Select and do any 4 of the 5 problems assigned during class time. The fifth problem is a take-home problem due class time on Friday, April 1, 2011.
(1) (20 points). DT LTI system. The unit impulse of a DT LTI system is given below. Find and sketch the zero-state response \( y[n] \) of this system due to input signal \( x[n] = 2\delta[n] - 3\delta[n - 2] \).

\[
h[n] = u[n] + 3\delta[n - 2] - u[n - 3]
\]

\[
\]

\[
= 2\delta[n] + 2\delta[n - 1] + 5\delta[n - 2] - 3\delta[n - 3] - 12\delta[n - 4]
\]
(2) (Total: 15 points) **LTI system.** For each of the following unit impulse responses corresponding to an LTI system, determine whether the system is (a) Memoryless, (b) Causal, and (c) BIBO stable. Provide a clear justification for each answer.

\[ x[n] \rightarrow \text{LTI system} \rightarrow y[n] \]

\[ h[n] \text{ or } h(t) \]

(a) (7.5 points) \( h[n] = (1 - n)u[n - 1] \)

\( h(n) \neq K \delta[n] \).
(b) Causal since \( h[n] = 0 \) for \( n < 0 \).
(c) Unstable since \( \sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=1}^{\infty} (k-1) \rightarrow \infty \)

(b) (7.5 points) \( h(t) = 2\sin(\pi t)u(t - 1) \)

\( h(t) \neq K \delta(t) \).
(b) Causal since \( h(t) = 0 \) for \( t < 0 \).
(c) Unstable since \( \int_{-\infty}^{\infty} |h(t)| \, dt = \int_{1}^{\infty} 2|\sin(\pi t)| \, dt \rightarrow \infty \)
(3) (Total: 25 points) **Electric circuits.** For the electric circuit shown:

(a) (10 points) Find its transfer function $H(s)$. Provide your work step by step.

\[
H(s) = \frac{Y(s)}{X(s)} = Z_{eq}(s) = \frac{5 \{(10s + 15)\}}{5 + 10s + 15} = \frac{5}{2} \left( \frac{2s + 3}{s + 2} \right)
\]

(b) (10 points) Find the impulse response $h(t)$. Show your work.

\[
h(t) = \int_{-\infty}^{\infty} H(s) \, ds = \left[ \delta(t) - \frac{5}{2} e^{-2t} u(t) \right]
\]

(c) (5 points) Find the unit step response $y_s(t)$. Provide your work.

\[
y_s(t) = \int_{-\infty}^{t} h(\tau) \, d\tau = \begin{cases} 
0, & t < 0 \\
\frac{15}{4} + \frac{5}{4} e^{-2t}, & t > 0
\end{cases}
\]

\[
= \left( \frac{15}{4} + \frac{5}{4} e^{-2t} \right) u(t)
\]
(4) (Total: 20 points) LTI system. Consider an LTI system.

\[
\begin{array}{c}
x[n] \xrightarrow{\text{LTI system}} y[n] \\
x(t) \xrightarrow{\text{LTI system}} y(t)
\end{array}
\]

\(h[n] \text{ or } h(t)\)

(a) (10 points) If the system is continuous time, given the unit-step response of the system to be

\[y_s(t) = (5te^{-t} + 11e^{-t} - 3)\delta(t)\]

Find the unit-impulse response \(h(t)\) of the system. What is \(h(2)\)?

\[
h(t) = \frac{dy_s(t)}{dt} = (5e^{-t} - 5te^{-t} - 11e^{-t}) u(t) + (5te^{-t} + 11e^{-t} - 3) \delta(t) \]

\[
= - (5t + 6)e^{-t}u(t) + 8\delta(t)
\]

\[
h(2) = -10e^{-2} - 6e^{-2} = -\frac{16}{e^2}
\]
(b) (10 points) If the system is discrete time, given the impulse response of the system to be

\[ h[n] = \left( \frac{1}{2} \right)^n u[n] \]

Find the unit-step response \( y_s[n] \) of the system. What is \( y_s[2] \)?

\[
y_s[n] = \sum_{k=0}^{\infty} h[n-k] = \sum_{k=0}^{\infty} \left( \frac{1}{2} \right)^{n-k} u[n-k] = \sum_{k=0}^{\infty} \left( \frac{1}{2} \right)^{n-k} u[n-k]
\]

\[
= \left( \frac{1}{2} \right)^n u[n] + \left( \frac{1}{2} \right)^{n-1} u[n-1] + \left( \frac{1}{2} \right)^{n-2} u[n-2] + \cdots + \left( \frac{1}{2} \right)^{n-3} u[n-3] + \cdots
\]

\[
y_s[2] = \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^1 + \left( \frac{1}{2} \right)^0 = \frac{1}{4} + \frac{1}{2} + 1 = \frac{7}{4}
\]
(Total: 20 points) **Electric circuits.** In the circuit shown, assuming \(v_c(0^+) = -3 \text{ V}\) and \(y(0^-) = 0.5 \text{ A}\), do the following:

\[
\begin{array}{c}
\text{+} \\
\text{---} \\
\text{+} \\
\end{array}
\begin{array}{c}
v_c\downarrow \\
0.4 \text{ F} \\
0.5 \text{ H} \\
\text{---} \\
3 \Omega \\
\text{+} \\
\end{array}
\begin{array}{c}
x(t) = u(t) \\
\text{---} \\
y(t) \uparrow \\
\end{array}
\]

(a) (10) Draw the Laplace domain circuit with all the numerical values provided.

\[
\begin{array}{c}
\text{+} \\
\text{---} \\
\text{+} \\
\text{---} \\
\text{+} \\
\end{array}
\begin{array}{c}
x(s) = \frac{1}{s} \\
\text{---} \\
Y(s) \\
\text{---} \\
\end{array}
\]

(b) (10) Using part (a) circuit, obtain \(Y(s)\) and perform inverse Laplace on it to obtain \(y(t)\) for \(t > 0\).

\[
Y(s) = \frac{\frac{4}{5} + \frac{3}{5} + \frac{1}{4}}{\frac{5}{2s} + \frac{5}{2} + 3} = \frac{s + 16}{4s} \cdot \frac{2s}{s^2 + 6s + 5}
\]

\[
\frac{(s+1)Y(s)}{s=1} = \frac{s+16}{2(s+4)(s+5)} = \frac{K_1}{s+1} + \frac{K_2}{s+5}
\]

\[
K_1 = \left. \frac{s+16}{2(s+5)} \right|_{s=-1} = \frac{15}{8}
\]

\[
K_2 = \left. \frac{s+16}{2(s+4)} \right|_{s=-5} = -\frac{41}{8}
\]

\[
y(t) = \frac{15}{8} e^{-t} u(t) - \frac{41}{8} e^{-5t} u(t)
\]