MTH 301
Hour Exam 1

Name: ______________________________ Date: ______________

11 Problems. 100 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 11 problems and 7 printed pages. Initial_____.

Print the name of the people sitting either side of you :- ____________________

Short Answer - minimum explanation and calculations necessary (5 points each).

1. Find a **unit** vector **perpendicular** to both $\vec{u} = 2\vec{i} + 2\vec{j}$ and $\vec{v} = 2\vec{j} + \vec{k}$.

2. Find the angle between $\vec{u} = 2\vec{i} + 2\vec{j}$ and $\vec{v} = 2\vec{j} + \vec{k}$.

3. Is the statement

   "There are just three unit vectors in 3-space, the vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$"

   true or false? Give a reason for your answer.

4. Write down inequalities which bound the region consisting of the hollow cylinder with inner radius 2 and outer radius 4 centered on the z-axis with base in the xy-plane and height 3.

5. Find a tangent vector to the curve with vector equation $\vec{r}(t) = t^2\vec{i} + (e^t - 1)\vec{j} + (2t + 1)\vec{k}$ at the point $(0, 0, 1)$.

6. Find a vector equation for the tangent line to the curve with vector equation $\vec{r}(t) = t^2\vec{i} + (e^t - 1)\vec{j} + (2t + 1)\vec{k}$ at the point $(0, 0, 1)$.

7. Find the center and radius of the sphere with equation

$$x^2 - 2x + y^2 + z^2 - 2z = 14.$$

8. Describe the graph of the equation $z = x^2$ in 3-space (sketch or describe in words).

Long Answer - show work and provide explanations, an answer without supporting work is not worth much (20 points each).

1. Let C be the curve with vector equation $\vec{r} = 3\sin(t)\vec{i} + 4t\vec{j} + 3\cos(t)\vec{k}$. Answer the following questions about C.

   (a) Find a tangent vector to C at the point $(3, 2\pi, 0)$.

   (b) Find an equation for the tangent line to C at the point $(3, 2\pi, 0)$.

   (c) Find an equation for the normal plane to C at the point $(3, 2\pi, 0)$ (Hint: The normal plane has the tangent vector as its normal vector).

2. Let $\vec{u} = a\vec{i} + b\vec{k} + b\vec{k}$ where a, b and c are constants.

   (a) Calculate $\vec{u} \cdot \vec{k}$.

   (b) Calculate $\vec{u} \times \vec{k}$.

   (c) Use your answers to find the vector $\vec{u}$ which satisfies **both** of the following two conditions simultaneously:

      i. $\vec{u} \cdot \vec{k} = 2$

      ii. $\vec{u} \times \vec{k} = 2\vec{i} + 3\vec{j}$.

3. Sketch and describe in words the following:

   (a) The region bounded by $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq \pi/2$, $0 \leq \rho \leq 1$ (spherical coordinates).

   (b) The graph of $r^2 = 1 - z^2$ (cylindrical coordinates).

   (c) The region bounded by $x^2 + z^2 = 2$ and $-2 \leq y \leq 2$.