several operations in one stroke at one die station. A progressive die performs several operations, one per stroke, at one die station (more than one stroke is necessary). A transfer die performs one operation at one die station.

7.12 It has been stated that the quality of the sheared edges can influence the formability of sheet metals. Explain why.

In many cases, sheared edges are subjected to subsequent forming operations, such as bending, stretching, and stretch flanging. As stated in Section 7.3 starting on p. 351, rough edges will act as stress raisers and cold-worked edges (see Fig. 7.6b on p. 353) may not have sufficient ductility to undergo severe tensile strains developed during these subsequent operations.

7.13 Explain why and how various factors influence springback in bending of sheet metals.

Plastic deformation (such as in bending processes) is unavoidably followed by elastic recovery, since the material has a finite elastic modulus (see Fig. 2.3 on p. 33). For a given elastic modulus, a higher yield stress results in a greater springback because the elastic recovery strain is greater. A higher elastic modulus with a given yield stress will result in less elastic strain, thus less springback. Equation (7.10) on p. 364 gives the relation between radius and thickness. Thus, increasing bend radius increases springback, and increasing the sheet thickness reduces the springback.

7.14 Does the hardness of a sheet metal have an effect on its springback in bending? Explain.

Recall from Section 2.6.8 on p. 54 that hardness is related to strength, such as yield stress as shown in Fig. 2.24 on p. 55. Referring to Eq. (7.10) on p. 364, also note that the yield stress, \( Y \), has a significant effect on springback. Consequently, hardness is related to springback. Note that hardness does not affect the elastic modulus, \( E' \), given in the equation.

7.15 As noted in Fig. 7.16, the state of stress shifts from plane stress to plane strain as the ratio of length-of-bend to sheet thickness increases. Explain why.

This situation is somewhat similar to rolling of sheet metal where the wider the sheet, the closer it becomes to the plane-strain condition. In bending, a short length in the bend area has very little constraint from the unbent regions, hence the situation is one of basically plane stress. On the other hand, the greater the length, the more the constraint, thus eventually approaching the state of plane strain.

7.16 Describe the material properties that have an effect on the relative position of the curves shown in Fig. 7.19.

Observing curves (a) and (c) in Fig. 7.19 on p. 364, note that the former is annealed and the latter is heat treated. Since these are all aluminum alloys and, thus, have the same elastic modulus, the difference in their springback is directly attributable to the difference in their yield stress. Likewise, comparing curves (b), (d), and (e), note that they are all stainless steels and, thus, have basically the same elastic modulus. However, as the amount of cold work increases (from annealed to half-hard condition), the yield stress increases significantly because austenitic stainless steels have a high \( n \) value (see Table 2.3 on p. 37). Note that these comparisons are based on the same \( R/T \) ratio.

7.17 In Table 7.2, we note that hard materials have higher \( R/T \) ratios than soft ones. Explain why.

This is a matter of the ductility of the material, particularly the reduction in area, as depicted by Eqs. (7.6) on p. 361 and (7.7) on p. 362. Thus, hard material conditions mean lower tensile reduction and, therefore, higher \( R/T \) ratios. In other words, for a constant sheet thickness, \( T \), the bend radius, \( R \), has to be larger for higher bendability.

7.18 Why do tubes have a tendency to buckle when bent? Experiment with a straight soda straw, and describe your observations.

Recall that, in bending of any section, one-half of the cross section is under tensile stresses and the other half under compressive stresses. Also, compressing a column tends to buckle it, depending on its slenderness. Bending of a tube subjects it to the same state of stress, and since
to be 8°. Therefore the exponent of the power curve is \( \tan 8° = 0.14 \). Furthermore, it can be seen that, for \( R = 1.0 \), we have LDR = 2.3. Therefore, the expression for the LDR as a function of the average strain ratio \( R \) is given by

\[
LDR = 2.3 R^{0.14}
\]

7.79 A steel sheet has \( R \) values of 1.0, 1.5, and 2.0 for the 0°, 45° and 90° directions to rolling, respectively. If a round blank is 150 mm in diameter, estimate the smallest cup diameter to which it can be drawn in one draw.

Substituting these values into Eq. (7.20) on p. 391, we have

\[
R = \frac{1.0 + 2(1.5) + 2.0}{4} = 1.5
\]

The limiting-drawing ratio can be obtained from Fig. 7.56 on p. 392, or it can be obtained from the expression given in the solution to Problem 7.78 as

\[
LDR = 2.3 R^{0.14} = 2.43
\]

Thus, the smallest diameter to which this material can be drawn is \( 150/2.43 = 61.7 \) mm.

7.80 In Problem 7.79, explain whether ears will form and, if so, why.

Equation (7.21) on p. 392 yields

\[
\Delta R = \frac{R_0 - 2R_{45} + R_{90}}{2} = \frac{1.0 - 2(1.5) + 2.0}{2} = 0
\]

Since \( \Delta R = 0 \), no ears will form.

7.81 A 1-mm-thick isotropic sheet metal is inscribed with a circle 4 mm in diameter. The sheet is then stretched uniaxially by 25%. Calculate (a) the final dimensions of the circle and (b) the thickness of the sheet at this location.

Referring to Fig. 7.63b on p. 399 and noting that this is a case of uniaxial stretching, the circle will acquire the shape of an ellipse with a positive major strain and negative minor strain (due to the Poisson effect). The major axis of the ellipse will have undergone an engineering strain of \( (1.25 - 1)/1 = 0.25 \), and will thus have the dimension \((4)(1+0.25) = 5 \) mm. Because we have plastic deformation and hence the Poisson’s ratio is \( \nu = 0.5 \), the minor engineering strain is \(-0.25/2 = -0.125\); see also the simple-tension line with a negative slope in Fig. 7.63a on p. 399. Thus, the minor axis will have the dimension

\[
\frac{x - 4 \text{ mm}}{4 \text{ mm}} = -0.125
\]

or \( x = 3.5 \) mm. Since the metal is isotropic, its final thickness will be

\[
\frac{t - 1 \text{ mm}}{1 \text{ mm}} = 0 - 0.125
\]

or \( t = 0.875 \) mm. The area of the ellipse will be

\[
A = \pi \left( \frac{5 \text{ mm}}{2} \right) \left( \frac{3.5 \text{ mm}}{2} \right) = 13.7 \text{ mm}^2
\]

The volume of the original circle is

\[
V = \frac{\pi}{4} (4 \text{ mm})^2 (1 \text{ mm}) = 12.6 \text{ mm}^3
\]

7.82 Conduct a literature search and obtain the equation for a tractrix curve, as used in Fig. 7.61.

The coordinate system is shown in the accompanying figure.

\[
\begin{align*}
x &= a \ln \left( \frac{a + \sqrt{a^2 - y^2}}{y} \right) - \sqrt{a^2 - y^2} \\
&= a \cosh^{-1} \left( \frac{a}{y} \right) - \sqrt{a^2 - y^2}
\end{align*}
\]

where \( x \) is the position along the direction of punch travel, and \( y \) is the radial distance of the surface from the centerline.
or $u = 127,000 \text{ in-lb/in}^3$. Therefore,

$$F_t = (127,000)(0.5)(0.1)(\sin 30^\circ) = 3190 \text{ lb}$$

and the maximum torque required is at the 15 in. diameter, hence

$$T = (3190 \text{ lb}) \left( \frac{12 \text{ in.}}{2} \right) = 19,140 \text{ in-lb}$$

or $T = 1590 \text{ ft-lb}$. Thus the maximum power required is

$$P_{\text{max}} = \frac{T\omega}{(100 \text{ rev/min}) \times 2 \pi \text{ rad/rev}}$$

$$= \frac{12.03 \times 10^5 \text{ in-lb/min}}{\pi}$$

or 30.3 hp. As stated in the text, because of redundant work and friction, the actual power may be as much as 50% higher, or up to 45 hp.

7.86 Obtain an aluminum beverage can and cut it in half lengthwise with a pair of tin snips. Using a micrometer, measure the thickness of the bottom of the can and of the wall. Estimate (a) the thickness reductions in ironing of the wall and (b) the original blank diameter.

Note that results will vary depending on the specific can design. In one example, results for an average can diameter of 2.6 in. and a height of 5 in., the sidewall is 0.003 in. and the bottom is 0.0120 in. thick. The wall thickness reduction in ironing is then

$$\%\text{red} = \frac{t_o - t_f}{t_o} \times 100\%$$

$$= \frac{0.0120 - 0.003}{0.012} \times 100\%$$

$$= 75\%$$

The initial blank diameter can be obtained by volume constancy. The volume of the can material after deep drawing and ironing is

$$V_f = \frac{\pi d_o^2 t_o}{4} + \pi d_{w,h} h$$

(a) A repeating unit cell for the part the upper illustration is shown below.

Since the initial blank has a thickness equal to the final can bottom (i.e., 0.0120 in.) and a diameter $d$, the volume is

$$0.1767 \text{ in}^3 = \frac{\pi d^2 t_o}{4} = \frac{\pi d^2}{4} \left( \frac{0.012}{4} \right)$$

or $d = 4.33$ in.

7.87 What is the force required to punch a square hole, 150 mm on each side, from a 1-mm-thick 5052-O aluminum sheet, using flat dies? What would be your answer if beveled dies were used instead?

This problem is very similar to Problem 7.71. The punch force is given by Eq. (7.4) on p. 353. Table 3.7 on p. 116 gives the UTS of 5052-O aluminum as UTS = 190 MPa. The sheet thickness is $t = 1.0 \text{ mm} = 0.001 \text{ m}$, and $L = (4)(150 \text{ mm}) = 600 \text{ mm} = 0.60 \text{ m}$. Therefore, from Eq. (7.4) on p. 353,

$$P_{\text{max}} = 0.7(\text{UTS})(t)(L)$$

$$= 0.7(190 \text{ MPa})(0.001 \text{ m})(0.60 \text{ m})$$

$$= 79,800 \text{ N} = 79.8 \text{ kN}$$

If the dies are beveled, the punch force could be much lower than calculated here. For a single bevel with contact along one face, the force would be calculated as 19,950 N, but for double-beveled shears, the force could be essentially zero.

7.88 Estimate the percent scrap in producing round blanks if the clearance between blanks is one tenth of the radius of the blank. Consider single and multiple-row blanking, as shown in the accompanying figure.
The area of the unit cell is \( A = (2.2R)(2.1R) = 4.62R^2 \). The area of the circle is \( 3.14R^2 \). Therefore, the scrap is

\[
\text{scrap} = \frac{4.62R^2 - 3.14R^2}{4.62R^2} \times 100 = 32\%
\]

(b) Using the same approach, it can be shown that for the lower illustration the scrap is 26%.

7.89 Plot the final bend radius as a function of initial bend radius in bending for (a) 5052-O aluminum; (b) 5052-H34 Aluminum; (c) C24000 brass and (d) AISI 304 stainless steel sheet.

The final bend radius can be determined from Eq. (7.10) on p. 364. Solving this equation for \( R_f \) gives:

\[
R_f = \frac{R_i}{4 \left( \frac{H_i Y}{Et} \right)^{\frac{3}{2}}} - 3 \left( \frac{H_i Y}{Et} \right) + 1
\]

Using Tables 2.1 on p. 32, 3.4, 3.7, and 3.10, the following data is compiled:

<table>
<thead>
<tr>
<th>Material</th>
<th>( Y ) (MPa)</th>
<th>( E ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5052-O Al</td>
<td>90</td>
<td>73</td>
</tr>
<tr>
<td>5052-H34</td>
<td>210</td>
<td>73</td>
</tr>
<tr>
<td>C24000 Brass</td>
<td>265</td>
<td>127</td>
</tr>
<tr>
<td>AISI 304 SS</td>
<td>265</td>
<td>195</td>
</tr>
</tbody>
</table>

where mean values of \( Y \) and \( E \) have been assigned. From this data, the following plot is obtained. Note that the axes have been defined so that the value of \( t \) is not required.

7.90 The accompanying figure shows a parabolic profile that will define the mandrel shape in a spinning operation. Determine the equation of the parabolic surface. If a spun part is to be produced from a 10-mm thick blank, determine the minimum blank diameter required. Assume that the diameter of the profile is 6 in. at a distance of 3 in. from the open end.

Since the shape is parabolic, it is given by

\[
y = ax^2 + bx + c
\]

where the following boundary conditions can be used to evaluate constant coefficients \( a \), \( b \), and \( c \):

(a) at \( x = 0 \), \( \frac{dy}{dx} = 0 \).
(b) at \( x = 3 \) in., \( y = 1 \) in.
(c) at \( x = 6 \) in., \( y = 4 \) in.

The first boundary condition gives:

\[
\frac{dy}{dx} = 2ax + b
\]

Therefore,

\[
0 = 2a(0) + b
\]

or \( b = 0 \). Similarly, the second and third boundary conditions result in two simultaneous algebraic equations:

\[
36a + c = 4
\]