MTH 301
Hour Exam 3

Name: ______________________ Date: __________

11 Problems. 100 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

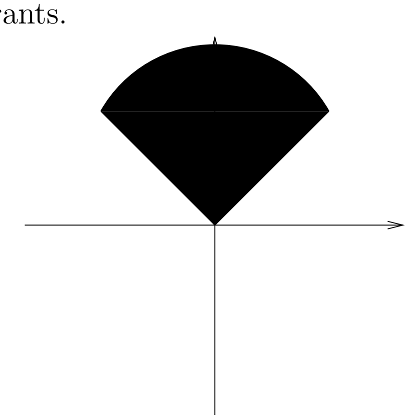
I have verified that this exam contains 11 problems and 11 printed pages. Initial_____.

Print the name of the people sitting either side of you :- ______________

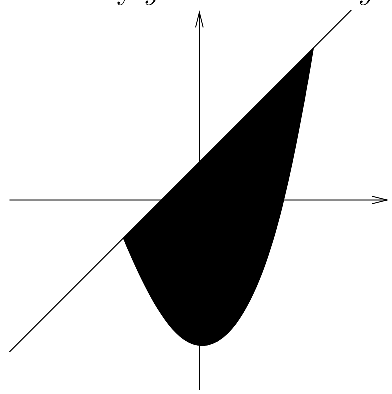
Short Answer - minimum explanation and calculations necessary (5 points each).

For the first four questions, **set up, but do not evaluate**, integrals for the function $f(x, y)$ or $f(x, y, z)$ over the described region. You may use any coordinate system you like, but you must convert the function f into the coordinate system you choose.

1. R is the region bounded between the lines $y = x$ and $y = -x$ and the circle $x^2 + y^2 = 2$ the circle of radius 2 centered at the origin in the upper two quadrants.



2. R is the region bounded by $y = x^2 - 1$ and $y = x + 1$.



3. R is the ice-cream cone region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 1$ and $f(x, y, z) = x^2 + y^2 + z^2$.

4. R is the quarter of the cylinder of radius 3, height 2 centered on the z-axis with base in the xy-plane in the first octant.

5. Is the vector field

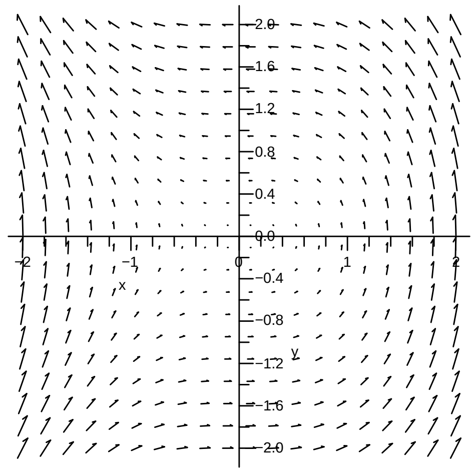
$$\vec{F} = (x^2 - y^2)\vec{i} - 2xy\vec{j}$$

conservative? Give a reason for your answer.

6. Calculate the divergence of the vector field

$$\vec{F} = x^2\vec{i} - (2xyz)\vec{j} + z^3x\vec{k}.$$

7. Is the following vector field conservative? Give a reason for your answer.



8. Suppose that $\vec{F} = x\vec{j}$. Explain why

$$\int_C \vec{F} \cdot d\vec{r} = 0$$

for any horizontal line segment C in the xy-plane (Hint: Sketch $\vec{F}$).

Long Answer - show work and provide explanations, an answer without supporting work is not worth much (20 points each).

NOTE: Long answer questions 2 and 3 are on **three** pages each, (though you won't necessarily need all three pages!).

1. Let $\vec{F} = x^3\vec{i} + (x + \sin^3(y))\vec{j}$.

   (a) Find the line integral
   $$\int_{C_1} \vec{F} \cdot d\vec{r}$$
   where C_1 is the line segment from $(0, 0)$ to $(\pi, 0)$.

(b) Use Green's Theorem to evaluate the integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the line segment from $(0, 0)$ to $(\pi, 0)$ followed by path from $(0, 0)$ to $(0, \pi)$ along the curve $y = \sin(x)$.

(c) Use the results of the last two problems to find

$$\int_{C_2} \vec{F} \cdot d\vec{r}$$

where C_2 is the path from $(0,0)$ to $(0,\pi)$ along the curve $y = \sin(x)$.

2. Calculate the following line integrals (for direct calculations, you need to show all your work, else you need to say what result you are using and why you can use it).

   (a)
   $$\int_C (z\vec{i} + z\vec{j} + (x+y)\vec{k}) \cdot d\vec{r}$$
   where C is the circle parameterized by
   $$\vec{r}(t) = \cos(t)\vec{i} + 2\vec{j} + \sin(t)\vec{k}$$
   with $0 \leq t \leq 2\pi$.

(b)

$$\int_C (3x\vec{i} + 5y\vec{j}) \cdot d\vec{r}$$

where C is the path consisting of the two line segments, the first from $(0, 4)$ to $(0, 1)$ and the second from $(0, 1)$ to $(3, 1)$.

(c)

$$\int_C ((\cos(x) + e^{x^2} + 3y)\vec{i} + (e^{\sin(y)} + x + \cos(y^3))\vec{j}) \cdot d\vec{r}$$

where C is the circle of radius $\sqrt{2}$ centered at $(0,0)$ oriented counterclockwise.

3. Rewrite the integral

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{18-x^2-y^2} \sqrt{x^2+y^2}\,dz\,dy\,dx$$

in cylindrical or spherical coordinates and use it to evauluate it.