MTH 301
Final Exam
Section A

Name: _____ Date: _____

14 Problems. 200 Points. Follow directions carefully, and show your work. Please do not leave any question blank, and turn off cell phones and other noisemakers to avoid disturbing your classmates.

I have verified that this exam contains 14 problems and 11 printed pages. Initial____.

Print the name of the people sitting either side of you :- _____

Short Answer - minimum explanation and calculations necessary (10 points each).

For the first five questions, let $f(x, y, z) = x^2y$.

1. Find the gradient vector ∇f of $f(x, y, z)$ at the point $(1, 1, -1)$.

2. Find the directional derivative $f(x, y, z)$ at the point $(1, 1, -1)$ in the direction of $\vec{u} = 4\vec{i} + 3\vec{k}$.

3. Calculate

$$\int_C \nabla f \cdot d\vec{r}$$

where C is the line segment from $(2, 1, 0)$ to $(1, 2, 3)$.

4. If $x = e^{-t}$, $y = t^2 - 1$ and $z = \ln(t)$, calculate

$$\frac{df}{dt}$$

using the chain rule.

5. Calculate

$$\int\int\int_B f(x,y,z)dV$$

where B is the box $[0,1] \times [0,1] \times [0,1]$.

6. Is the vector field

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$$

a curl field? Give a reason for your answer.

7. Evaluate

$$\int\int_D (x^2 + y^2) dA$$

where D is the unit disc centered at the origin.

8. Is the vector field

$$\vec{F} = e^z \vec{i} + \vec{j} + xe^z \vec{k}$$

conservative? Give a reason for your answer.

9. Write down parametric equations for the line segment from $(\pi, 0)$ to $(0, 0)$.

10. Sketch the contour $f(x, y) = 1$ where

$$f(x, y) = \frac{y}{x}.$$

Long Answer - show work and provide explanations, an answer without supporting work is not worth much (25 points each).

1. Find the equation of the tangent plane to $z = y\ln(x)$ at the point $(1, 4, 0)$.

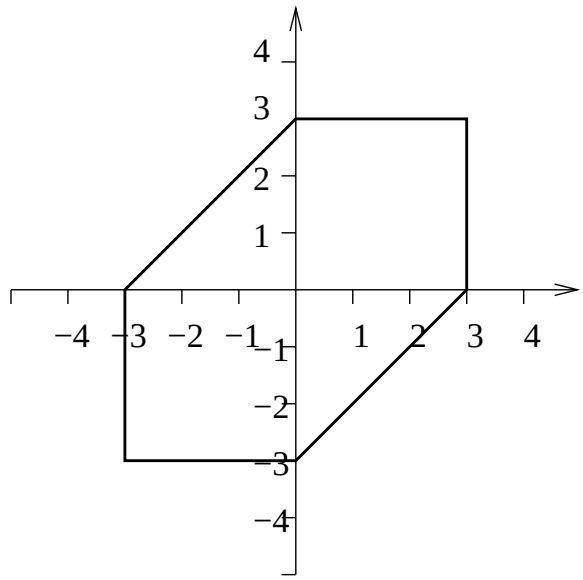
2. Classify the critical points of the function

$$f(x, y) = x + y + \frac{9}{x} + \frac{1}{y}.$$

3. Evaluate

$$\int_C (-y\vec{i} + x\vec{j}) \cdot d\vec{r}$$

where C is the closed curve illustrated below oriented clockwise.



4. (Note that this question continues on the next two pages) Let

$$\vec{F} = -3xe^z\vec{i} - y(3z-2)e^z\vec{j} + (3z-2)e^z\vec{k},$$

let S_1 be the part of the parabolioid $z = 4 - x^2 - y^2$ oriented outward and let S_2 be the disk $x^2 + y^2 \leq 4$ in the xy-plane oriented downward and let S_3 be the closed surface consisting of S_1 and S_2 stuck together.

(a) Calculate

$$\int\int_{S_2} \vec{F} \cdot d\vec{S}$$

(be careful with the orientation).

(b) Calculate

$$\int\int_{S_3} \vec{F} \cdot d\vec{S}$$

(c) Use your answers to part (a) and (b) to evaluate

$$\int\int_{S_1} \vec{F} \cdot d\vec{S}$$