

MTH 301
Hour Exam 2

Name: _____

Date: _____

11 Problems. 100 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 11 problems and 7 printed pages.
Initial_____.

Print the name of the people sitting either side of you :- _____

Short Answer - minimum explanation and calculations necessary (5 points each).

For the first four questions, let $f(x, y) = x^2y$

1. Calculate the partial derivatives f_x and f_y at the point $(1, 2)$.

2. Find the gradient vector ∇f at $(1, 2)$.

3. Find the rate of change of $f(x, y)$ at the point $(1, 2)$ in the direction of $\vec{u} = \vec{i} - \vec{j}$.

4. Find the linear approximation to $f(x, y)$ at the point $(1, 2)$.

5. Explain why the graph of $g(x, y) = \sqrt{x^2 + y^2}$ does not have a tangent plane at the point $(0, 0)$.

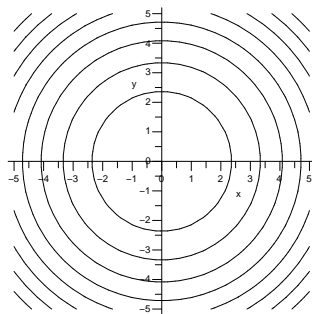
6. Does the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$$

exist? Give a reason for your answer.

7. Draw a tree diagram for $f(x, y)$ where x and y are both functions of a single variable t and use it to write down a formula for $\frac{df}{dt}$.

8. Describe in words or sketch the graph with the following contour diagram assuming the contours are equally spaced and getting larger as you move away from the origin.



Long Answer - show work and provide explanations, an answer without supporting work is not worth much (20 points each).

1. Find and classify the critical points of the function

$$f(x, y) = x^3 - 3x - y^2.$$

2. Use Lagrange multipliers to find the minimum and maximum values of the function $f(x, y) = xy$ subject to the constraint $x^2 + y^2 \leq 8$.

3. Let $f(x, y, z) = x^2 + y^2 + z^2$. Answer the following questions about $f(x, y, z)$.

(a) Describe the level set $f(x, y, z) = 1$.

(b) Find the gradient vector ∇f .

(c) Find the gradient vector at the point $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$.

(d) Use your answers to find an equation for the tangent plane to the sphere of radius 1 centered at the origin at the point $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$.