Fatigue Loading - Note, fatigue is now in "Stress Analysis" section of FE, not "Mechanical Design".

\[ \sigma_a = \frac{1}{2} (\sigma_{\text{max}} + \sigma_{\text{min}}) \]

\[ \sigma_m = \frac{1}{2} (\sigma_{\text{max}} - \sigma_{\text{min}}) \]

For \( \sigma_m = 0 \),

For steel,

- \( S_e \),
- \( \log(S) \),
- \( 10^3 \), \( 10^6 \)

For non-ferrous,

- \( S \),
- \( N \)

linear relation between \( S \) (stress amplitude) and \( N \) (number of cycles until failure).
FE REVIEW: MACHINE DESIGN

PROBLEMS 1 & 2: ref MECHANICAL ENGINEERING

VARIABLE LOADING THEORIES (in FE book)

1. Given: Yield strength, \( S_y = 75 \text{ ksi} \)
   Tensile strength, \( S_{ut} = 100 \text{ ksi} \)
   Endurance limit strength, \( S_e = 35 \text{ ksi} \)
   Stress amplitude, \( \sigma_a = 20 \text{ ksi} \)

   Find: max mean stress, \( \sigma_m \), for infinite life

Solution: use Soderberg Theory:

\[
\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}
\]

\( n \) = factor of safety, fatigue

Let \( n = 1 \)

\[
\frac{20\text{ ksi}}{35\text{ ksi}} + \frac{\sigma_m}{75\text{ ksi}} = \frac{1}{1} \Rightarrow \sigma_m = 32.1 \text{ ksi}
\]
2. Given: Cold drawn steel with:
   \[ S_{tu} = 100 \text{ksi} = 690 \text{MPa} \]
   \[ S_y = 75 \text{ksi} = 518 \text{MPa} \]
   The steel bar is 25mm in diameter
   Load: cyclic bending
   Temperature: 200°C
   Find: Modified endurance limit, \( S_e' \)

Solution:
\[
S_e' = \begin{cases} 
\frac{1}{2} S_{tu} & \text{if } S_{tu} < 200 \text{ksi} (1400 \text{MPa}) \\
100 \text{ ksi} (700 \text{MPa}) & \text{if } S_{tu} > 200 \text{ksi}
\end{cases}
\]
\[
S_{tu} = 690 \text{ MPa}, \quad \therefore \quad S_e' = \frac{1}{2} S_{tu}
\]
\[
S_e = k_a k_b k_c k_d k_e S_e'
\]

Determine modifying factors, \( k \)

Surface factor, \( k_a \)
\[
k_a = a S_{tu}^b
\]

\( a \) & \( b \) are based on surface condition and are given in a table in the FE book.

For cold drawn \( a = 4.51, b = -0.265 \)

be sure to use appropriate units

(if MPa, don't use Pa, etc).

\[
\frac{4.50 (690 \text{MPa})}{-0.265} = \frac{-0.265}{4.50 (690 \times 10^6 \text{Pa})} = \text{no!}
\]
2. cont.

\[ k_a = 0.8 \]

Size factor, \( k_b \) (bending)

\[ k_b = 1.189(d)^{-0.097} \]

if \( 8 \text{mm} \leq d \leq 250 \text{mm} \)

\[ d = 25 \text{mm} \quad (\text{given}) \]

\[ k_b = 1.189(25 \text{mm})^{-0.097} \quad \text{NOT} \quad 1.189(0.025 \text{m})^{-0.097} \]

\[ k_b = 0.87 \]

Load factor, \( k_c \)

\[ k_c = 1 \quad \text{for bending} \]

Temp factor, \( k_d \)

\[ k_d = 1 \quad \text{if} \ T < 450^\circ C \]

Misc effects factor, \( k_e \)

\[ k_e = 1 \quad (\text{unless} \ k_e \text{ is given, assume it is} 1) \]

\[ S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_{e'} \]

\[ = (0.8)(0.87)(1)(1)(1) \left\{ \frac{1}{2} S_{ut} \right\} \quad S_{ut} = 690 \text{MPa} \]

\[ S_e = 240 \text{ MPa} \]
3. See in FE book: MECHANICAL ENGINEERING, Power Transmission, Fatigue Loading

3. Determine the diameter of a solid steel rotating shaft with the following loading:

\[ \begin{align*}
\downarrow F & \quad \uparrow T \\
\uparrow R_A & \quad \downarrow R_B
\end{align*} \]

As the shaft rotates, the material experiences reverse bending stress (tension on bottom, becomes compression as it rotates to the top). The mean bending stress, therefore, is zero. The torsion load is constant; therefore, the torque amplitude is zero. The magnitudes of the loading are:

Given: \( M_a = 2000 \text{ N-m} \) (amplitude of bending moment)  
\( M_m = 0 \) (as explained above)  
\( T_a = 0 \) (" " " )  
\( T_m = 1000 \text{ N-m} \) (given)

Factor of safety, \( n = 2 \)
Stress concentration factors given:

\[ K_f = 1.87 \] (fatigue stress concentration factor for bending & axial loads in this example)
\[ K_{fs} = 1.60 \] (stress conc. factor for torque or shear fatigue loading)

Material properties given as:

Yield stress, \( \sigma_y = 500 \) MPa
Tensile, \( \sigma_t = 700 \) MPa
Fatigue endurance limit, \( \sigma_e = 140 \) MPa

Equation in FE book for shaft fatigue loading:

\[
\frac{\pi d^3}{32} = n \left[ \left( \frac{M_{\text{max}}}{\sigma_y} + \frac{K_f M_a}{\sigma_e} \right)^2 + \left( \frac{T_{\text{max}}}{\sigma_y} + \frac{K_{fs} T_a}{\sigma_e} \right)^2 \right]^{\frac{1}{2}} 
\]

\[
= Z \left[ \left( \frac{0}{\sigma_y} + \frac{(1.87)(2000 \text{ N}\cdot\text{m})}{140 \text{ MPa}} \right) + \left( \frac{1000 \text{ N}\cdot\text{m}}{500 \text{ MPa}} + \frac{(1.60)(0)}{\sigma_e} \right) \right]^{\frac{1}{2}}
\]
\[
\frac{\pi d^3}{32} = 2.0 \left[ (0 + 2.67 \times 10^{-5} \text{ m}^3)^2 + (2 \times 10^{-6} \text{ m}^3 + 0)^2 \right]^{1/2}
\]

\[
\frac{\pi d^3}{32} = 2.67 \times 10^{-5} \text{ m}^3
\]

\[
d = 0.065 \text{ m} = 65 \text{ mm}
\]
Bolted joints

Tension, bolt generally carry small (~15%) of ex. load.

Determine relative stiffness of B and M

Basic:

\[ A_d = \text{nominal cross-sectional area} \]
\[ A_t = \text{tensile stress area} \quad A_t < A_d \]

SAE Grade \rightarrow \text{strength ISO Class}

↑ Grade    ↑ Strength
4. Given: $\frac{1}{2}$-13 UNC Grade 5, bolt

2 1/2" long
Clamping 2" of steel
Tighten to 75\% of Proof
External tension load of 5000 lb

Find! Factors of safety against joint separation.

Proof Strength? It is effectively the "proportional limit" - the maximum stress before $\frac{\sigma}{E}$ becomes non-linear.

Proof strength

$\epsilon = 0.002$ in/in (0.2\% strain)

$\sigma_{ys}$ - determined using 0.2\% offset method
For a Grade 5 bolt: Proof Strength = 85 ksi
Tensile Strength = 120 ksi
Yield Strength = 92 ksi

\[ \frac{1}{2} - 13 \text{ UNC} \]

- Coarse threads (F = fine)
- Unified National Standard (aka "English")
- Number of threads per inch
- Nominal diameter, (this is a \( \frac{1}{2} \) inch bolt)

\[ A_2 = 0.1419 \text{ in}^2 \text{ for } \frac{1}{2} - 13 \text{ UNC} \]

\( A_2 \) is the "tensile stress area"—effectively, it is the cross-sectional area in the threaded region. It is "given".

\[ A_d = \frac{\pi}{4} d^2 \] - this is the nominal cross-section (through non-threaded shank).

\[ A_d = \frac{\pi}{4} \left( \frac{1}{2} \text{ in} \right)^2 = 0.1963 \text{ in}^2 \]

Total thread length = 1.25" (given)

\[ L_d \]

\[ L_{\text{thread}} = 1.25" \]

\[ L_g = \text{"Grip"} \]
Solution

Several things need to be calculated.

Factor of safety for joint separation, $n_s$

$$n_s = \frac{F_i}{P(1-c)}$$

($n_s$ is what this problem has requested to be determined)

$P =$ external tension load

$P = 5000 \text{ lb (given)}$

$F_i =$ preload (tension in bolt after being tightened)

$C =$ fraction of external load carried through the bolt

-Determine preload force, $F_i$

$$F_i = (% \text{ Proof })( \text{ Press strength })(A_t)$$

$% \text{ Proof} = 75\% = 0.75 \ (\text{given})$

$\text{Proof strength} = 85 \text{ksi} \ (\text{for Grade 5})$

$A_t = 0.149 \text{ in}^2 \ (\text{for } \frac{1}{2}-13\text{ UNC})$
\( F_i = 0.75 (85 \text{ ksi}) (0.1419 \text{ in}^2) = 9050 \text{ lb} \)

Determine \( C \) ("joint coefficient")

\[ C = \frac{k_b}{k_b + k_m} \]

\( k_b \) and \( k_m \) are the stiffnesses of the bolt and clamped members, respectively.

\[ R_m = dEAe^{b(d/l)} \quad \text{(eqn in FE book)} \]

\( A \) and \( b \) are parameters based on material being clamped.

For steel, the FE book gives:

\[ A = 0.78715 \quad (5 \text{ is fig. } 7.29) \]
\[ b = 0.62873 \]
\[ d = \text{diameter } (\frac{1}{8} \text{ in}) \]
\[ l = \text{grip length } (2 \text{ in}) \]
\[ E = \text{Young's modulus } (30 \times 10^6 \text{ psi}) \]

\[ R_m = (0.5 \text{ in})(30 \times 10^6)(0.62873)(0.78715)e^{0.62873(0.5 \text{ in})/2 \text{ in}} \]

\[ R_m = 17.55 \times 10^6 \text{ lb/in} \]
$R_b$ - Bolt stiffness. There are two common methods. The FE book uses the more complicated method, which is a bit more precise. Both methods are presented here.

FE Book: 

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} \quad (2 \text{ springs in series})$$

All are given except $l_d$ and $l_t$.

Given: Length = 2.5

$l_{thd} = 1.25$

$l_{grip} = 2.0$

$$l_d = \text{length} - l_{thd} = 2.5'' - 1.25'' = 1.25''$$

$$l_t = \text{grip} - l_d = 2.0'' - 1.25'' = 0.75''$$

$$R_b = \frac{(0.1963 \text{ in}^2)(0.1419 \text{ in}^2)(30 \times 10^6 \text{ psi})}{(0.1963 \text{ in}^2)(0.75 \text{ in}) + (0.1419 \text{ in}^2)(1.25 \text{ in})}$$

$$R_b = 2.57 \times 10^6 \text{ psi}$$
Simpler method, less precise, to find bolt stiffness: (not in FE book)

The bolt is an axial loaded member. The deflection is:

\[ S = \frac{PL}{EA} \]

\[ P_b = \frac{P}{S} = \frac{EAd}{L} \]

where \[ A_d = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.5 \text{ in})^2 = 0.1963 \text{ in}^2 \]
\[ E = 30 \times 10^6 \text{ psi} \]
\[ L = g_c = 2 \text{ in} \]

\[ P_b = \frac{(30 \times 10^6 \text{ psi}) (0.1963 \text{ in}^2)}{2 \text{ in}} \]

\[ P_b = 2.94 \times 10^6 \text{ lb/in} \quad (14\% \text{ different than book}) \]

We will use book's method so \[ P_b = 2.57 \times 10^6 \text{ lb/in} \]

\[ C = \frac{P_b}{k_b + k_m} = \frac{2.57 \times 10^6 \text{ lb/in}}{2.57 \times 10^6 \text{ lb/in} + 17.55 \times 10^6 \text{ lb/in}} \]

\[ C = 0.128 \quad (\text{bolt carrying 12.8\% of external tension}) \]
4(7) cont

Finally...

\[ F_c \]

\[ N_s = \frac{F_c}{P(1-c)} \]

\[ N_s = \frac{9050 \text{ lb}}{5000(1-0.128)} = 2.07 \quad \text{(Answer)} \]

This example problem was far longer than to be expected, but it included several steps that by themselves could be EE questions.
4(b) "J O I N T" C O N T I N U E D

F O R T H E A B O V E B O L T E D J O I N T,
D E T E R M I N E T H E "L O A D F A C T O R", \( n_b \).

The load factor is a factor of safety applied for the external load only:

\[
S_p = \frac{CP n_b}{A_t} + \frac{F_t}{A_t} \quad \text{(in FE book)}
\]

\[
C P \cdot A_t - F_t
\]

\[ n_b = \frac{S_p A_t - F_t}{C P} \]

\[
S_p = 85 \, k \text{psi} \quad \text{(proof strength, given)}
\]

\[
A_t = 0.1419 \, \text{in}^2 \quad \text{(tensile stress area, given)}
\]

\[
F_t = 9050 \, \text{lb} \quad \text{(calculated above, force in bolt)}
\]

\[
p = 5000 \, \text{lb} \quad \text{(external load, given)}
\]

\[
C = 0.128 \quad \text{(calculated above, joint coefficient)}
\]

\[
(85,000 \, \text{psi})(0.1419 \, \text{in}^2) - 9050 \, \text{lb}
\]

\[
= \frac{n_b}{0.128(5000 \, \text{lb})}
\]

\[
n_b = 4.7
\]
Given

A bracket is attached to a structure using bolts at A & B.

Determine force on the 2 bolts

Solution: Make this an equivalent load with all forces and moments at the centroid of the fastener group.

\[ V = 5 \text{kN} = 5000 \text{N} \]
\[ M = (5 \text{kN})(100 \text{mm}) = 500,000 \text{ N-mm} \]

Equilibrium:
\[ F_v-A + F_v-B = V \]

Assume \[ F_v-A = F_v-B \]

Therefore \[ F_v-A = F_v-B = \frac{V}{n} = \frac{5000}{2} = 2500 \text{N} \]

See MECHANICAL ENGINEERING

Fastener Groups:
\[ F_v = F_{x-A} = F_{v-B} = \frac{5000N}{2} = 2500N \]

\[ F_{M-A} = F_{M-B} = F_M = \frac{Mr_i}{r_A^2 + r_B^2} \]

\( r \) - distance from centroid to bolt

\[ F_M = F_{M-A} = \frac{Mr_i}{r_A^2 + r_B^2} \quad \quad r_A = r_B = 40\text{mm} \]

\[ F_M = \frac{(5000\text{N-m})(40\text{mm})}{(40\text{mm})^2 + (40\text{mm})^2} = 6250\text{ N} \]

**Vector addition of forces:**

\[ F_{\text{tot-A}} = F_{\text{tot-B}} = \sqrt{F_v^2 + F_M^2} \]

\[ = \sqrt{(2500N)^2 + (6250N)^2} \]

\[ = 6730N \]
5b) What diameter must the two bolts be such that the shear stress is one-half yield strength — yield strength being 100 MPa. \( \left( T_{ys} = \frac{1}{2} S_{ys} \right) \) - Max shear stress static failure theory

\[ F_{tor} = \frac{6730 \text{ N}}{2} = 6730 \text{ N} \quad \text{(given)} \]

\[ T = \frac{V}{A} \quad A = \frac{\pi}{4} d^2 \]

\[ V = 6730 \text{ N} \quad (= F_{tor}) \]

\[ T_{allowed} = \frac{1}{2} S_{ys} = \frac{1}{2} (100 \text{ MPa}) = 50 \text{ MPa} \]

\[ 50 \text{ MPa} = \frac{6730 \text{ N}}{\frac{\pi}{4} d^2} \]

\[ d = \left( \frac{6730 \text{ N}}{\frac{\pi}{4} (50 \text{ MPa})} \right)^{1/2} = 13 \text{ mm} \]

5c) What would the diameter need to be for factor of safety against yield is 1.5 \((n)\)?

\[ T_{allowed} = \frac{1}{n} \left( \frac{1}{2} S_{ys} \right) = \frac{1}{1.5} \times 50 \text{ MPa} \]
The allowed value is 33 MPa.

\[ d = \left( \frac{6730 \text{ N}}{\frac{\pi}{4} \times 33 \text{ MPa}} \right)^{\frac{1}{2}} = 16 \text{ mm} \]

Remember

\[ \tau_{ys} = \frac{1}{2} \sigma_{ys} \]

Mohr's circle

Uniaxial load:

So it is common for allowable shear stress to be \( \frac{1}{2} \sigma_{ys} \).
STATIC FAILURE THEORIES

- PRINCIPAL STRESSES
  - always 3, always orthogonal!

  By convention \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \)

  FOR PLANE STRESS, 1 Princ. stress will be 0

  Brittle Material: use Max Normal stress Theory:
  \( \sigma_1 \geq \sigma_{\text{UT}} \) Failure (fracture)

  Ductile: use Max Shear Stress or Distortion Energy
  (almost the same theory)

  \[
  \text{Max Shear} \quad I_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{1}{2} S_{\text{YS}} \quad \text{Failure (Yield)}
  \]

  \[
  \text{Dist. Energy} \quad \left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2} \right]^{1/2} \geq S_{\text{YS}} \quad \text{Yield Failure}
  \]
6. Given: Oil tempered steel, helical spring, compression spring, ends are squared & ground (to allow it to rest flat), 18 total coil turns \( (N_{tor}) \), wire dia: 0.080 in, mean spring dia: 1.0 in.

Determine allowable force for factor of safety against yielding of 3.0.

Soln: \( T = K_5 \frac{8FD}{\pi d^3} \)

\[
K_5 = \frac{2C+1}{2C} \quad \text{(stress concentration)}
\]

\[
C = \frac{D}{d} = \frac{1.0''}{0.080''} = 12.5
\]

\[
K_5 = \frac{2(12.5)+1}{2(12.5)} = 2.08
\]

\[
T = \frac{8F(1.0\text{ in})}{\pi (0.080\text{ in})^3} = 10,345 \text{ F (psi)}
\]


\[ T_{allowed} = 0.5 S_{UT} \quad \text{from FE book} \]

\[ S_{UT} = \frac{A}{d^{m}} \quad \text{From FE book,} \]

\[ A = 1610 \quad m = 0.193 \]

for oil tempered steel

Units of diameter must be mm

\[ \text{and } S_{UT} \text{ is MPa.} \]

\[ d = 0.080 \text{in} \left(\frac{25.4 \text{mm}}{\text{in}}\right) = 2.03 \text{mm} \]

\[ S_{UT} = \frac{1610}{(2.03)^{0.193}} = 1404 \text{ MPa} \]

\[ 1404 \text{ MPa} \left(\frac{1 \text{ksi}}{6.89 \text{ MPa}}\right) = 204 \text{ ksi} \]

\[ T_{allowed} = \frac{1}{2} S_{UT} = 102 \text{ ksi} \]

\[ N = \frac{T_{allowed}}{T_{applied}} \]

\[ T_{applied} = \frac{T_{allowed}}{N} = \frac{102 \text{ ksi}}{3} = 34 \text{ ksi} \]

\[ T_{applied} = 10,345 \text{ F = 34,000 ps} \]

\[ F = 33.15 \]
6b) Determine spring constant for this spring.

From FE book: 

\[ k = \frac{d^4 G}{8 D^3 N} \]

\[ N_{\text{active}} = N = N_{\text{tot}} - 2 \]

\[ N = 18 - 2 = 16 \]

\[ r = \frac{1}{2} \] because the first and last coil are squared and ground (flattened) and don't "bend" during compression so they do not affect stiffness.

\[ k = \frac{(0.080 \text{ in})^4 (11.5 \times 10^6 \text{ psi})}{8 (1 \text{ in})^3 (16)} \]

\[ k = 3.68 \text{ lb/in} \]
Bearings:

- $C$ - load factor from catalog
- $C_0$ - static
- $e$ - catalog factor, defines minimum
  $\frac{F_a}{F_r}$ ratio below which $F_a$ can be ignored ($e^2 = \phi$)

$$e = 0.513 \left( \frac{F_a}{C_0} \right)^{0.236} \geq \frac{F_a}{V \bar{F}_r} \quad \text{then} \quad x = 1, \ y = 0$$
$$V = 1 \quad \text{inner ring} \quad \text{soleto}, \ V = 1.2 \quad \text{outer ring}$$

$\begin{bmatrix} \text{Ball} \end{bmatrix} \quad \begin{bmatrix} \text{Roller} \end{bmatrix}$

If both axial and radial loads, make into an equivalent load

Notes:
See MECHANICAL ENGINEERING,
Ball/Roller Bearing Selection

7. Given: Ball bearing on rotating shaft
   Dynamic load rating, \( C = 21,200 \) lb
   Static load rating, \( C_0 = 18,000 \) lb
   (These load ratings are properties of the bearing - provided by the manufacturer).

   Axial load, \( F_a = 1000 \) lb
   Radial load, \( F_r = 1700 \) lb
   Shaft speed: 2000 rpm

Find: Expected bearing life.

**Note:** Most ball bearings are designed to carry radial loads only. Their ratings are based on that. However, they can carry axial (thrust) loads, but the loads must be analyzed by combining them into an effective radial load (so-called “equivalent” load).

Equivalent load: \( P_{eq} = XVF_r + YF_a \) (from FE Book)

\( V = 1 \) (inner race rotates)

We need to determine if \( F_a \) is large enough to affect life.
\[
\frac{F_a}{V_{Fr}} = \frac{1000 \text{ lb}}{(1)(1700 \text{ lb})} = 0.5882
\]

From FE Book:
\[
e = 0.513 \left( \frac{F_a}{C_0} \right)^{0.236} = 0.513 \left( \frac{1000 \text{ lb}}{18,000 \text{ lb}} \right)^{0.236} = 0.26
\]

so \[
\frac{F_a}{V_{Fr}} > e \quad \text{Therefore, } F_a \text{ is significant.}
\]

Therefore
\[
X = 0.576 \quad Y = 0.84 \left( \frac{F_a}{C_0} \right)^{-0.247}
\]

\[
Y = 0.84 \left( \frac{1000 \text{ lb}}{18,000 \text{ lb}} \right)^{-0.247}
\]

\[
Y = 1.71
\]

\[
\text{Note if } \frac{F_a}{V_{Fr}} < e, \text{ then } X = 1, Y = 0 \}
\]

\[
P_{eb} = XV_{Fr} + YF_a
\]

\[
= (0.576)(1)(1700 \text{ lb}) + (1.71)(1000 \text{ lb}) = 2662.1 \text{ lb}
\]

\[
L = \left( \frac{C}{P_{eb}} \right)^a = \left( \frac{21,200 \text{ lb}}{2662.1 \text{ lb}} \right)^3 = 505 \times 10^6 \text{ revolutions}
\]
Columns (Buckling)

MECHANICAL ENGINEERING
(FE Manual)

Slenderness ratio: \( S_r = \frac{L_{eff}}{r} \)

\[
r = \sqrt{\frac{E}{A}}
\]

where \( E \) is area moment of inertia

* Note: For buckling, determine minimum of \( I \)!

Example: Column cross-section is 10 mm by 20 mm

\[ I = \frac{1}{12} b h^3 \]

- Select \( h < b \)
- \( h = 10 \text{ mm} \)
- \( b = 20 \text{ mm} \)

\[ I = \frac{1}{12} (20 \text{ mm})(10 \text{ mm})^3 = 1667 \text{ mm}^4 \]

Effective length \((L_{eff})\) depends upon actual length \((L)\) and end conditions
Columns Cont.

End Conditions:

- Pinned-pinned
  \[ l = \frac{2}{3} l \]

- Fixed-free
  \[ l_{ef} = 2.1 l \]

- Fixed-pinned
  \[ l_{ef} = 0.8 l \]

- Fixed-fixed
  \[ l_{ef} = 0.65 l \]

The verbal & numerical descriptions are provided in the FE manual, but not the sketches.
Columns (cont)

2 eqns are provided.

\[ \frac{\pi \sqrt{2E}}{8y} > \frac{I}{r} \quad (r = \text{rad of gyr}) \]

Then it is "Intermediate" otherwise, "long".
Pitch Circle: Pitch circle is the apparent circle that two gears can be taken like smooth cylinders rolling without friction. The diameter of the pitch circle is the "effective diameter" of the gear.

Diametral Pitch: Diametral pitch (p) is the number of teeth per unit volume.

\[ p = \frac{\text{Number of Teeth}}{\text{Diameter of Pitch circle}} \]

Module: Module (m) is the inverse of diametral pitch. \( m = 1/p \)

Circular Pitch: Circular pitch is the space in pitch circle used by each teeth.

Pressure Line: Pressure line is the common normal at the point of contact of mating gears along which the driving tooth exerts force on the driven tooth.

Pressure Angle: Pressure angle is the angle between the pressure line and common tangent to pitch circles. It is also called angle of obliquity. High pressure angle requires wider base and stronger teeth.
Gear Train

\[ M_v = \frac{\text{product of \# teeth on driver}}{\text{product of \# teeth on driven}} \]

- \( \Rightarrow \) odd \# of gears (output rotates same direction)
- \( \Rightarrow \) even \# of gears (" rotates opposite direction)

ex

<table>
<thead>
<tr>
<th>Gear</th>
<th># teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

1 drive 2, 2 drives 3, 3 drives 4:

\[ M_v = \frac{-10 \cdot 20 \cdot 22}{20 \cdot 22 \cdot 12} = -0.83 \quad (= \frac{\omega_{\text{out}}}{\omega_{\text{in}}}) \]

1 drives 2
2 & 3 rotate at same \( \omega \)
3 drives 4

\[ M_v = \frac{(\#1)(\#3)}{(\#2)(\#4)} = \frac{(10)(10)}{(20)(20)} = +0.25 \]
Gears
\[ P_{out} = P_{in} = (T_w)_{out} = (T_w)_{in} \quad \omega \rightarrow \text{rad/sec} \]

Torque
velocity ratio:
\[ m_v = \frac{\omega_{out}}{\omega_{in}} = \frac{-N_{in}}{N_{out}} \]

\[ m_v = \frac{12}{30} = 0.4 \]

Driver output
"output is 40% speed of input"

\( (\text{torque})(\text{speed}) = \text{const} \)

Transmission
\[ T_{trans} = \frac{30}{12} = 2.5 \quad \text{increase in torque} \]

Planetary

Planetary gear system

\[ \omega_F = \omega_{in} \]
\[ \omega_L = \omega_{in} \]

\[ \omega_F - \omega_{in} = m_v \]

\[ \omega_L - \omega_{in} = m_v \]

\[ \omega_F = \text{ring or sun} \]
\[ \omega_L = \text{sun or ring} \]