following: a) squares. b) semi-circles

20. The circumference of a tree at different heights above the ground is given in the table below. Assuming all of the horizontal cross-sections are circular, estimate the volume of the tree.

Height (inches)	0	10	20	30	40	50
Circumference (inches)	26	22	18	12	6	2

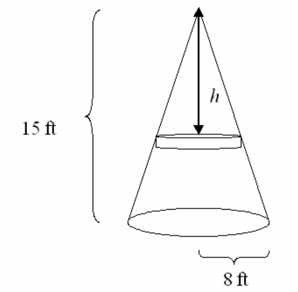
21. The soot produced by a garbage incinerator spreads out in a circular pattern. The depth, $H(r)$, in millimeters, of the soot deposited each month at a distance of *r* kilometers from the incinerator is given by $H(r) = 0.115e^{-2r}$. Write a definite integral giving the total volume of soot deposited within 5 kilometers of the incinerator each month. Evaluate the integral and express your final answer in cubic meters.

22. A metallic rod 5 cm in length is made from a mixture of several materials so that its density changes along its length. Suppose the density of the rod at a point *x* cm from one end is given by $\delta(x) = 2 + 0.5\cosh x$ grams per cm of length. Find the total mass of the rod.

23. A cylindrical form is filled with slow-curing concrete to form a column. The radius of the form is 10 feet and the height is 25 feet. While the concrete hardens, gravity causes the density to vary so that the density at the bottom is 90 pounds per cubic foot and the density at the top is 50

pounds per cubic foot. Assume that the density varies linearly from top to bottom. Find the total weight (in pounds) of the concrete column.

24. A tank of water has the shape of a right circular cone shown below. In each case, set up the integral for the amount of work needed to pump the water out of the tank under the given conditions. Your integral must correspond to the variable indicated in the picture. (The density of water is 62.4 pounds per cubic foot.)



a) The tank is full, the tank will be emptied, and the water is pumped to a point at the top of the tank.

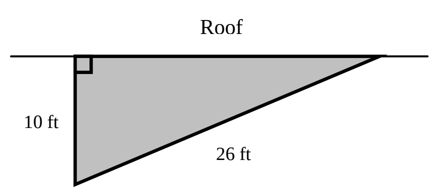
b) The tank is full, the tank will be emptied, and the water is pumped to a point 3 feet above the top of the tank.

c) The tank is full, the water will be pumped until the level of the water in the tank drops to 5 feet, and the water is pumped to a point at the top of the tank.

d) The initial water level of the tank is 12 feet, the tank will be emptied, and the water is pumped to a point 3 feet above the top of the tank.

25. Workers on a platform 45 feet above the ground will lift a block of concrete weighing 500 pounds from the ground to the platform. The block is 5 feet tall and is attached to a 40 foot chain that weighs 3 pounds per foot. Find the amount of work required.

26. A flag in the shape of a right triangle is hung over the side of a building as shown at the right. It has a total weight of 240 pounds and uniform density. Find the work needed to lift the flag onto the roof of the building.



27. a) Find a formula for the general term of the sequence $\frac{-2}{9}, \frac{4}{16}, \frac{-6}{25}, \frac{8}{36}, \frac{-10}{49}, \cdots$

b) Determine if the sequence $a_n = \frac{3n^2+2}{2-5n^2}$ converges or diverges. If it converges, find its limit.

28. Find the following sums: a) $\sum_{n=3}^{10} 3\left(\frac{1}{4}\right)^n$ b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^n$

29. A 200 mg dose of a particular medicine is given every 24 hours. Suppose 5% of the dose remains in the body at the end of 24 hours. Let P_n represent the amount of medicine that is in the body right before the n^{th} dose is taken. Let Q_n represent the amount of medicine that is in the body right after the n^{th} dose is taken. Express P_n and Q_n in closed-form.

30. Use the integral test to determine if the series converges or diverges:

a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ b) $\sum_{n=1}^{\infty} \frac{3n^2+2n}{\sqrt{n^3+n^2+1}}$

31. Use the ratio test to determine if the series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{e^{n+1}}{n^2 2^n}$ b) $\sum_{n=0}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}$

32. Indicate whether the following statements are True or False.

a) If $0 \le a_n \le b_n$ and $\sum a_n$ converges, then $\sum b_n$ converges.

b) If $0 \le a_n \le b_n$ and $\sum a_n$ diverges, then $\sum b_n$ diverges.

c) If $\sum a_n$ converges, then $\sum |a_n|$ converges.

d) If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

e) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

33. Determine the radius of convergence and the interval of convergence (you do not need to investigate convergence at the endpoints):

a) $\sum_{n=1}^{\infty} \frac{(2n+1)(x+4)^n}{3^{n+1}}$ b) $3 - \frac{3x^2}{2!} + \frac{3x^4}{4!} - \frac{3x^6}{6!} + \frac{3x^8}{8!} + \cdots$

c) $1 + \frac{(x-1)}{2} + \frac{2!(x-1)^2}{4} + \frac{3!(x-1)^3}{8} + \frac{4!(x-1)^4}{16} + \cdots$

34. Suppose that $\sum_{n=0}^{\infty} C_n (x-2)^n$ converges when $x = 4$ and diverges when $x = 6$. Which of the following are True, False, or impossible to determine?

a) The power series diverges when $x = -3$.
b) The power series converges when $x = 1$.
c) The power series diverges when $x = 5$.

35. Find the Taylor polynomial of degree two that approximates the function $f(x) = \sqrt{x^3+1}$ near $x = 2$.

36. Suppose $P_2(x) = c_0 + c_1 x + c_2 x^2$ is the second degree Taylor polynomial for a function $f(x)$ where $f(x)$ is always increasing and concave down. Determine the signs of $c_0, c_1,$ and c_2.

37. Consider the function given by $f(x)=\sum_{k=0}^{\infty}(-1)^{k+1}\frac{k!}{(2k)!}(x-3)^k$.

a) Find $f(3)$. b) Find $f'(3)$. c) Find $f''(3)$.

d) Find the Taylor series for $f(3x)$ about $x=1$. Include an expression for the general term of the series.

38. Find the exact value of $\int_0^{1/13} f(x)dx$ if $f(x)=\sum_{k=0}^{\infty}(k+1)x^k = 1+2x+3x^2+4x^3+5x^4+\cdots$

39. Match the function with its Taylor series near $x=0$.

a) $f(x)=\frac{1}{1-x}$ i) $\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!}$

b) $f(x)=e^x$ ii) $\sum_{k=0}^{\infty}x^k$

c) $f(x)=\cos x$ iii) $\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1}$

d) $f(x)=\arctan x$ iv) $\sum_{k=0}^{\infty}\frac{x^k}{k!}$

40. By recognizing each series as a Taylor series evaluated at a particular value of *x*, find the following sums, if possible.

a) $1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\cdots$ b) $\sum_{k=0}^{\infty}\frac{(-1)^k(0.5)^{k+1}}{k+1}$ c) $\sum_{k=0}^{\infty}\left(\frac{\pi}{e}\right)^{k+1}$

41. a) State the Taylor series for $f(x)=\sin x$ near $x=0$.

b) Use the series in part a) to find the value of $g^{(10)}(0)$ for the continuous function

$$g(x)=\begin{cases}\frac{\sin x}{x} & x\neq 0\\ 1 & x=0\end{cases}.$$

42. Find the Taylor series about 0 for the following functions (include the general term):

a) $f(x)=x\ln(1+2x)$ b) $f(x)=e^{-x^2}$

43. Expand $\frac{a}{(a+r)^2}$ about 0 in terms of the variable $\frac{r}{a}$ where *a* is a positive constant and *r* is very large when compared to *a*.

44. Indicate whether the following statements are True or False.

a) If $f(x)$ and $g(x)$ have the same Taylor polynomial of degree two near $x=0$, then $f(x)=g(x)$.

b) The Taylor series for $f(x)g(x)$ about $x=0$ is $f(0)g(0)+f'(0)g'(0)x+\frac{f''(0)g''(0)}{2!}x^2+\cdots$.

c) The Taylor series for *f* converges everywhere *f* is defined.

45. a) Write Euler's Formula for $e^{i\theta}$.

b) Express the complex number $\frac{\sqrt{3}}{2}-\frac{1}{2}i$ in the form $Re^{i\theta}$.

c) Express the complex number $2e^{\frac{-\pi}{4}i}$ in the form $a+bi$.

d) Express the complex number $e^{(3+4i)t}$ in the form $a(t)+b(t)i$.

46. Match the following differential equations with one of its solutions.

a) $\frac{dy}{dx}=\sqrt{|y^2-4|}$ i) $y=x^3+3x^2$

b) $\frac{dy}{dx}=\frac{y-5}{x^2y^2+1}$ ii) $y=e^x+e^{-x}$

c) $\frac{dy}{dx}=y-x^3+6x$ iii) $y=5$

d) $\frac{dy}{dx}=y-\ln|y-x|+1$ iv) $y=e^x+x$

47. Consider the initial value problem $\frac{dy}{dx}=10x^2+y$, $y(0)=3$. Use Euler's method to approximate $y(0.4)$. Complete the following table of values.

x	*y*	Δ*x*	Δy
0.0			
0.1			
0.2			
0.3			
0.4			

48. Solve the differential equation subject to the initial condition: $\frac{dy}{dx} = x\left(y^2+4\right)$, $y(0)=2$.

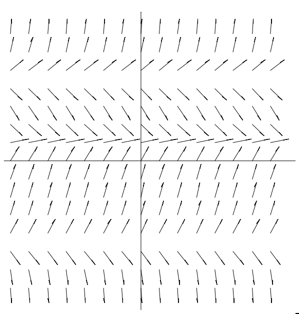
49. A particular drug is known to leave a patient's system at a rate directly proportional to the amount of the drug in the bloodstream. Previously, a physician administered 9 mg of the drug and estimated that 5 mg remained in the patient's bloodstream 7 hours later.
a) Write a differential equation for the amount of drug in the patient's bloodstream at time *t*.
b) Solve the differential equation in part a).
c) Find the approximate time when the amount of drug in the patient's bloodstream was 0.1 mg.

50. Match the differential equation with the slope field (assume *a* is a positive constant):

a) $\frac{dy}{dx} = ax^2 + y^2$ b) $\frac{dy}{dx} = x - a$ c) $\frac{dy}{dx} = (y^2-4)(y-a)$

d) $\frac{dy}{dx} = (x^2-4)(y-a)$ e) $\frac{dy}{dx} = a$ f) $\frac{dy}{dx} = y + a$

(*i*)



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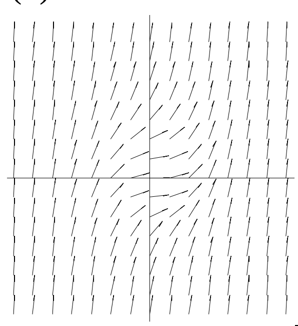

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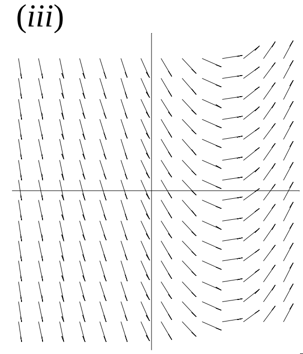

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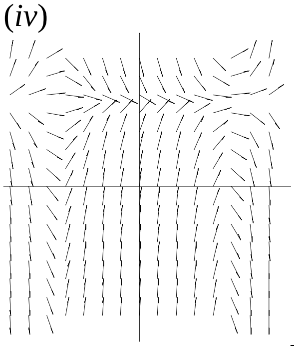

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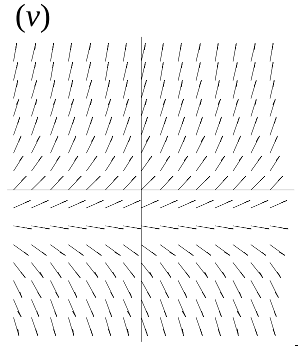

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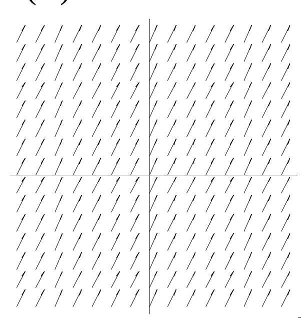

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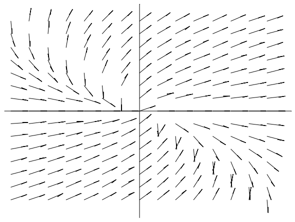
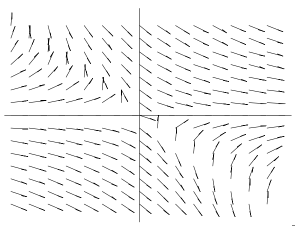
51. The differential equation $\frac{dy}{dx} = f(x, y)$ has slope field:



Match the related differential equation with its slope field:

a) $\frac{dy}{dx} = -f(x, y)$ b) $\frac{dy}{dx} = -\frac{1}{f(x, y)}$ c) $\frac{dy}{dx} = \left(f(x, y)\right)^2$ d) $\frac{dy}{dx} = -f(x, -y)$

(*i*)



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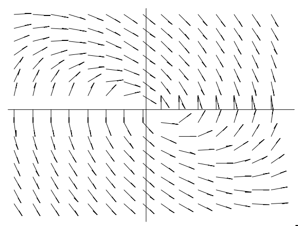

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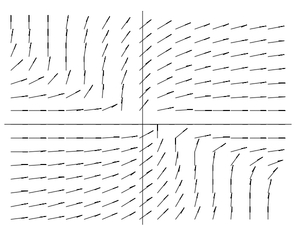

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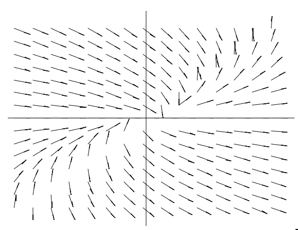

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52. Dead leaves accumulate on the floor of a forest at a continuous rate of 4 grams per square centimeter per year. At the same time, these leaves decompose continuously at the rate of 60% per year.
a) Write a differential equation for the quantity of leaves (in grams per square centimeter) at time *t*. Solve this differential equation.
b) Find the equilibrium solution and give a practical interpretation. Is the solution stable?

53. A room with a southern exposure heats up during the morning. The temperature of the room increases linearly so that it rises 1°F for every 15 minutes. Early in the morning, a cup of coffee with a temperature of 180°F is placed in the room when the room temperature is 60°F. Newton's Law of Cooling states that the rate of change in the temperature of the coffee should be proportional to the difference in temperature between the coffee and the room.
a) Write a formula for the temperature of the room *t* minutes after the coffee is placed there.
b) Write an initial value problem for the temperature of the coffee as a function of time.

54. The area that a bacteria colony occupies is known to grow at a rate that is proportional to the square root of the area. Assume the proportionality constant is $k = 0.06$ Write a differential equation that represents this relationship. Solve the differential equation.

55. It is of considerable interest to policy makers to model the spread of information through a population. Two models, based on how the information is spread, are given below. Assume the population is of a constant size *M*.

a) If the information is spread by mass media (TV, radio, newspapers), the rate at which information is spread is believed to be proportional to the number of people not having the information at that time. Write a differential equation for the number of people having the information by time *t*. Sketch a solution assuming that no one (except the mass media) has the information initially.

b) If the information is spread by word of mouth, the rate of spread of information is believed to be proportional to the product of the number of people who know and the number who don't. Write a differential equation for the number of people having the information by time *t*. Sketch the solution for the cases in which i) no one knows initially, ii) 5% of the population knows initially, and iii) 75% of the population knows initially.

56. A population is modeled by a function $P(t)$ that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{6}\left(1 - \frac{P}{15}\right)$.

a) Consider the solution with the condition $P(0) = 18$. Is the solution increasing or decreasing? Find $\lim_{t \to \infty} P(t)$.

b) Consider the solution with the condition $P(0) = 7$. For what value of *P* is the population growing the fastest?