MATH 129
FINAL EXAM REVIEW PACKET ANSWERS
(Revised Spring 2008)

1. $\int_0^3 \left(10\pi \sin\left(\frac{\pi}{3}t\right)+30\right)dt = 150$ people.

2. $\int_1^2 f(5-2x)dx = \frac{7}{2}$. Let $u = 5-2x$ and change the endpoints.

3. $\int \frac{t}{\sqrt{t+1}}dt = \frac{2}{3}(t+1)^{3/2} - 2(t+1)^{1/2} + c$. By the method of substitution with $u = t+1$.
You can also use integration by parts with $u = t$ and $v' = (t+1)^{-1/2}$. The result is equivalent, just written in a different form. $\int \frac{t}{\sqrt{t+1}}dt = 2t(t+1)^{1/2} - \frac{4}{3}(t+1)^{3/2} + c$.

4. a) $\int \frac{\ln\left(z^2+1\right)}{z^2}dz = -\frac{1}{z}\ln\left(z^2+1\right) + 2\arctan z + c$. Let $u = \ln(z^2+1)$ and $v' = \frac{1}{z^2}$.

b) $\int x\arcsin(x^2)dx = \frac{1}{2}x^2\arcsin(x^2) + \frac{1}{2}\sqrt{1-x^4} + c$. First make a substitution with $w = x^2$.
Then let $u = \arcsin(w)$ and $v' = 1$.

c) $\int_0^1 x \cdot g''(x)dx = 3$. Let $u = x$ and $v' = g''$ for the first integration by parts.

5. a) $\int \cos^2(3\theta+2)d\theta = \frac{1}{6}\cos(3\theta+2)\sin(3\theta+2) + \frac{1}{6}(3\theta+2) + c$. Let $u = 3\theta+2$ before using table formula # 18. If you use another approach, your answer will look different.

b) $\int \frac{2}{4t^2-9}dt = \frac{1}{6}\left(\ln|2t-3| - \ln|2t+3|\right) + c$. Let $u = 2t$ and factor the denominator before using table formula # 26. If you use another approach, your answer will look different.

c) $\int \frac{dy}{\sqrt{y^2+8y+15}} = \ln\left|(y+4)+\sqrt{y^2+8y+15}\right| + c$. Complete the square before using table formula # 29.

6. $\int \frac{3y^3+5y-1}{y^3+y}dy = 3y - \ln|y| + \frac{1}{2}\ln|y^2+1| + 2\arctan(y) + c$. First do long division.
Then use partial fractions $\frac{A}{y} + \frac{By+C}{y^2+1}$.

7. $\int \frac{dx}{(5-x^2)^{3/2}} = \frac{1}{5}\frac{x}{\sqrt{5-x^2}} + c$ Let $x = \sqrt{5}\sin(\theta)$.

8. $v = k(R^2 - r^2)$. $\frac{1}{R-0}\int_0^R k(R^2-r^2)dr = \frac{2}{3}kR^2$.

9. $E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} xf(x)dx + \int_0^{\infty} xf(x)dx = 0 + \int_0^{\infty} x\frac{1}{7}e^{-x/7}dx = 7$. Use integration by parts or the table of integrals.

10. a) $\int_0^1 e^{-t^2}dt \approx \frac{1}{2}e^{-1/16} + \frac{1}{2}e^{-9/16} \approx 0.75459794$

b) Not clear because $f(t) = e^{-t^2}$ changes concavity on the given interval. When $f(t)$ is concave down, the midpoint rule provides an overestimate.

11. a) The integral converges. $\int_0^{\infty} \frac{1}{x^2+4}dx = \frac{\pi}{4}$. Use table formula # 24.

b) The integral converges. $\int_1^{\infty} \frac{1}{2^x}dx = \frac{1}{2\ln 2}$.

c) The integral diverges. $\int_0^1 \frac{e^x}{\left(e^x-1\right)^2}dx = \infty$. Let $u = e^x - 1$.

d) The integral converges. $\int_{\pi/6}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}}dx = 2\sqrt{\frac{\sqrt{3}}{2}}$. Let $u = \cos x$

12. $\int_m^{\infty} e^{-\left(\frac{x-m}{s}\right)^2}dx = \frac{s\sqrt{\pi}}{2}$. Let $u = \frac{x-m}{s}$ and change the endpoints.

13. a) The integral converges. Rewrite as $\int_0^{\infty} a\cdot f(x)dx = a\int_0^{\infty} f(x)dx$.

b) The integral converges. Let $u = ax$.

c) The integral diverges. Rewrite as $\int_0^{\infty}\left(a + f(x)\right)dx = \int_0^{\infty} adx + \int_0^{\infty} f(x)dx$.

d) The integral converges. Let $u = a + x$.

14. a) The integral converges. $\int_2^\infty \frac{d\theta}{\sqrt{\theta^3+2}} < \sqrt{2}$. Use the comparison $\frac{1}{\sqrt{\theta^3+2}} < \frac{1}{\theta^{3/2}}$.

b) The integral converges. $\int_1^\infty \frac{1+\sin^2 x}{(x+3)^3}\,dx < \frac{1}{16}$. Use the comparison $\frac{1+\sin^2 x}{(x+3)^3} < \frac{2}{(x+3)^3}$.

c) The integral diverges. $\int_1^\infty \frac{(1+\sin^2 x)x^2}{x^3+3}\,dx$. Use the comparison $\frac{(1+\sin^2 x)x^2}{x^3+3} > \frac{x^2}{x^3+3}$.

15. $\int_0^6 16\sqrt{36-(6-x)^2}\,dx$ Use Pythagorean Theorem to find the width of the slice.

16. a) $\int_0^1 3\sqrt{x}\,dx + \int_1^2 (6-3x)\,dx$. b) $\int_0^3 \left(\frac{6-y}{3} - \frac{y^2}{9}\right) dy$.

17. a) $\pi \cdot 5^2 \cdot 3 - \int_0^3 \pi (5e^{-x})^2\,dx = \frac{(125+25e^{-6})}{2}\pi$.

b) $\int_0^3 \pi\left(5-5e^{-x}\right)^2 dx = \frac{(75+100e^{-3}-25e^{-6})\pi}{2}$.

18. $\int_0^8 \pi\left(y^{1/3}\right)^2 dy = \frac{96}{5}\pi$.

19. a) $\int_0^\pi (\sin x)^2\,dx = \frac{\pi}{2}$. b) $\int_0^\pi \frac{1}{2}\pi\left(\frac{\sin x}{2}\right)^2 dx = \frac{\pi^2}{16}$

20. Left hand rule:

$$10\cdot\pi\left(\frac{26}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{22}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{18}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{12}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{6}{2\pi}\right)^2 = \frac{4160}{\pi} \text{ cubic inches.}$$

Right hand rule:

$$10\cdot\pi\left(\frac{22}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{18}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{12}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{6}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{2}{2\pi}\right)^2 = \frac{2480}{\pi} \text{ cubic inches.}$$

Trapezoid rule: $\frac{3320}{\pi}$ cubic inches. Use the average of the left and right hand rules.

21. $\int_0^5 2\pi r(0.115)e^{-2r}\,dr = 0.0575\pi\left(1-11e^{-10}\right)$. Approximately 190.55 cubic meters of soot.

22. $\int_0^5 \left(2+0.5\cosh x\right)dx = 10+0.5\sinh 5$ grams.

23. $\int_0^{25}\left(-\frac{8}{5}h+90\right)\pi(10)^2\,dh = 175{,}000\pi$ pounds.

24. a) $\int_0^{15} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ b) $\int_0^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

c) $\int_0^{10} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ d) $\int_3^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

25. $500\cdot 45+\int_0^{40} 3(40-x)dx = 24{,}900$ foot-pounds.

26. $\int_0^{10}\left(\frac{240}{0.5\cdot 24\cdot 10}\right)(10-h)\left(\frac{12}{5}h\right)dh = 800$ foot-pounds.

27. a) $a_n = \frac{(-1)^n 2n}{(n+2)^2}$ b) Converges to $\frac{-3}{5}$.

28. a) $\frac{(3/64)\left(1-(1/4)^8\right)}{1-1/4} = \frac{65535}{1048576}$ b) $\frac{3/4}{1-1/4} = 1$

29. $P_n = \frac{(0.05)(200)\left(1-0.05^{n-1}\right)}{1-0.05}$ $Q_n = \frac{(200)\left(1-0.05^n\right)}{1-0.05}$ $n = 1,2,3,\cdots$

30. a) The series converges. $\int_2^{\infty}\frac{1}{x(\ln x)^2}dx = \frac{1}{\ln 2}$.

b) The series diverges. $\int_1^\infty \frac{3x^2+2x}{\sqrt{x^3+x^2+1}}\,dx = \infty$.

31. a) The series diverges. $\lim_{n\to\infty} \frac{e(n)^2}{2(n+1)^2} = \frac{e}{2} > 1$.

b) The series converges. $\lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{4} < 1$

32. a) False b) True c) False d) True e) False

33. a) The radius of convergence is $R = 3$. The interval of convergence is $(-7, -1)$.
b) The radius of convergence is $R = \infty$. The interval of convergence is $(-\infty, \infty)$.
c) The radius of convergence is $R = 0$. The series only converges for $x = 1$.

34. a) True b) True c) Impossible to determine.

35. $P_2(x) = 3 + 2(x-2) + \frac{1}{3}(x-2)^2$

36. The sign of c_0 cannot be determined, $c_1 > 0$, $c_2 < 0$.

37. a) $f(3) = -1$. b) $f'(3) = \frac{1}{2}$. c) $f''(3) = -\frac{1}{6}$.

d) $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{k!}{(2k)!} 3^k (x-1)^k$ Substitute $3x$ into the series, then simplify.

38. $\frac{1}{12}$ Integrate term by term. The result is recognizable as the series for $\frac{x}{1-x}$.

39. a) *ii* b) *iv* c) *i* d) *iii*

40. a) $\sin 1$ b) $\ln(1.5)$ c) Series diverges because $\frac{\pi}{e} > 1$.

41. a) $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$

b) $-\frac{1}{11}$ Use a) to find the series for $\frac{\sin x}{x}$, consider the term containing x^{11}.

42. a) $x\ln(1+2x) = 2x^2 - \frac{4x^3}{2} + \frac{8x^4}{3} - \frac{16x^5}{4} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{n+2}}{n+1}$

b) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

43. $\frac{a}{(a+r)^2} = \frac{1}{a}\left(1+\frac{r}{a}\right)^{-2} = \frac{1}{a}\left(1 - 2\left(\frac{r}{a}\right) + 3\left(\frac{r}{a}\right)^2 - 4\left(\frac{r}{a}\right)^3 + \cdots\right)$

44. a) False b) False c) False

45. a) $e^{i\theta} = \cos\theta + i\sin\theta$ b) $\frac{\sqrt{3}}{2} - \frac{1}{2}i = e^{-(\pi/6)i}$ c) $2e^{\frac{-\pi}{4}i} = \sqrt{2} - i\sqrt{2}$

d) $e^{(3+4i)t} = e^{3t}\cos(4t) + i\cdot e^{3t}\sin(4t)$

46. a) *ii* b) *iii* c) *i* d) *iv*

47.

x	y	Δx	Δy
0.0	3.0	0.1	$(10(0)^2 + 3)(0.1) = 0.3$
0.1	3.3	0.1	$(10(0.1)^2 + 3.3)(0.1) = 0.34$
0.2	3.64	0.1	$(10(0.2)^2 + 3.64)(0.1) = 0.404$
0.3	4.044	0.1	$(10(0.3)^2 + 4.044)(0.1) = 0.4944$
0.4	4.5384		

$y(0.4) \approx 4.5384$

48. $y(x) = 2\tan\left(x^2 + \frac{\pi}{4}\right)$

49. a) $\frac{dQ}{dt} = -\alpha Q$ where $\alpha > 0$. b) $Q(t) = Ae^{-\alpha t}$. c) $t = \frac{7\ln(90)}{\ln(9/5)} \approx 53.59$ hours.

50. a) *ii*　　b) *iii*　　c) *i*　　d) *iv*　　e) *vi*　　f) *v*

51. a) *i*　　b) *ii*　　c) *iii*　　d) *iv*

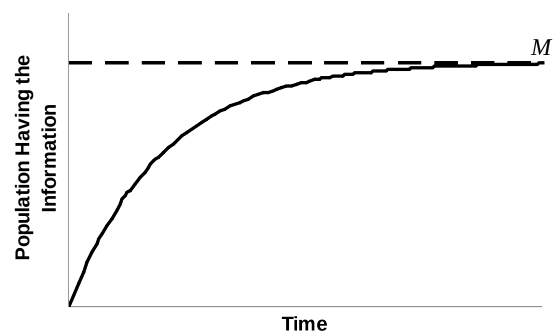
52. a) $\frac{dL}{dt} = 4 - 0.6L$,　$L(t) = \frac{Ae^{-0.6t} + 4}{0.6}$

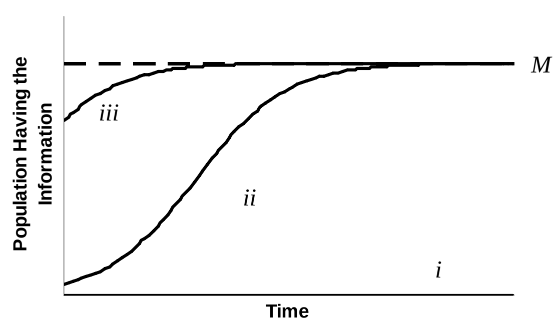
b) The stable equilibrium solution is $L = \frac{20}{3}$. If we start with 20/3 grams per square centimeter of leaves, we will always have that amount.

53. a) $r(t) = \frac{1}{15}t + 60$　　b) $\frac{dH}{dt} = k\left(H - \left(\frac{1}{15}t + 60\right)\right)$ $H(0) = 180$ where $k < 0$

54. $\frac{dA}{dt} = 0.06\sqrt{A}$,　$A(t) = (0.03t + c)^2$

55. a) $\frac{dP}{dt} = k(M - P)$ where $k > 0$.





b) $\frac{dP}{dt} = kP(M - P)$ where $k > 0$.

56. a) Decreasing. $\lim_{t \to \infty} P(t) = 15$.

b) $P = \frac{15}{2}$.